

LECTURE NOTES

ON

STRUCTURAL MECHANICS

Compiled by

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CHAPTER-1

Review of Basic Concepts

FORCE

FORCE SYSTEM

Force is that which changes or tends to change the state of rest or uniform motion of a body along a straight line. It may also deform a body changing its dimensions. The force may be broadly defined as an agent which produces or tends to produce, destroy or tends to destroy motion. It has a magnitude and direction.

Mathematically: $\text{Force} = \text{Mass} \times \text{Acceleration}$.

Where $F = \text{force}$, $M = \text{mass}$ and $A = \text{acceleration}$.

UNITS OF FORCE

In C.G.S. System: In this system, there are two units of force: (i) Dyne and (ii) Gram force (gmf).

In M.K.S. System: In this system, unit of force is kilogram force (kgf).

In S.I. Unit: In this system, unit of force is Newton (N). One Newton is that force which acting on a mass of one kilogram produces in it an acceleration of one m/sec^2

$1 \text{ Newton} = 10^5 \text{ Dyne}$.

EFFECT OF FORCE

A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or decelerate it.
2. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
3. It may give rise to the internal stresses in the body, on which it acts.
4. A force can change the direction of a moving object.
5. A force can change the shape and size of an object.

CHARACTERISTICS OF A FORCE

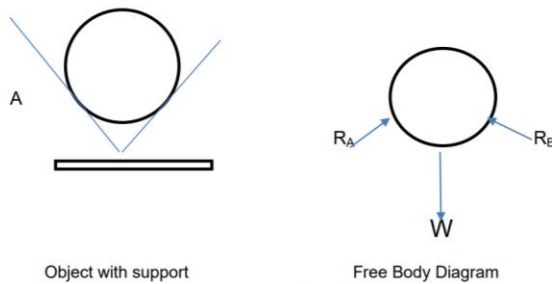
In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

1. Magnitude of the force (i.e., 50 N, 30 N, 20 N etc.)
2. The direction of the line, along which the force acts (i.e., along West, at 30° North of East etc.). It is also known as line of action of the force.

3. Nature of the force (push or pull).
4. The point at which (or through which) the force acts on the body.

FREE BODY DIAGRAM:

The representation of reaction force on the body by removing all the support or forces act from the body is called free body diagram.



MOMENT OF A FORCE

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment,

$$M = P \times l$$

where, P = Force acting on the body,

and, l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

Moment of a force about a point is the product of the force and the perpendicular distance of the point from the line of action of the force.

UNIT OF MOMENT

Unit of moment depends upon unit of force and unit of length.

If, however, force is measured in Newton and distance is measured in meter, the unit of moment will be Newton meter (Nm).

TYPES OF MOMENTS

Broadly speaking, the moments are of the following two types:

1. Clockwise moments.
2. Anticlockwise moments.

Clockwise moment is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move.

Anticlockwise moment is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move.

EQUILIBRIUM OF FORCES:

if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces..

A body can be said to be in equilibrium when all the force acting on a body balance each other or in other word there is no net force acting on the body.

Equilibrium of a body is a state in which all the forces acting on the body are balanced (cancelled out), and the net force acting on the body is zero.

i.e $\Sigma F = 0$

PRINCIPLES OF EQUILIBRIUM

1. Two force principle. As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. Three force principle. As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

3. Four force principle. As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Thus, the necessary and sufficient conditions of equilibrium for a system of co-planar and nonconcurrent forces are:

(i) The algebraic sum of the resolved parts of the forces along any direction is equal to zero (i.e., $\Sigma X = 0$),

(ii) The algebraic sum of the resolved parts of the forces along a directional right angles to the previous direction is equal to zero (i.e. $\Sigma Y = 0$),and

(iii) The algebraic sum of the moments of the forces about any point in their plane is equal to zero (i.e. $\Sigma M = 0$).

Stable equilibrium

A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. This happens when some additional force sets up due to displacement and brings the body back to its original position.

