KIITPOLYTECHNIC

LECTURE NOTES

ON

DIGITAL SIGNAL PROCESSING

Compiled by

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CHAPTER -1

INTRODUCTION OF SIGNALS, SYSTEMS & SIGNAL PROCESSING

1.1. Discuss Signals, Systems & Signal Processing

1.1.1. Explain basic elements of a digital signal processing

Signal: A signal is defined as any physical quantity that varies with time, space or any other independent variable or variables.

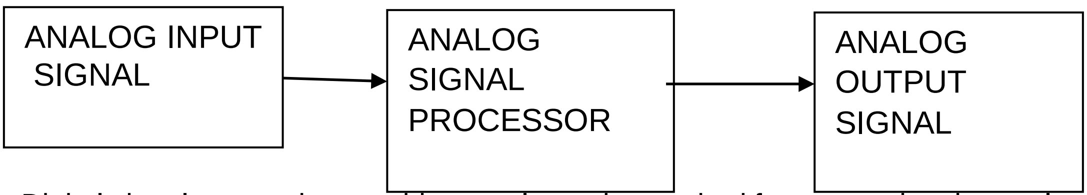
System: It is defined as the physical device that performs an operation upon the signal.

Signal Processing: - The operation which one carried out in the signal by the system one called as signal processing.

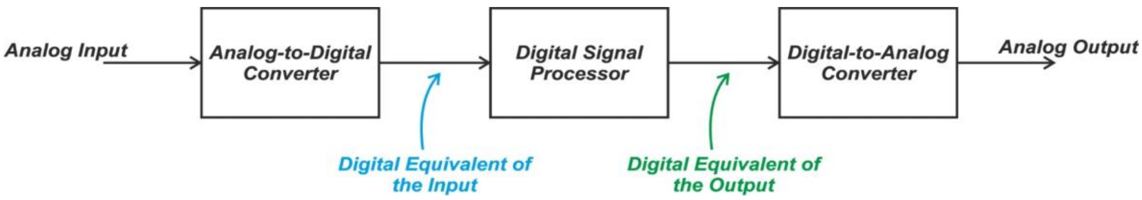
1.1.2. Explain basic elements of digital signal processing.

- Most of the signals encountered in science and engineering are analog in nature.
- That is, the signals are functions of a continuous variable, such as time or space. And usually take on values in a continuous range.
- Such signals are processed directly by appropriate analog systems.

DIAGRAM



- Digital signal processing provides an alternative method for processing the analog signal.



ADC (Analog to digital converter)

- It is the interface between analog signal and digital signal processor.
- It converts analog signal to digital data.

Digital signal processor

- It is nothing only a programmable microprocessor programmed to perform the desired operation on the inputted signal.
- The digital signal processor may be a large programmable digital computer or a small microprocessor programmed to perform the desired operations on the input signal.
- It may also be a hardwired digital processor configured to perform a specified Set of operations on the input signal.

DAC (digital to Analog converter)

- It is the interface between processed digital data and output analog signal.

1.2. Classify signals

- The methods we use in processing a signal or in analyzing the response of a system to a signal depend heavily on the characteristic attributes of the specific signal.
- There are techniques that apply only to specific families of signals. Consequently, any investigation in signal processing should start with a classification of the signals involved in the specific application.

Multi channel signal

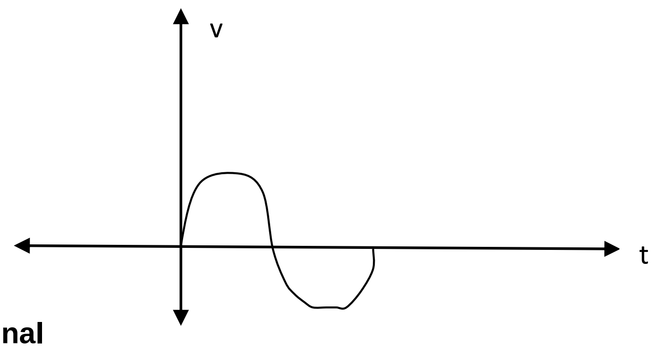
- The signal which is generated by multiple sources or multiple sensors and are represented in vector form is called as multichannel signal.
- Example : Earth quake generated wave , Electrocardiogram(ECG) 3-channel ,12-channel .

Multi-dimensional signal

- The signal which is a function of more than one independent variables are called as multidimensional signal.
- Example : $f(x,y) = x^2 + 2y+3$

Continuous time signal

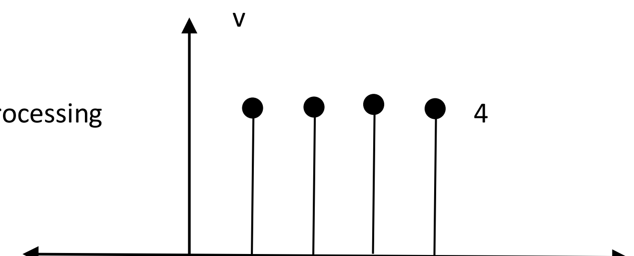
- The signal which can be defined for every point of time in an interval is called as continuous time signal.
- Example: $X (t) = \cos\pi t$.



Discrete time signal

- The signal which is only defined on specific point of time is called discrete time signal.
- Example: $X(n) = 2^n$, $n \geq 0$

  $= 1$, $n < 0$



Continuous valued signal:

- The signal which takes on all possible values on a finite or an infinite range is called as continuous valued signal.

Discrete valued signal:

- The signal which takes on values from a finite set of possible values is called as discrete-valued signal.

1.3 Concept of frequency

- The concept of frequency is closely related to concept of time.
- The nature of frequency is affected by nature of time (continuous or discrete).
- Frequency is an inherently positive physical quantity, but for mathematical convenience we use +ve and –ve frequency.
- Frequency range for analog sinusoids is $-\infty < F < \infty$.
- Frequency range for discrete sinusoidal is $-1/2 < f < 1/2$.

1.4 Analog to Digital & Digital to Analog conversion & explain the following.

Sampling of Analog signal

- Sampling is defined as selection of values of an analog signal at discrete-time instants.
- Sampling can be done in many ways but uniform sampling or periodic sampling is most used.
- The sampled signal $X(n) = X_a(nT)$, $-\infty < n < \infty$

Sampling theorem

- The theorem states that any signal X(t) having finite energy, which has no frequency components higher than f_h Hz , can be completely described and reconstructed from its samples per second.
- This sampling rate of $2f_h$ samples per second is called as the Nyquist rate and its reciprocal ($1/2f_h$) is called as the Nyquist period .

Quantization of continuous amplitude signals

- The process of converting a discrete –time continuous amplitude signal into a digital signal by expressing each sample value to a finite number of digits is called as quantization.
- The error resulted in representing the continuous valued signal by a finite set of discrete values is called quantization error quantization noise.

Coding of quantized sample

- It is the process of assigning unique value to each level.
- The word length of 'b' bits we can create 2^b different binary numbers.
- Hence $2^b \geq L$.

Digital to analog conversion

- To convert a digital signal into its corresponding analog signal a digital to analog (D/A) converter.
- By interpolating the samples, a rough sketch of analog signal can be obtained.

CHAPTER-2

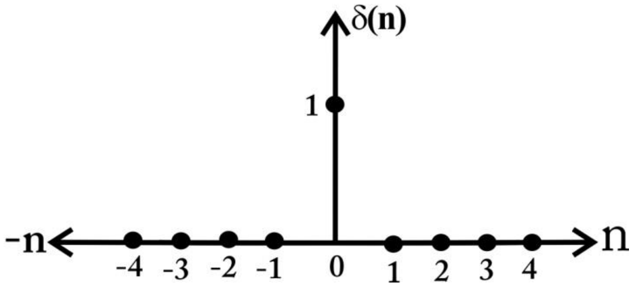
DISCRETE TIME SIGNALS & SYSTEMS

2.1 State and explain discrete time signals.

2.1.1. Some elementary discrete time signals.

Unit sample sequence is denoted as $\delta(n)$

$\delta(n) = \begin{cases} 1, & for\ n=0 \\ 0, & for\ n \neq 0 \end{cases}$

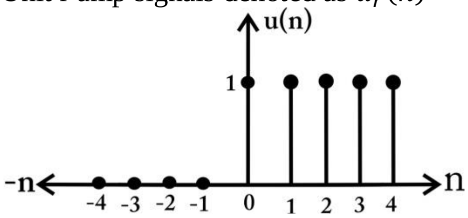


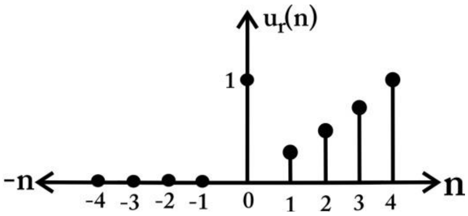
In words the unit sample sequence is a signal which give only u nit value at $n = 0$ and Zero value except $n \neq 0$.

Unit Step signal is denoted as u(n)

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Unit r amp signals denoted as $u_r(n)$





The exponential signal

$$x(n) = a^n$$

If a is real, then $x(n)$is a real signal

If a is complex valued then $x(n)$ is an exponential signal.

$$a = re^{j\theta}$$

$$x(n) = r^n e^{jn\theta} a$$

$$= r^n(\cos\theta\, n + j\sin\theta\, n)$$

2.1.2. Classify discrete time signal.

Depending upon the various characteristics of the signals the discrete time signals are can be classified as.

