

1ST SEM./COMMON TO ALL /2023(W) NEW

Th-3 Engineering Mathematics - I

Full Marks: 80

Time- 3 Hrs

Answer any five Questions including Q No.1& 2
Figures in the right hand margin indicates marks

1. Answer **All** questions 2 x 10

- a. Evaluate $\sin 4365^\circ$
- b. Find the value of $\frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$
- c. Find the co-ordinates of the center of the circle $2x^2 + 2y^2 - 6x + 8y - 4 = 0$.
- d. Find the distance between the points (2, 3, 4) and (3, 5, 4).
- e. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ then evaluate $A + 2B$.
- f. Find the equation of the sphere with its center at (1, 2, 3) and radius 7 unit.
- g. At $x = 0^\circ$, evaluate $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix}$
- h. Find the intercepts cut off by the line $2x + 3y + 1 = 0$.
- i. Find the order of the matrix B if $[3 \ 4 \ 2] \times B = [2 \ 1 \ 0 \ 3 \ 6]$.
- j. Evaluate $\sin(\tan^{-1} x + \cot^{-1} x)$

2. Answer **Any Six** Questions 6 x 5

- a. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$
- b. If $A + B = \frac{\pi}{4}$, prove that $(1 + \tan A)(1 + \tan B) = 2$
- c. Prove without expanding, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

- d. Find the equation of the line which passes through the point (1,2) and perpendicular to the line $4x + 3y + 5 = 0$.

- e. Find the equation of the circle passing through the point (7,3) having radius 3 units and whose center lies on the line $y = x - 1$.
- f. Find the equation of the plane which passes through the point (3, 4, -1) and parallel to the plane $2x - 3y + 5z + 7 = 0$.
- g. Find the equation of sphere if the end points of its diameter are (-1, 2, 3) and (2, 5, 6).

Answer **Any Three** Questions

- 3 a. Solve by matrix inverse method, 7
 $2x - y = 2, 3x + 2y = 17$
- b. If $\begin{bmatrix} x-2 & 3 \\ z-5 & 0 \end{bmatrix} = \begin{bmatrix} 2 & y+1 \\ -2 & 0 \end{bmatrix}$, Find the value of x, y, z . 3
- 4 a. Find the equation of the plane passing through the point (3, 2, 1) and 5
the intersection of planes $2x + 3y - 4z + 1 = 0$ and $3x - y + z + 2 = 0$.
- b. Find the equation of the circle passing through the points (0, 0), (3,0) 5
and (0, 4).
- 5 a. If $A+B+C = \pi$, prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \cdot \sin B \cdot \sin C$ 5
- b. Prove that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{3}{16}$ 5
- 6 a. Verify that $(AB)^T = B^T \cdot A^T$, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}$ 5
- b. Solve by Crammer's rule, $4x + 5y = 3, 3x - 2y = 8$. 5
- 7 Find the distance of the point (1, 2) from $x + 2y + 1 = 0$ measured 10
parallel to the line $3x - y + 1 = 0$.