Unstable equilibrium

A body is said to be in an unstable equilibrium, if it does not return back to its original position, and heels farther away, after slightly displaced from its position of rest.

Neutral equilibrium

A body is said to be in a neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest.

Free body: A body is said to be free body if it is isolated from all other connected members

FREE BODY DIAGRAM

Free body diagram of a body is the diagram drawn by showing all the external forces and reactions on the body and by removing the contact surfaces.

Steps to be followed in drawing a free body diagram

- a. Isolate the body from all other bodies.
- b. Indicate the external forces on the free body. (The weight of the body should also be included. It should be applied at the centre of gravity of the body)
- c. The magnitude and direction of the known external forces should be mentioned.
- d. The reactions exerted by the supports on the body should be clearly indicated.
- e. Clearly mark the dimensions in the free body diagram.

CENTRE OF GRAVITY (C.G):

Centre of Gravity of a body is a fixed point with respect to the body, through which resultant of weights of all particles of the body passes, at any plane .

Centroid is the centre point or geometric centre of a plane figure like triangle, circle, quadrilateral, etc. The method of finding centroid is same as finding C.G of a body.

METHODS FOR CENTRE OF GRAVITY

The centre of gravity (or centroid) may be found out by any one of the following two methods:

1. By geometrical considerations
2. By moments
3. By graphical method

MOMENT OF INERTIA:

MOMENT OF INERTIA OF A COMPOSITE SECTION

The moment of inertia of a composite section may be found out by the following steps :

1. First of all, split up the given section into plane areas (i.e., rectangular, triangular, circular, etc., and find the centre of gravity of the section).

2. Find the moments of inertia of these areas about their respective centres of gravity.

3. Now transfer these moment of inertia about the required axis (AB) by the Theorem of Parallel Axis, i.e., $I_{AB} = I_G + ah^2$

where I_G = Moment of inertia of a section about its centre of gravity and parallel to the axis.

a = Area of the section,

h = Distance between the required axis and centre of gravity of the section.

4. The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

CHAPTER-2

Simple and Complex Stress, Strain

Simple Stress

Consider a force is acting on a body on a unit area. a simple stress is defined as the internal force which is resisting the external force per unit area. It is denoted with σ

$$\text{Stress } (\sigma) = F/A$$

where,

F = Force acting on the body

A = Cross-sectional Area where the force is acting

Types of Stress

Stress applied to a material can be of two types as follows:

Tensile Stresses

When a body subjected to two equal and opposite axial pulling forces(Tensile Load) then the stress produced in every section of the body is called Tensile stresses.

Tensile strain Is the change in length by its original length.

Compressive Stresses

When a body subjected to two equal and opposite axial pushing forces(Compression Load) then the stress produced in every section of the body is called Compressive stresses.

Compressive strain Is the change in length by its original length.

Simple Strain

Due to the external forces, the body may undergo some deformation. This deformation per unit length is said as the Strain. Strain represented by ϵ

$$\epsilon = \delta l / l$$

Where

δl = Change in the length of the body

l = Original length of the body

Types of Strain

Strain experienced by a body can be of two types depending on stress application as follows:

Tensile Strain

The deformation or elongation of a solid body due to applying a tensile force or stress is known as Tensile strain. In other words, tensile strain is produced when a body increases in length as applied forces try to stretch it.