(i) Energy signal & process signals.

(ii) Periodic and a periodic singles.

(iii) Symmetric (even) and antisymmetric (odd) signals.

(i) Energy signal & process signals

→ The signal which has finite energy and zero average process.

→ $x(t)$ is an energy signals if $0 < E < \infty$ and $p = 0$ where $E \rightarrow$ energy and $P \rightarrow$process of the signals $x(t)$.

$$E = \int_{-\infty}^{\infty} x^2(t)dt \quad \text{for real signal.}$$

$$E = \int_{-\infty}^{\infty} x^2(t)dt \quad \text{for complex valued signal.}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{for discret} - \text{time signal x(n)}$$

→ Energy of 0 signals must be finite.

→ Non periodic signals are energy signals .

Power signal. → These signals are fine limited
→ Power energy signal is Zero.

→ The signal which has finite average power and infinite energy.

→ $x(t)$ is a power signal of $0 < p < \infty$ and $E = \infty$

For real signal average power

$$P = \lim_{T\to\infty} \frac{I}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} x^2(t)dt.$$

For complex valued signal average power P is

$$P \lim_{T\to\infty} \frac{I}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} (xt)^2 dt.$$

For discrete time signal

$$P \lim_{N\to\infty} \frac{I}{N} \sum_{n=\frac{-n}{2}}^{\frac{N}{2}} |x(n)|^2$$

→ Practical periodic signals are power signals.

→ These signals can exist over infinite time.

→ Energy of the power signal is infinite.

(ii) **riodic signals** **periodic signals**

→ The signal which exhibits a definite → The signal which does

pattern and repeats after certain amount of time (T) → fundamental period.	not repeat → Period is infinity

→ $x(t+T) = x(t), -\infty < t < \infty$

$x(n+N_0) = x(n), -\infty < n < \infty$

N_0 = Sampling period.

(iii) **Even** **Odd**

Even	**Odd**
→ The signal which exhibits symmetry in time domain is called even signal	→ The sign
→ Mathematically	

$x(t) = x(-t)$ or $\hat{x}(n) = \dot{x}(-n)$

2.1.3. Discuss simple manipulation of discrete time signal.

Manipulation or modification of a signal always involves independent variable and dependant variable (signal amplitude)

→ Transformation of the independent variable (Time)

Transformation involves shifting of a signal. $x(n)$ by replacing independent variable n. by n-k, where k is $+ve$ or $-ve$ constant. When k is $+ve$ constant for example. $x(n-2)$ is called delay operation.

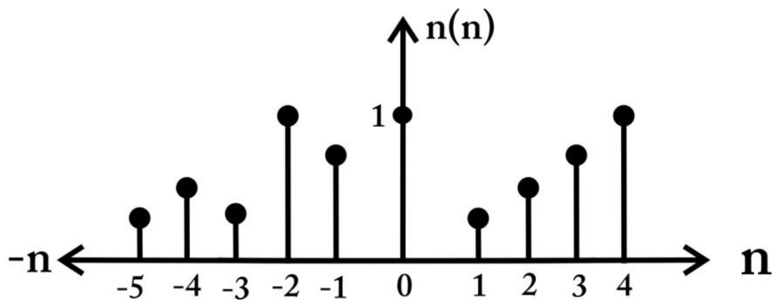
→ Time delay operation is denoted as TD.

Advancing of signal

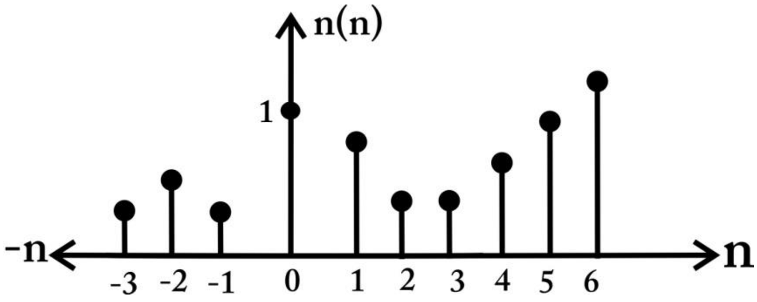
When K becomes $-ve$ constant the signal operation is called advancing operation :

Ex. $x(n+2)$

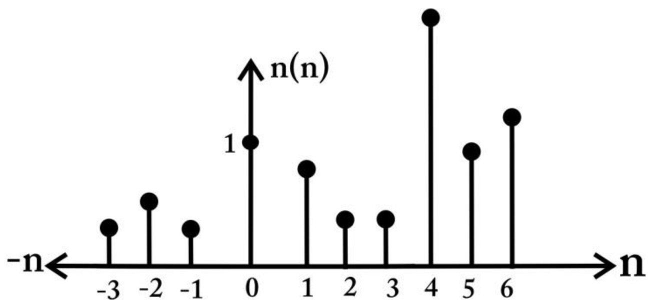
$x(n)$



$x(n-2)$



$x(n+2)$



Folding or reflection

It $x(n)$ is n is replaced by $-n$ then this operation is called folding on reflection of the signal about the time origin n=0.

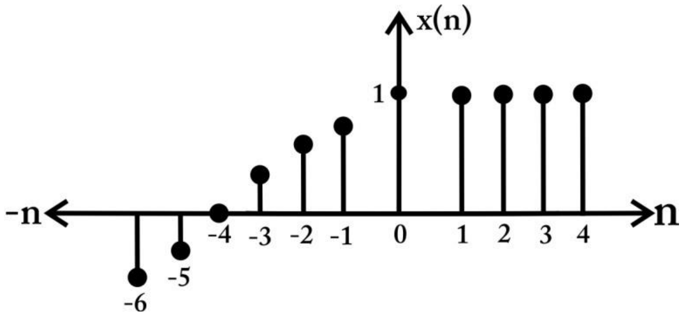
→ The time folding operation is denoted as FD

→ The TD & FD are not commutative

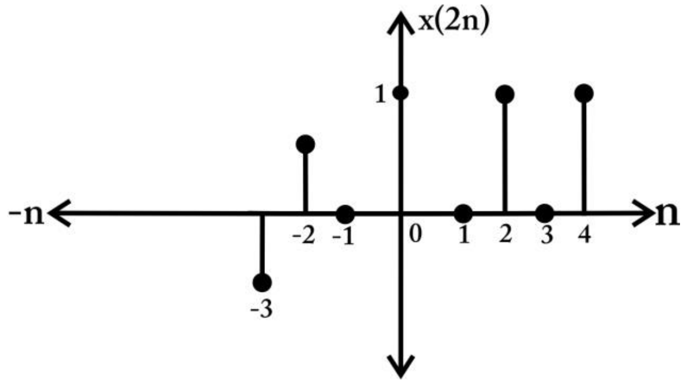
Time scaling and down sampling

It the independent variable of $x(n), n$ is replaced by un, whre u is an integral called as time scaling or down sampling.

$x(n)$



$x(2n)$



Amplified scaling

This operation is done by simply multiplying a constant value to every signal sample.

$$y(n) = \; At\, x\,(n) - \; \infty < n < \infty$$

Addition

The seam of two signals $x_1(n)$ & $x_2(n)$ is a signal $y(n)$, whose value of any instant of time is addition of $x_1(n)$ & $x_2(n)$ at that same instant.

$$y(n) = x_1(n) + \; x_2(n) - \infty < n < \infty$$

Product

The product of two signals is similarly defined on product of sample to sample basis

$$y(n) = \; x_1(n)x_2(n). -\infty < n < \infty$$

2.2 Discrete time system

2.2.1 Describe input –output of the system

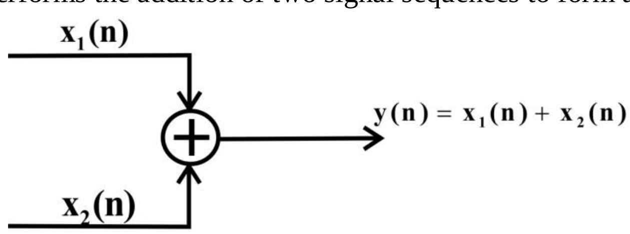
In a discrete time system, the output and input are always described by a rule or a mathematical relationship.

This mathematical relationship denoted by $x(n) \xrightarrow{T} y(n)$.

When $y(n) \rightarrow$ response at the system to the excitation or input $x(n)$

2.2.2 Block diagram representation of discrete time system.

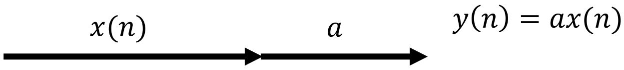
Addition: An addition performs the addition of two signal sequences to form another (sum) sequence.



$\rightarrow$ It is a memory less operation

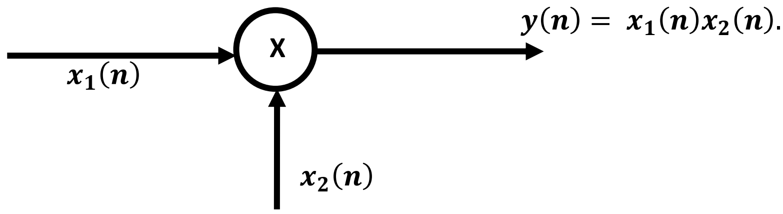
Multiplication (constant)

It is a memoryless operation where a constant is multiplied into every sample and depicted as follows.



Signal multiplier:

It is a memory less operation where we get multiplication of two signals as another sequence ad depicted as follows.

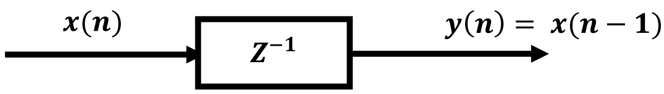


Unit delay element

It is a system which only delays the sample of any sequence by one sample.

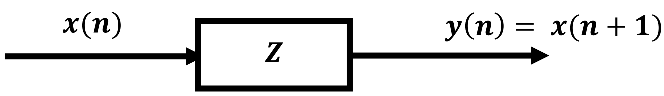
If $x(n)$ is the input the output will be $x(n-1)$

$$\therefore y(n) = x(n-1)$$

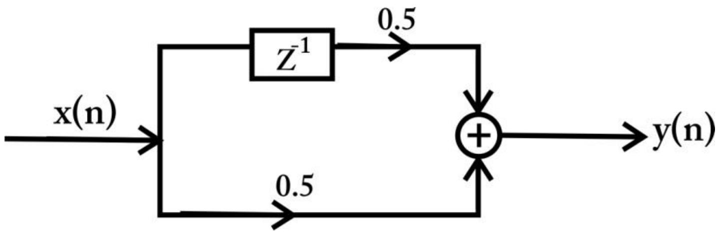


Unit advance element

A unit advance system moves the input sequence $x(n)$ ahead by one sample in time to yield $x(n+1)$



$$y(n) = x(n) + \frac{1}{2}x(n-1)$$



2.2.3 <u>Classify discrete time system</u>

Discrete time system can be classified or categorized depending upon it's general properties.

(i) Static systems and dynamic system.

(ii) Time invariant system and time variant.

(iii) Linear system and non linear system.

Since for analyzing and designing the designer heavily depends upon the general characteristics of the discrete systems hence for knowing their properties clearly we have to classify them as follows.

(iv) Causal system and non-causal system.

(v) Stable system and unstable system.

The signal $x(t)$ defined for $t \in R$ is causal if and only it is zero negative t, otherwise, the signal is non-causal.

$x(t) = 0,$ for $t < 0$

(i) (a) Static System: (Memory Less System)

If output of a discrete time system at any instant 'n' depends only on that the sample of input at that instant not on future or past input samples in called static system.

Ex.:- $y(n) = ax(n)$

$y(n) = 5x(n) + bx^3(n)$

(b) Dynamic System: (System With Memory)

If output of a discrete time system at any instant 'n' depends upon future or past input samples then the system is called as dynamic system or system with memory.

Ex. $y(n) = x(n-1) + 3x(n+1)$

(ii) (a) Time Invariant System: (Shift Invariant System)

If input-output characteristic of a system does not change with time is called time invariant system or shift invariant system.

(b) Time Variant System: (Shift Variant System)

If input-output characteristic of a system changes with time is called time invariant system or shift invariant system.

(iii) (a) Linear System.

A linear system is the type of the system which satisfies the superposition principle.

Superposition principle

It states that response of the system to a weighted sum of signals be equal to the corresponding weighted sum of the responses of the system to each of the individual input signals.

$$T[a, x, (n) + a_2 x_2(n)] = a_1 T\,[x_1(n)] + a_2 T\,[x_2(n)]$$

Where a_1& a_2 are the arbitrary constants and $x_1(n)$ & $x_2(n)$ are the arbitrary input sequences.

(b) Non liner system:

The system which does not satisfy the superposition principle is called as nonlinear system.

(iv) Causal system & Non-Causal System

Causal signal:- The signal $x(n)$ is said to be causal if it's value is zero for $n < 0$ otherwise the signal is non causal.

Example of causal signal.

$x_1(n) = a^n u(n)$

$x_2(n) = \left\{\underset{\uparrow}{1}, 2, -3, -1, 2\right\}$

Non-Causal Signal:- The signal $x(n)$ is said to be non-causal if it's value is zero for $n > 0$ otherwise the signal is causal.

Example of non-causal signal.

$x_1(n) = a^n u(-n+1)$

$x_2(n) = \left\{1, 2, -\underset{\uparrow}{3}, -1, 2\right\}$

Anti-causal:- A signal that is zero for all $n \geq 0$ is called anti-causal signal.

Causal System:-

→ A system is said to be causal if the output of the system at any time 'n' depends only at present and post inputs, but does not depend on future inputs.

→ Mathematically causal system is represented as

$$y(n) = F[x(n), x(n-1), x(n-2) \dots \dots]$$

Ex:- $y\ (n) = x(n) + x(n-1)$

→ Any practical system is a causal system.

<u>Non-Causal System</u> :-

A System is said to be non-causal if the output of the system depends on future inputs.

→ Mathematically non-causal system is represented as:

$$y(n) = \ F\ [x(n), x(n+1), x(n+2), \dots \dots]$$

Ex:- $y(n) = \ x(n+1) + x\ (2n) + x(n-1)$

→ It is non practical system.

→ In signal processing the study of non-causal system is important.

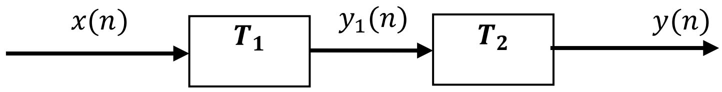
(v) Stable System and Unstable System.

2.2.4 <u>Discuss inter connection of discrete time system.</u>

→ Interconnection presents a great facility to DSP unlikely ASP.

→ There are two basic ways in which systems can be interconnection. (i) series (cascade) (ii) parallel (cascode)

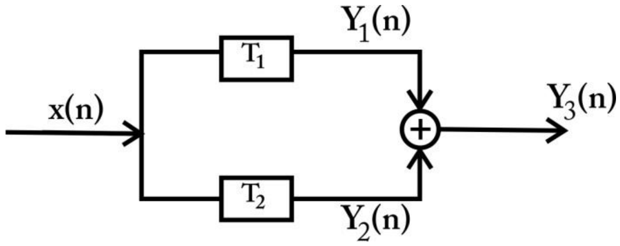
Series (Cascade)



The output of the second system is

$$y(n) = T_2[y_1(n)]$$
$$= T_2[T_1\{x(n)\}]$$
$$= T_c[x(n)]$$
$$T_c = T_2T_1 \neq T_1T_2$$

Parallel



$$y_3(n) = \ y_1(n) + y_2(n) = T_1[x(n)] + T_2[x(n)]$$
$$= (T_1 + T_2)[x(n)]$$
$$= T_p[x(n)]$$
$$T_p = T_1 + T_2$$

2.3 Discuss discrete time linear time – invariant system

2.3.1 Discuss different technique for the analysis of linear system.

A linear time invariant system can be analyzed by two ways

→ One technique involves direct solution of input –output equation for the system.

→ Another technique involves decomposition or resultant of the input signal into elementary signals.

➢ **In First type of technique**

$$y(n) = F\,[y(n-1), y(n-2), \dots \dots y(n-M), x(n), x(n-1)$$
$$\dots .., x(n-M)$$
$$y(n) = -\sum_{K=1}^{N} Q_k\, y(n-k) + \sum_{K=0}^{M} b_k\,(n-l) \dots \dots ..(1)$$

→ Where Q_k & b_k are constant parameters state specify the system and are independent of $x(n)$ & $y(n)$.

→ Equation (1) is called as difference equation

➢ **In Second type of technique**

Decomposition of input signal into a weighted sum of elementary signal is given by

$$x(n) = \sum_k C_k\, x_k(n). \dots \dots \dots ..(2)$$

c_k → set of amplitudes
$y_k(n)$ →let response of the system to the elementary signal component $x_k(n)$.
$y(n)$ =Total response of the system to the total signal $x(n)$

$$y(n) = T[x(n)] = T\left[\sum_k l_k\, x_k(n)\right] [from\ (02)]$$
$$= \sum_k c_k\, T\,[x_k(n)]$$
$$= \sum_k c_k\, y_k(n).$$

2.3.2 Discuss the resolution of a discrete time signal into impulses :-

→ Any discrete time signal can be decomposed or resolved into impulses.

→ Let $x(n)$be the discrete time signal to be decomposed. Which have non zero values over infinite duration .

→ $x_k(n)$ = Elementary signal to $x(n)$.

$$x_k(n) = \delta(n-k).$$

$$x(K) =$$
$$K = 0,1,2,3 \ldots\ldots n$$

$x(n)\delta(n-p) = x(p)\delta\ (n-p)$for delay $n = p$.

→ If we repeat this multiplication over all possible delays $-\infty < K < \infty$

The sum of all possible products will give

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\, \delta(n-k)$$

2.3.3 Discuss the response of LTI system to orbiter I/Ps using convolution theorem.

Set the systems response given by $y(n) = T[x(n)]$ ……..(1)
Impulse sample sequence at $n = K$
Math erotically $h(n,k) = T[\delta(n-k)]$
Let $x(n)$be the arbitrary input signal.
Expression of $x(n)$ as sum of weighted impulses is given by

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\, \delta(n-K)$$

Where $x(k)$ = sample value for $x(n)$ at $n = K$.
$\delta(n-k) =$ unit impulse sample at $n = K$.

→ The response of the system to $x(n)$ is given by

$$y(n) = T\ [x(n)]$$

$$= T\left[\sum_{K=-\infty}^{\infty} x(K)\delta(n-k)\right]$$

$$= \sum_{K=-\infty}^{\infty} x(K)\, T\ [\delta(n-k)]$$

$$\boxed{y(n) = \sum_{K=-\infty}^{\infty} x(K)\ \ h(n,k).} \ldots\ldots.(2)$$

Let $h(n)$ be the unit impulse $\delta(n)$ response of LTI system.
i.e.$h(n) = T\ [\delta(n)]$
Since the system is time invariant the response of the system to delayed unit impulse sample $\delta(n-K)$ is

$$h(n-K) = T\ [\delta(n-k)] \quad \ldots\ldots\ldots(3)$$

Hence from equation (2) and using equation (3)

$$\boxed{y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)} \ldots.(4)$$

This equation (4) states that, the response $y(n)$ of a signal $x(n)$ by LTI system, is the convolution sum of input signal $x(n)$ & unit impulse response $h(n)$.

2.3.4 Explain properties of convolution and interconnection of LTI system.

* → Symbol for convolution

$$y(n) = x(n) * h(n) \equiv \sum_{K=-\infty}^{\infty} x(K)\ \ h(n-k)$$

$$y(n) = h(n) * x(n) \equiv \sum_{K=-\infty}^{\infty} h(K)\ x(n-K)$$

Commutative Law :-

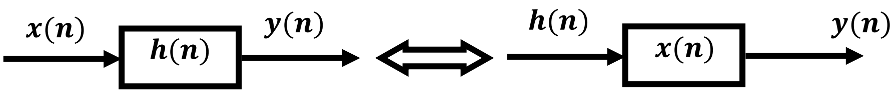
$$x(n) * h(n) = h(n) * x(n)$$

Associative Law :-

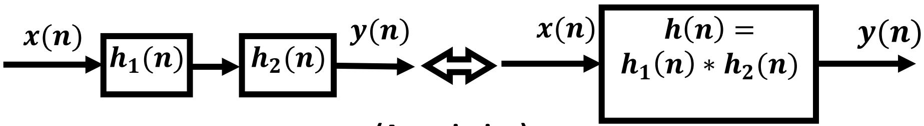
$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

Distributive Law:-

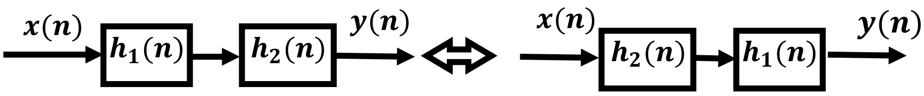
$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)]$$



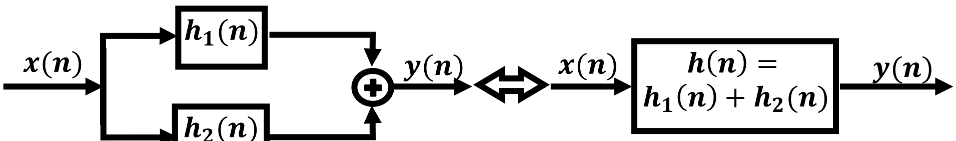
(Interpretation of commutative property)



(Associative)



(Associative & Commutative)



(parallel configuration)

2.3.5 Study systems with finite duration and infinite duration impulse response .

Linear time – invariant system one classified as two types depending on response towards impulse

(i) FIR System

(ii) IIR System

(i) FIR (Finite-duration impulse Response LTI) System

→ The LTI System which response a finite duration inpulse sequence is called as FIR system.

→ The response or output to such sequence is given by

$$y(n) = \sum_{K=0}^{M-1} h(K)\ \ x(n-K)$$

Since $h(n) = 0, n < 0$ and $n \geq M$

→ FIR System has a finite memory of length –M samples.

(ii) IIR (Infinite-duration impulse response LTI) system

→ The LTI system which responds to infinite duration impulse sequence is called as IIR system.

→ The response or output to such sequence is given by

$$y(n) = \sum_{K=0}^{\infty} h(K)\ x\,(n-K)$$

→ IIR system has infinite memory length.

2.4 Discuss Discrete time system described by difference equation.

2.4.1 Explain recursive and non-recursive discrete time system.

→ An IIR System can be easily described by difference equation.

Recursive- Involving a process that is applied repeatedly

Recursive system:

→ The system whose output / response $y(n)$ at time n depends on any number of post output values like $y(n-1), y(n-2)$ .... and also presented post inputs are called as recursive system.

→ Response or output of a recursive system can be given by

$$y(n) = F\,[y(n-1), y(n-2) \ldots.\ y(n-N), x(n), x(n-1) \ldots..\, x(n-M)]$$

→ In terms of difference equation . if can be as follows

$$y(n) = \frac{n1}{n+1} \sum_{K=0}^{n} x(K) \quad n = 0,1, \ldots..$$

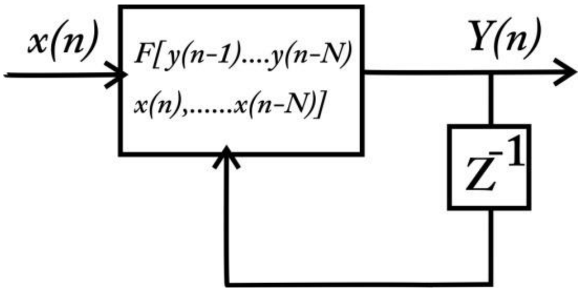
$$(n+1)y(n) = \sum_{K=0}^{n} x(K)$$

$\boxed{y(n) = \frac{n}{n+1}\ y(n-1) + \frac{1}{n+1}\ x(n)}$------(1)

Equation (1) is difference equation for recursive system.

→ All recursive system are IIR system also

→ The basic realization of recursive system is



→ The recursive system consists of a loop and delay element

→ The output of a recursive system can only be computed in order like $y(n), y(1), \ldots \ldots..$

→ Memory cant required

Non recursive system:-

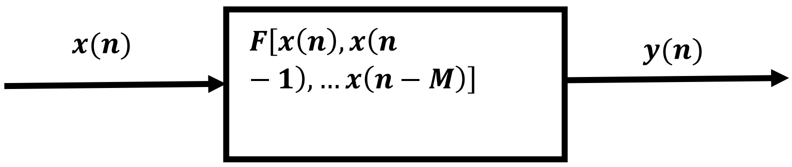
→ The system whose output or response $y(n)$ at time n. depends on only past and present input voters are called as non recursive system.

→ Output of non recursive system can be given by

$$y(n) = F[x(n), x(n-1), \dots x(n-m)]$$

→ All non recursive system are causal FIR system.

→ The basic realization of non recursive system is



→ There is no delay and feedback in non delay and feedback in non recursive system.

→ Output of a non recursive system can be computed in any order like $y(20), y(15)$ … ..

→ Memory less.

2.4.2 Determine the impulse response of linear time invariant recursive system.

Zerostate = (forced response) :- If a system is initially relaxed at time $n = 0$, then it's memory is zero hence the system is called in zero state and it's response is called zero state response of forced response.

CHAPTER -3

THE Z- TRANSFORM AND IT'S APPLICATION TO THE ANALYSIS OF LTI SYSTEM.

3.1. Discuss Z-transform and its application to LTI system:

3.1.1. State and explain direct Z-transform

→ Z-transform is a mathematical tool which transformer changes time domain desire the time signal $x(n)$. Into Z-domain.

→ Mathematically Z-transform is defined as.

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

→ Z-transform of a signal $x(n)$ is denoted by $x(Z) = Z\{x(n)\}$

and this relationship denoted by $x(n) \xleftrightarrow{Z} x(Z)$

→ Z- transform is an infinite power series and it's values exist for those values of Z for which this series converges.

ROC (Region of convergence)

ROC of $x(Z)$. Is defined as the regional set of values of z for which $x(Z)$ attains a finite value.

Problem finding Z-transform

Q-1. Determine Z-transform of following finite duration signals.

(a) $x(n) = \left\{\left(1,5,6,\underset{\uparrow}{7},0,8\right)\right\}$

(b) $x(n) = \{2,3,\underset{\uparrow}{6},4,0,1\}$

(c) $x(n) = \{0,1,2,3\}$

(d) $x(n) = \delta(n)$

(e) $x(n) = \delta(n-K), K > 0$

(f) $x(n) = \delta(n+K), K > 0$

(g) $x(n) = \{0,0,2,5,7\}$

(a) $\boldsymbol{x(n)} = \left\{\left(\mathbf{1,5,6,\underset{\uparrow}{7},0,8}\right)\right\}$

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

$$= \sum_{n=-3}^{2} x(n)\, Z^{-n}$$
$$= x(-3)Z^3 + x(-2)Z^2 + x(-1)Z^1 + x(0)Z^0 + x(1)Z^{-1} + x(2)Z^{-2}$$
$$= (1)Z^3 + 5Z^2 + 6Z + 7 + 0 + 8Z^{-2}$$

(d) $\boldsymbol{x(n) = \delta(n) = \ 1 \ \ n = 0}$

$\boldsymbol{= 0 \ \ n \neq 0}$

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$
$= (1)Z^0 = 1$..................(Ans)

(e) $\boldsymbol{x(n) = \delta(n - K)}$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\ Z^{-n}$$
$$= (1)Z^{-K}$$

Q. $x(n) = \delta(n)$ $\qquad$ $u(n) = 1, n \geq 0$

$= 0\ n < 0$

$$x(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} u(n)\, Z^{-n} = \sum_{n'=0}^{\infty} (1)\ Z^{-n}$$
$= 1 + (Z^{-1})^1 + \ (Z^{-1})^2 + \ (Z^{-1})^3 + \cdots \infty$ $\qquad$ $S = \frac{1}{a-r}, r < 1$
$$= \frac{1}{1 - Z^{-1}} = \frac{Z}{Z - 1}$$

Q- $x(n) = x^n u(n)$

$$x(Z) = \sum_{n=-\infty}^{\infty} [a^n\, u(n)]\, Z^{-n}$$
$$\sum_{n=0}^{\infty} x^n = 1 + r + r^z + \cdots = \frac{1}{1 + r}$$
$$= \sum_{n=0}^{\infty} a^n\ Z^{-n}$$
$$= \frac{Z}{Z - a}$$

Q. $\boldsymbol{x(n) = \ cos\ \omega_0 n = cos\ \theta}$

Q. $\boldsymbol{x(n) = a^n}$

$$x(Z) = \sum_{n=-\infty}^{\infty} a^n Z^{-n}\ = \sum_{n=-\infty}^{\infty} (aZ^{-1})^n$$

Q. $\boldsymbol{x(n) = \ e^{j\omega n}}$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)\ Z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} \left(e^{jw} Z^{-1}\right)^n \qquad n > 0$$
$$= \sum_{n=0}^{\infty} \left(e^{jw} Z^{-1}\right)^n$$

$$= 1 + \left(e^{jw}Z^{-1}\right)^2 + \left(e^{jw}Z^{-1}\right)^3 + \cdots + > \infty$$
$$= \frac{1}{1 - e^{jw}Z^{-1}} = \frac{Z}{Z - e^{jw}}$$

Q. $\boldsymbol{x(n) = cas\ \omega_0 n \qquad n \geq 0}$

3.2. Properties of Z-Transform

(1) Linearity Property

$$x_1(n) \xleftrightarrow{Z} X_1(Z)$$
$$x_2(n) \xleftrightarrow{Z} X_2(Z)$$
$$x(n) = a_1x_1(n) + a_2x_2(n) \xleftrightarrow{Z} X(Z) = a_1X_1(Z) + a_2X_2(Z)$$

a_1 & a_2 are two arbitrary constants.

Problem:- $\boldsymbol{x(n) = cos\ \omega_0 n, \qquad n \geq 0}$

$$cos\ \omega_0 n = \frac{1}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)$$
$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\sin\omega_0 n = \frac{1}{2}\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right)$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

(2) Time Reversal :-

If $x(n) \xleftrightarrow{Z} X(Z)$

Then $x(-n) \xleftrightarrow{Z} X(Z^{-1})$

Ex: $x(n) = 2^n,\ n < 0$

$$= \left(\frac{1}{2}\right)^n, n = 0, 2,4 \ldots.$$
$$= \left(\frac{1}{2}\right)^n, n = 1,3,5 \ldots.$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\,Z^{-n} = \sum_{n=-\infty}^{-1} 2^n\,Z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n Z^{-n} + \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n Z^{-n}$$

$$= \sum_{m=1}^{\infty} 2^{-m}.Z^m + \sum_{P=0}^{\infty}\left(\frac{1}{2}\right)^2.Z^{-2P} + \sum_{q=0}^{\infty}\left(\frac{1}{3}\right)^{2q+1} Z^{-(2q+1)}$$

$$= -n,\quad p = \frac{n}{2},\quad q = \frac{n-1}{2}$$

$$= \sum_{m=1}^{\infty}\left(\frac{Z}{2}\right)^m + \sum_{P=0}^{\infty}\left(\frac{Z^{-1}}{2}\right)^{2P} + \sum_{q=0}^{\infty}\left(\frac{Z^{-1}}{3}\right)^{2q+1}$$

$$= \left(\frac{Z}{2}\right) + \left(\frac{Z}{2}\right)^2 + \ldots\infty \quad +$$

$$= (\because\quad a + a^2 + a^3 + \cdots\infty) = \frac{a}{1 - r}$$

$$= \frac{\frac{z}{2}}{1 - \frac{z}{2}} +$$

Q. $\boldsymbol{x(n) = a^n \quad / \quad x(n) = a^{-n}}$

$$\boldsymbol{x(n) = a^n}$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)Z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n . Z^{-n}$$

$$= \sum_{n=0}^{\infty} (a\, Z^{-1})^n$$

$$= 1 + (aZ^{-1})^1 + (aZ^{-1})^2 + \cdots \infty$$

$$= \frac{1}{1 - aZ^{-1}}$$

$\boldsymbol{x(n) = a^{-n}}$

$$X(Z) = \sum_{n=-\infty}^{\infty} a^{-n} . Z^{-n}$$

$$= \sum_{n=0}^{\infty} a^{-n} . Z^{-n} = \sum_{n=0}^{\infty} [(aZ)^{-1}]^n$$

$$= 1 + \frac{1}{(aZ)} + \left(\frac{1}{aZ}\right)^2 + \left(\frac{1}{aZ}\right)^3 + \ \ldots$$

$$= \frac{1}{1 - \frac{1}{aZ}}$$

$$= \frac{1}{1 - a^{-1}Z^{-1}}$$

$$= \frac{1}{1 - (aZ^{-1})^{-1}}$$

(3) Time shifting

If $x(n) \xleftrightarrow{Z} X\,(Z)$

Then $x\,(n - n_0) \xleftrightarrow{Z} Z^{-n_0} X\,(Z)$

(a) $\boldsymbol{x(n) = 2^n}, \quad \boldsymbol{n} > 0$

$$X(Z) = \sum_{n=-\infty}^{\infty} 2^n\ Z^{-n}$$

$$= 1 + (2Z^{-1})^1 + (2Z^{-1})^2 + \cdots \infty$$

$$= \frac{1}{1 - 2Z^{-1}}$$

$(Not\ Suitable)$

(b) $\boldsymbol{x(n) = 2^{n-4}}, \quad \boldsymbol{n} > 0$

$$X(Z) = \sum_{n=-\infty}^{\infty} (2^{n-4})\ \ Z^{-n}$$

$$= \sum_{n=0}^{\infty} 2^{n-4} \,.\, Z^{-n}$$

$$= 2^{-4} + 2^{-3}Z^{-1} + 2^{-2}Z^{-2} + 2^{-1}Z^{-3} + Z^{-4} + \cdots$$

(c) $\boldsymbol{x(n) = U(n)}$

$$X(Z) = \sum_{n=-\infty}^{\infty} U(n)Z^{-n}$$

$$= \sum_{n=0}^{\infty} Z^{-n}$$

$$= 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)}$$

(d) $\boldsymbol{x(n) = U(n-1)}$

$$X(Z) = \sum_{n=-\infty}^{\infty} U\,(n-1)\,Z^{-n}$$

$$= \sum_{n=1}^{\infty} Z^{-n}$$

$$= +\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots\infty$$

$$= \left(\frac{1}{2}\right)\,[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right) = Z^{-1}\left(\frac{1}{1-\frac{1}{2}}\right)$$

(4) Scaling Property:

If $x(n) \xleftrightarrow{z} X(Z)$

$a^n x(n) \longleftrightarrow X(a^{-1}Z)$

(a) $\boldsymbol{x(n) = u(n)}$

$$X(Z) = \sum_{n=-\infty}^{\infty} u(n)Z^{-n}$$

$$= \sum_{n=0}^{\infty} Z^{-n}$$

$$= \frac{1}{1-\left(\frac{1}{2}\right)} = \frac{Z}{Z-1}$$

(b) $\boldsymbol{x(n) = 2^n u(n)}$

$$X(Z) = \sum_{n=-\infty}^{\infty} 2^n u(n) Z^{-n}$$
$$= \sum_{n=0}^{\infty} 2^n Z^{-n}$$
$$= 1 + 2^1 Z^{-1} + 2^2 Z^{-2}$$
$$= 1 + (2Z^{-1})^1 + (2.Z^{-1})^2 + \cdots$$
$$= \frac{1}{1 - \frac{2}{Z}} = \frac{1}{1 - 2Z^{-1}}$$

G.P $S = a\left(\frac{1-r^n}{1-r}\right), s = \frac{a}{1-r} = (r) < 1$

$$s = \frac{a}{r-1}$$

(5) Differentiation property

If $x(n) \xleftrightarrow{z} X(Z)$

Then $n\, x(n) \xleftrightarrow{Z} -Z\frac{d(Z)}{dz}$

Or $n\, x(n) \xleftrightarrow{Z} Z^{-1}\frac{d\times(Z)}{dz^{-1}}$

Ex:- $x(n) = nu(n)$

$$x\ (Z) = \sum_{n=-\infty}^{\infty} n\, u\, (n) Z^{-n} = \sum_{n=-0}^{\infty} n.Z^{-n}$$
$$= 0 + Z^{-1} + 2Z^{-2} + 3Z^{-3} + \cdots \infty$$
$$= Z^{-1}(1 + 2Z^{-1} + 3Z^{-2} + \cdots\ \infty)$$
$$= \frac{d}{dz}\ Z^{-1}\frac{d}{dZ^{-1}}\left(\frac{1}{1-Z^{-1}}\right) = Z^{-1}\,(1 - Z^{-1})^{-2}$$
$$= Z^{-1}\,(n_{c_0}(1)^n (Z^{-1})^0 + n_{c_1}$$

(6) Convolution property

If $x,(n) \xleftrightarrow{Z} X_1(Z)$ and $x_2(n) \xleftrightarrow{Z} X_2(Z)$

Then

$$x(n) = x,(n) \times x_2\,(n) = \sum_{K=-\infty}^{\infty} x_1(K) x_2(n-K)$$
$$\xleftrightarrow{Z} X(Z) = x_1(Z).x_1(Z)$$

3.3. Discus Z- transform

3.3.1. Explain poles and Zones:

$X(Z)$ is a national function if this is ratio of two polynomials in $Z^{-1} or Z$

$$X(Z) = \frac{x_1(Z)}{D(Z)} = \frac{b_0 + b_1 Z^{-1} + \cdots + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + \cdots + a_n Z^{-N}}$$
$$= \frac{\sum_{K=0}^{M} b_k Z^{-K}}{\sum_{K=0}^{N} a_K Z^{-K}}$$

Zeros : The values of Z at which $x(Z)Q = 0$ are eared as Zero

Poles: The values of z at which $x(Z) = \infty$ are called as poles

→ In representation of $x(z)$graphically by a pole –Zero plot (pattern) in the complier which shows the location of poles by crosses (x) and zeros by circles (0)

→ Roc should not contain any pole.

3.3.2. Determine Pole location and time domain behavior for causal signals.

→ Here we will discuss Z-plane location of pole pair and the form (Shope) of the corresponding signal in the time domain.

	Transform		**Transform**
(1)	$\delta(n)$	----------	
(2)	$\delta(n-K)$	----------	Z^{-K}
(3)	$u(n)$	----------	$\frac{1}{1-Z^{-1}} = \frac{Z}{Z-1}$
(4)	$-u(n-1)$	----------	$\frac{1}{1-Z^{-1}} = \frac{Z}{Z-1}$
(5)	$nu(n)$	----------	$\frac{Z^{-1}}{(1-Z^{-1})^2} = \frac{Z}{(Z-1)^2}$
(6)	$a^n u(n)$	----------	$\frac{1}{1-aZ^{-1}} = \frac{Z}{Z-1}$
(7)	$a^n u(-n-1)$	----------	$\frac{Z}{Z-a}$
(8)	$na^n u(n)$	----------	$\frac{aZ}{(Z-a)^2}$
(9)	e^{-an}	----------	$\frac{Z}{Z-e^{-a}}$
(10)	$n^2 u(n)$	----------	$\frac{Z(Z+1)}{(Z-1)^3}$
(11)	ne^{-an}	----------	$\frac{Ze^{-a}}{(Z-e^{-a})^2}$
(12)	$\sin\omega_0 n$	----------	$\frac{Z\sin\omega_0}{Z^2 - 2Z\cos\omega_0 + 1}$
(13)	$\cos\omega_0 n$	----------	$\frac{Z(Z-\cos\omega_0)}{Z^2-2Z\cos\omega_0+1}$
(14)	$a^n \sin\omega_0 n$	----------	$\frac{Z\,a\sin\omega_0}{Z^2-2Z\,a\cos\omega_0+a^2}$
(15)	$a^n \cos\omega_0 n$	----------	$\frac{Z(Z-a\cos\omega_0)}{Z^2-2Za\cos\omega_0+a^2}$

Q-1 Find inverse Z-transform of

$$X(Z) = \frac{Z}{3Z^2-4Z+1}$$

By fraction method

$$X(Z) = \frac{\frac{1}{2}Z}{Z-1} + \frac{-\frac{1}{2}}{Z-\frac{1}{3}}$$

$$\therefore x(n) = \frac{1}{2}(1)^n u(n) - \frac{1}{2}\left(\frac{1}{3}\right)^n u(n)$$

Q-2 Find inverse Z transform of $X(Z) = \frac{1+3Z^{-1}}{1+3Z^{-1}+2Z^{-2}}$, $|Z|72$

$$X(Z) = \frac{1-3Z^{-1}}{1+3Z^{-1}+2Z^{-2}} = \frac{Z(Z+3)}{(Z+1)(Z+2)}$$

$$\Rightarrow X(Z) = \frac{2Z}{Z+1} - \frac{Z}{Z+2} \ (By\ portial\ fraction)$$

$$\therefore x(n)2(-1)^n u(n) - \ (-2)^n u(n)$$

Q-3 Find inverse Z transform of $X(Z) = \frac{Z(Z^2-4Z+5)}{(Z-1)(Z-2)(Z-3)}$

∴ By partial fraction

$$x(Z) = \frac{Z}{Z-1} - \frac{Z}{Z-2} + \frac{Z}{Z-3}$$

$$\therefore x(n) = u(n) - \ (2)^n u(n) + (3)^n u(-n-1)$$

Q-4 $X(Z) = \frac{1}{(1-2Z^{-1})(1-Z^{-1})^2}$

Q-5 $X(Z) = \frac{Z^2+Z}{(Z-1)(Z-3)}$

Q-1 Find inverse –Z Transform of

$$X(Z) = \frac{Z}{3Z^2-4Z+1}$$

$$\frac{X(Z)}{Z} = \frac{1}{3Z^2-4Z+1}$$

$$\Rightarrow \frac{X(Z)}{Z} = \frac{1}{3Z^2-3Z-Z+1}$$

$$= \frac{1}{3Z(Z-1)-1\,(Z-1)}$$

$$= \frac{1}{(Z-1)(3Z-1)}$$

Now by partial fraction method

$$\frac{1}{(Z-1)(3Z-1)} = \frac{A}{Z-1} + \frac{B}{3Z-1}$$

$$\Rightarrow \frac{1}{(Z-1)(3Z-1)} = \frac{A(3Z-1) + \ B(Z-1)}{(Z-1)(3Z-1)}$$

$$\Rightarrow 1 = A\,(3Z-1) + B\,(Z-1)$$

$$\Rightarrow 1 = A3Z - A + BZ - B$$

$$= Z\,(3A+B) - A - B$$

By Comp

$$\Rightarrow \mathbf{1 = Z(3A+B) - A - B}$$

By Comparing coefficient of Z & constants

We get

$$\mathbf{3A+B=0} \qquad \& \qquad \mathbf{-A-B=1}$$

$$3(1-B)+B=0$$

$$\Rightarrow \ -3B+B+3=0$$

$$\Rightarrow \ -2B = \ -3$$

$$\Rightarrow B = \frac{3}{2}$$

$$\Rightarrow -A = B+1$$

$$\Rightarrow \boxed{A = 1-B}$$

$$A = 1 — \frac{3}{2}$$

$$= 1 - \frac{3}{2} = \frac{1}{2}$$

$$\mathbf{X1\delta\omega} \ \ \frac{X(Z)}{Z} = \frac{\left(\frac{1}{2}\right)}{Z-1} - \frac{\left(\frac{3}{2}\right)}{3Z-1}$$

$$\Rightarrow \frac{X(Z)}{Z} = \left(\frac{1}{2}\right)\frac{1}{Z-1} - \left(\frac{3}{2}\right)\left(\frac{1}{3Z-1}\right)$$
$$\Rightarrow X(Z) = \left(\frac{1}{2}\right)\frac{Z}{Z-1} - \left(\frac{3}{2}\right)\left(\frac{Z}{3Z-1}\right)$$
$$\Rightarrow X(Z) = \left(\frac{1}{2}\right)\frac{Z}{Z-1} - \frac{3}{2\times 3}\left(\frac{Z}{Z-\frac{1}{3}}\right)$$
$$= \left(\frac{1}{2}\right)\left(\frac{Z}{Z-1}\right) - \frac{1}{2}\left(\frac{Z}{Z-\frac{1}{3}}\right)$$

By formula

$$\boldsymbol{x(n) = \left(\frac{1}{2}\right)(1)^n u(n) - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)^n u(n)}$$

CHAPTER- 4

DISCUSS FOURIER TRANSFORM: ITS APPLICATIONS PROPERTIES

4.1. Discuses Discrete furriers transform.

→ Discrete Fourier Transform is a computational or mathematical tool for analyzing discreet time signal in frequency domain.

→ DFT consents $x(n)$ (Discrete time domain signal of infinite length to discrete frequency sequence $X(K)$ of finite length.

→ DFT is obtained by sampling one period of the Fourier transform at finite number of frequency points.

→ $x(n) \xrightarrow{DFT} X(K) = X(e^{jw})$

$$X(K) = \sum_{n-0}^{N-1} x(n)\, e^{-j\,(2\pi n)\frac{K}{M}}$$
$$\Rightarrow X(K) = DFT\,[X(n)]$$
$$x(n) = \frac{1}{N}\sum_{n=0}^{M-1} X\,(K)\, e^{j(2\pi n)\frac{K}{N}}$$
$$\Rightarrow x(n) = I\,Dft\,[X(n)]$$

Both n & K are ranging from 0 to N-1

n → time index since if denotes time constant $0 \le n \le N-1$

K →frequency index since if denotes frequency constant $0 \le K \le N-1$

$W_N = e^{-J2\frac{\pi}{N}}$ = Twiddle factor

$N - Noof$ Equally spaced sample points.

Ex 3:1 = Find DFT

$$x(n) = \{1,1,0,0\}$$
$$M = 4$$
$$X(K) = \sum_{n=0}^{M} x(n)e^{-j}(2\pi n)\frac{K}{N}\ , K = 0 \ldots N-1$$
$$K = 0$$
$$X(0) = \sum_{n=0}^{3} x(n).\, e^{0} = \sum_{n=0}^{3} x(n)\ (1)$$
$$= x(0) + x(1) + x(2) + x(3) = 1 + 1 + 0 + 0 = 2$$
$$K = 1, X(1) = \sum_{n=0}^{3} x(n)\,.\, e^{-j(2\pi n)\frac{1}{4}} = \sum_{n=0}^{3} x(n)\,.\, e^{-j\left(\frac{\pi n}{2}\right)}$$
$$= x(0).\, e^{0} + x(1).\, e^{-j\left(\frac{\pi}{2}\right)} + x(2) - e^{-j\left(\frac{\pi 12}{2}\right)} + x(3).\, e^{j\left(\frac{\pi.3}{2}\right)}$$
$$=$$

$x(2) =$

$x(3) =$

Ex.3.2 Find DFT of $x(n) = 1$ for $0 \le n \le 2$ $\quad n = 4, 8\, point$

$= o$ otherwise

Ex.33 Find 6 point DFT of $x(n) = \{1,1,1,1,1,,\}$

4.2. Relate DFT to other transform

Relate to fierier transform

The Fourier transform $x(e^{jw})$ of a finite duration sequence $x(n)$ having length N is given by

$$x(e^{jw}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n} \;.....(1)$$

Where $x\,(e^{j\omega})$ is a continuous transition of ω.

The DFT of $x(n)$ is given by

$$X(K) = \sum_{n=0}^{N-1} x(n)\;\; e^{-j\left(\frac{2\pi K}{N}\right)^{n}} \;------(2)$$

$$K = 0,1,2 \quad N-1$$

By comparing with (1) & (2) we get.

$X(K) = \; x(e^{jw})|_{\omega=\frac{2\pi K}{N}}$

<u>Relate to Z- transform :-</u>

Z- transform of finite duration 'N' sequence $x(n)$ is given by.

$$x(Z) = \sum_{n=0}^{N-1} x(n)\, Z^{-n} \;------(1)$$

But by IDFT

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X\,(K)\, e^{j\frac{2\pi Kn}{N}}$$

By putting $x(n)$ in equation (1) from equation (2) we get

$$X(Z) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{K=0}^{N-1} X\,(K)\, e^{j\frac{2\pi Kn}{N}}\right] Z^{-n}$$

$$\Rightarrow X(Z) = \frac{1}{N}\sum_{K=0}^{N-1} X\,(K) \sum_{n=0}^{N-1}\left(\, e^{j\frac{2\pi K/N}{N}}\; Z^{-1}\right)^{n}$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X\,(K)\left[1 + e^{j\frac{2\pi K}{N}}.\; Z^{-1} + \left(e^{j\frac{2\pi K}{N}}.Z^{-1}\right)^{2} + \cdots + \left(e^{\frac{j2\pi K}{N}}Z^{-1}\right)^{N-1}\;\right]$$

$$= \frac{1}{N}\sum_{K=0}^{N-1}\left[\frac{(1)\left[1-\left(e^{j\frac{2\pi k}{N}}.Z^{-1}\right)^{N}\right]}{1-e^{\frac{j2\pi K}{N}}.Z^{-1}}\right] X(K)$$

$$= \frac{1}{N}\sum_{K=0}^{N-1} X\,(K)\;\left[\frac{(1)\left(1-e^{j2\pi}Z^{-N}\right)}{1-e^{\frac{j2\pi K}{N}}.Z^{-1}}\right]$$

$$\Rightarrow \boxed{X(Z) = \frac{1}{N}\;(1-Z^{-N})\sum_{K=0}^{N-1}\frac{X(K)}{1-e^{\frac{j2\pi K}{N}}Z^{-1}}}$$

$\rightarrow$ Sum of first 'n' terms of geometries sequence is

$s_n = \frac{a_1(1-r^n)}{1-r}$, x_1 = Finet number

$r = common\ ration$

→ Sum of infinite G.P. series, is

$s_\infty = \frac{a}{1-r}$, $a = First\ number$

4.3. Discuss property of DFT

4.4. Discuss periodicity linearity & symmetry property

Symmetry – if DFT $[x(n)] = X(K)$

Then DFT $[x^*(n)] = X^*(N-K) = X^*\big((-K)\big)_N$

It is also called as complex conjugate property

Periodicity :- If $x(n)$ & $X(K)$ one on N point DFT pair then.

$x(n+N) = x(n)$ for all n.

$X(K+N) = X(K)$ for all K

Linearity :- if $x_1(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} X_1(K)$ & $x_2(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} X_2(K)$

$a_1x_1(n) + a_2x_2(n) \underset{N}{\overset{DFT}{\longleftrightarrow}} a_1x_1(K) + a_2x_2(K)$

Where a_1& a_2 are two arbitrary constants.

Multiplication of tow DFTS:

Let $x_1(n)$ & $x_2(n)$ be two finite duration sequences of length N . and their DN-point DFTS are.

$$x_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\,2\pi n\frac{K}{n}}\ , K = 0, 1, \ldots . N-1$$

$$x_2(K) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j\,2\pi n\frac{K}{n}}\ , K = 0, 1, \ldots . N-1$$

Let $x_3(K) = x_1(K).x_2(K)$.

By IDFT of $\{x_3(K)\}$ is

$$x_3(m) = \frac{1}{N}\sum_{K=0}^{N-1} X_3\,(K)e^{-\frac{j2\pi Km}{n}}$$

$$\Rightarrow x_3(m) = \frac{1}{N}\sum_{K=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n)\, e^{-\frac{j2\pi Km}{n}}\right]\left[\sum_{l=0}^{N-1} x_2(l)\, e^{-\frac{j2\pi Klm}{n}}\right] e^{\frac{j2\pi Km}{n}}$$

$$\Rightarrow x_3(m) = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \left[\sum_{K=0}^{N-1} e^{j2\pi K\frac{(m-n-l)}{N}}\right]$$

$$\Rightarrow \boxed{x_3(m) = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n)\, x_2\big((m-n)\big)_N} --- (1)$$

This above expression continually that multiplication of two DFTS of two sequences is equivalent to the circular convolution of two sequences in time domain.

$\big((m-n)\big)_N$ →circular convolution.

4.5. Explain multiplication of two DFT. & circular convolution

4.6. Let $x_1(n)$ & $x_2(n)$ one finite duration sequence both of length N.

$X_1(K)$ & $x_2(K)$ be DFTs of $x_1(n)$ & $x_2(n)$ respecivally

Let $x_3(n)$ be another sequence whose DFT is $X_3\ (K)$

Where $X_3(K) = X_1(K)x_2(K)$

From convolution theorem we know

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m).x_2(n-m) --- (1)$$

(For N number of Samples)
The equation (1) can be represented as.

$$x_3(n) = x_1(n)(N)x_2(n)$$

Hence DFT $[x_1(n)(N)x_2(n)] = X_1(K)X_2(K)$
Multiplication
If DFT $[x_1(n)] = X_1(K)$
DFT $[x_2(n)] = X_2(K)$
Then DFT $[x_1(n)x_2(n)] = \frac{1}{N}[X_1(K)(n)x + 2\ (K)]$

$$\boxed{\begin{aligned} d^{j\theta} &= \cos\theta - j\sin\theta \\ e^{-j\theta} &= \cos\theta + j\sin\theta \end{aligned}}$$

CHAPTER- 5

FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS

5.1 Compute DFT & FFT. Algorithm

→ FFT is a providence for computing

→ DFT of a finite series easily

→ It is nothing only set of algorithm

→ It is used in digital spectral analysis filter simulation, auto connection and pattern recognition.

→ FFT is based on decomposition and breaking the transform into smaller transforms and combining them to get the total transform.

→ $\boldsymbol{W_N^0 = e^{-\frac{j2\pi}{N}}}$ = **Twoddle factor**

→ FFT algorithm basically bits two properties of twiddle factor.

(i) $W_N^{K+\frac{N}{2}} = 1W_N^K$

(ii) $W_N^{K+N} = W_N^K$

→ There are two types of FFT algorithms

(i) Decimation in –time

(ii) Decimation in frequency

→ In decimation –in time algorithm, the sequence for which we need the DFT is successively divided into smaller sequences and the DFTs of these entire sequences are combined in a certain portion to obtain the required DFT of entire sequence.

→ In Decimation –in- frequency algorithm the frequency sample of the DFT are decomposed into smaller and smaller subsequences in a certain pattern.

5.2 Explain direct computation of DFT.

DFT of a sequence $x(n)$ is evaluated as follows

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nK}{N}} \quad , \quad 0 \le K \le N-1$$

Since $W_N = e^{-\frac{j2\pi}{N}}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\ W_N^{nk} \quad , \qquad 0 \le K \le N-1$$

$$= \sum_{n=0}^{N-1} \{R_e[x(n)] + j\, I_m[x(n)]\}\, \{R_e[W_N^{xK}] + j\, I_m[W_N^{nK}]\}$$

$$= \sum_{n=0}^{N-1} \left\{ R_e[x(n)] R_e[W_N^{xK}] - \sum_{n=0}^{N-1} I_m\, [x(n)] I_m[W_N^{nK}] \right\} + j \left\{ \sum_{n=0}^{N-1} I_m\ [x(n)] R_e[W_N^{xK}] + \sum_{n=0}^{N-1} R_e[x(n)]\, I_m[W_N^{nK}] \right\}$$

By using the above formula we can complete DFT directly .

5.3 Discuss the Radix – 2 algorithm

→ Radix -2 algorithm is also known as radix -2 decimation –in-time (DIT) algorithm.

→ In Radix-2 algorithm number of output points (N). can be expressed as power of 2. i.e. $N = 2^M$ we have M is an integer

The N-Point DFT of $x(n)$ is

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}, \quad K = 0,1, \ldots . N-1$$

By separating $x(n)$ into even and add values of $x(n)$ we get

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nK} + \sum_{n=0}^{N-1} x(n)\, W_N^{nK}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, W_N^{(2n+1)K}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, W_N^{2nK} + W_N^K \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, W_N^{2nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n)\, W_N^{2nK} + W_N^K \sum_{n=0}^{\frac{N}{2}-1} x_0(n)\, W_N^{2nk} ----(3)$$

Since $x(n) = x_0(n) + x_e(n)$

$$= x(2n+1) + x(2n)$$

$$\&\ W_N^2 = \left(e^{-\frac{j2\pi}{N}}\right)^2$$

$$= e^{-\frac{j2\pi}{\frac{N}{2}}} = W_{\frac{N}{2}}$$

From equation (1)

$$X(K) = \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_e(n)\ W_{\frac{N}{2}}^{nK}}_{\frac{N}{2}\,Point\ even\ indexed} + W_N^K \underbrace{\sum_{n=0}^{\frac{N}{2}-1} x_0(n)\ W_{\frac{N}{2}}^{nK}}_{\frac{N}{2}\,Point\ odd\ indexed}$$

$$= X_e(K) + W_N^K x_0(K)$$

For $K \geq \frac{N}{2}$, $\quad W_N^{K+\frac{N}{2}} = -W_N^K$

Now $X(K)$ for $K \geq \frac{N}{2}$ is given by

$$X(K) = X_e\left(K - \frac{N}{2}\right) - W_N^{K-\frac{N}{2}} X_0\left(K - \frac{N}{2}\right)$$

$$\text{for} K = \frac{N}{2} + \frac{N}{2} + 1, \ldots \ldots .., N-1$$

Steps of Radix – 2 DIT FFT . algorithm :-

Twiddle factor $W_N = e^{-\frac{j2\pi}{N}}$

(1) $W_N^K = W_N^{K+N}$

(2) $W_N^{K+\left(\frac{N}{2}\right)} = -W_N^K$

(3) $W_N^2 = W_{\frac{N}{2}}$

The competing formulas for FFT is given by

$X(K) = X_e(K) + W_N^K X_0(K)$ for $0 \leq K \leq \frac{N}{2} - 1$

$$= X_e\left(K - \frac{N}{2}\right) - W_N^{K-\frac{N}{2}} X_0\left(K - \frac{N}{2}\right), \frac{N}{2} \leq K \leq N-1$$

For a 8 point DFT/FFT for $\boldsymbol{K = 0, 1, 2, 3, 4, 5, 6, 7}$ the FFT values are as follows.

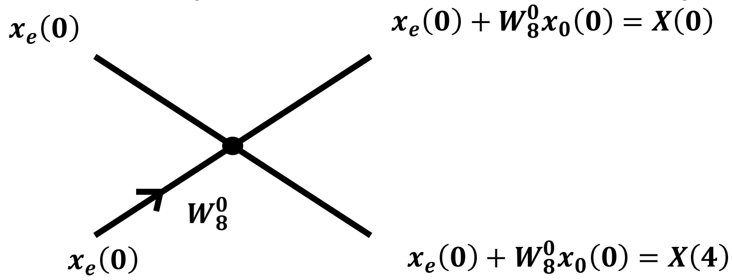
$\boldsymbol{X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)}$

$X(0) = x_e(0) + W_8^0 x_0(0)$ $\quad$ $X(4) = x_e(0) + W_8^0 x_0(0)$

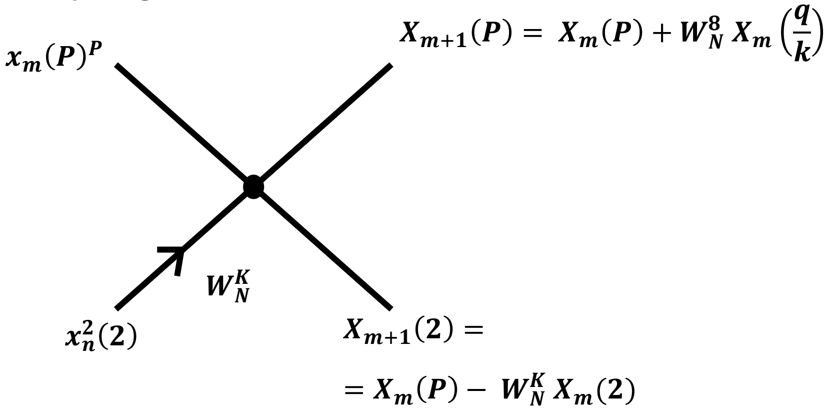
$X(1) = x_e(1) + W_8^1 x_0(1)$ $\quad$ $X(5) = x_e(1) + W_8^1 x_0(1)$

$X(2) = x_e(2) + W_8^2 x_0(2)$ $\quad$ $X(6) = x_e(2) + W_8^2 x_0(2)$

$X(3) = x_e(3) + W_8^3 x_0(3)$ $\quad$ $X(7) = x_e(3) + W_8^3 x_0(3)$



Butterfly Diagram



Bit reversal

Bit reversal is useful in arranging the samples for calculating DIT algorithm for N=8

For N=8

out Sample	presentation	versal	mple
0	000	000	0

1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

Radix – 2

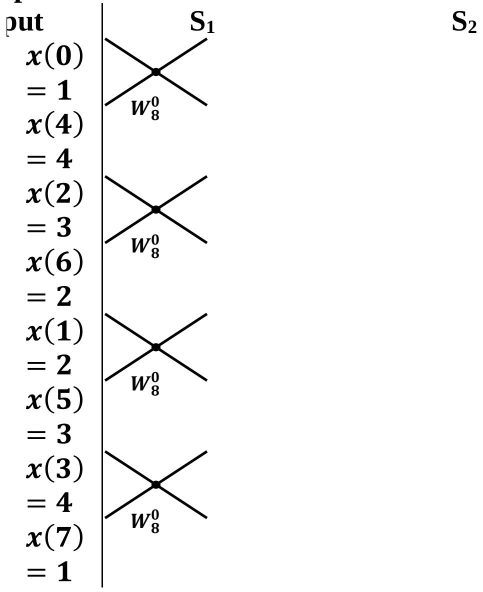
Divide the number of input samples by 2 till we reach minimum two samples.

Q- Draw and find FFT for a 8-point sequence

$x(n) = \{1,2,3,4,4,3,2,1\}$

$x(0) = 1, x(1) = 2, x(2) = 3, x(3) = 4, x(4) = 4, x(5) = 3, x(6) = 0,$
$x(7) = 1$

As per bit reversal



5.4 Introduction to digital filter

→ Filter is defined as a device which rejects unwanted frequencies from the input signal and allow the desired frequencies

→ When this input signal is a discrete time sequence then this filter is a digital filter

→ A filter is a LTI discrete time system.

→ Basically, two types (i) FIR filter

(ii) IIR Filter

(i) FIR Filter

This filter whose present output sample depends on the present input sample and previous input samples.

(ii) IIR Filter

This filter whose present output sample depends on present input, past input samples and output samples.

5.5 Introduction to DSP. Architecture, familiarization of different types of processor.

Ans: Digital signal processors one of two types

1) General purpose digital signal processor

2) Special purpose digital signal processor

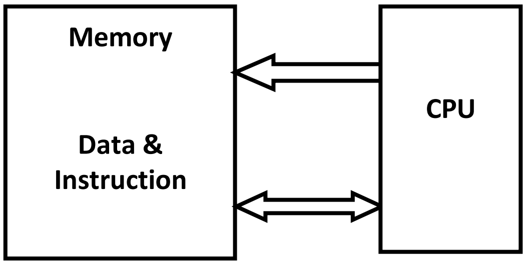
Introduction to DSP Architecture

DSP Architecture one of following types

1) Von Neumann architecture
2) Havard architecture
3) Super Havard architecture

1) Von Neumann Architecture :-

Advantage:



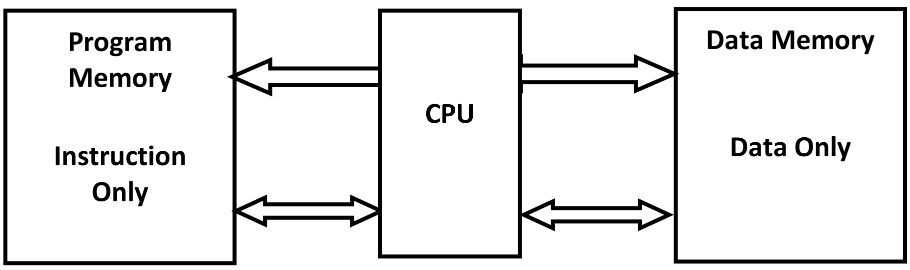
→ It is cheap and requires lesser number of pins than the Havard Architecture.

→ It is simple to use

Disadvantage:

→ It doesn't permit multiple memory access.

Havard Architecture



→ The Havard architecture physically separates memory for their instruction & data requiring dedicated buses for each of them.

→ Instructions and operands can therefore be fetched simultaneously

→ Most DSP processors are modified Havard architecture with two or three memory buses

→ It has multiport memor