Compressive Strain

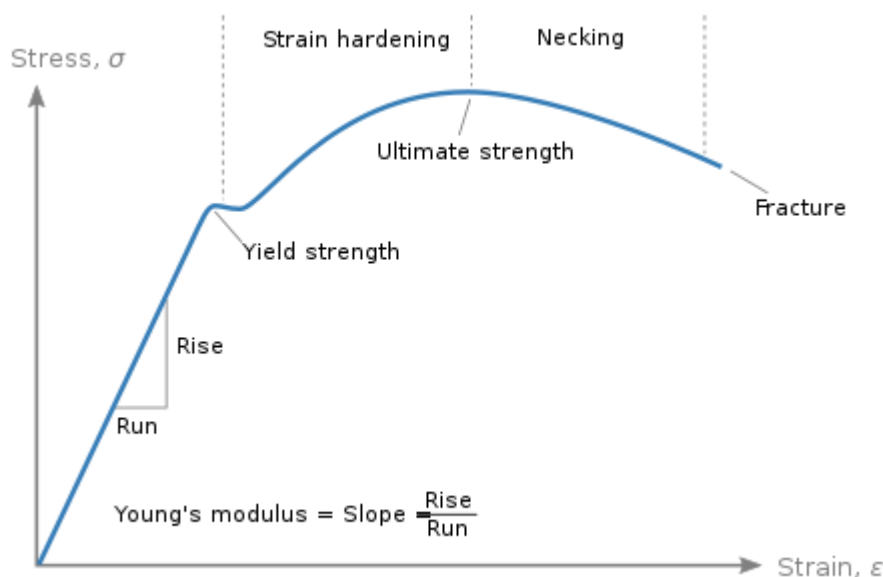
Compressive strain is the deformation in a solid due to the application of compressive stress. In other words, compressive strain is produced when a body decreases in length when equal and opposite forces try to compress it.

Hooke's Law

This states that strain is proportional to the stress producing it. A material is said to be elastic if all the deformations are proportional to the load.

Stress-Strain Curve Explanation

The material's stress-strain curve represents the relationship between stress and strain for materials. The strain values are plotted on the curve corresponding to the stress incurred by different loads on the object.



The stress-strain diagram has different points or regions as follows:

- i. Proportional limit
- ii. Elastic limit
- iii. Yield point
- iv. Ultimate stress point
- v. Fracture or breaking point

(i) Proportional Limit

The region in the stress-strain curve that observes Hooke's Law is known as the proportional limit. According to this limit, the ratio of stress and strain provides us with the proportionality constant known as young's modulus. In the graph point, OA is known as the proportional limit.

(ii) Elastic Limit

Elastic limit is the maximum stress that a substance can endure before permanently being deformed. When the load acting on the object is completely removed and the material returns to its original position, that point is known as the object's elastic limit.

(iii) Yield Point

The point at which the material starts showing to deform plastically is known as the yield point of the material. Once the yield point of an object is crossed, plastic deformation occurs. There are two types of yield points (i) upper yield point (ii) lower yield point.

(iv) Ultimate Stress Point

The point at which a material endures maximum stress before failure is known as the Ultimate Stress point. After this point, the material will break.

(v) Fracture or Breaking Point

In the stress-strain curve, the point at which the failure of the material takes place is known as the breaking point of the material.

Young's Modules

Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain produced is called young's modules or the modules of Elasticity.

Modules of rigidity

For elastic material shear strain is proportional to the shear stress.

Relation between K and E

The above figure represents a unit cube of material under the action of a uniform pressure P. It is clear that the principle stresses are -P, -P and -P and the linear strain in each direction.

$$-P/E + \mu P/E + \mu P/E = \frac{-P}{A} (1-2\mu)$$

But we know

Volumetric strain = sum of linear strain

$$\text{By definition } K = \frac{-P}{\delta V / V}$$

$$\text{or } K = \frac{-P}{\frac{-3P}{E} (1-2\mu)}$$

$$\text{or } K = \frac{E}{3(1-2\mu)}$$

$$\text{or } E = 3K(1-2\mu)$$

Complex Stress and Strain:

3.1 Determination of normal stress, shear stress and resultant stress on oblique plane.

In many instances, however, both direct and shear stresses are brought into play, and the resultant stress across any section will be neither normal nor tangential to the plane.

If σ_r is the resultant stress making an angle ϕ with the normal to the plane on which it acts.

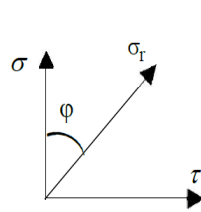


Fig 3.1

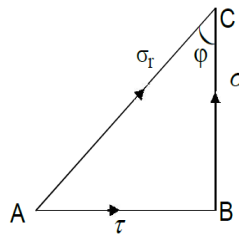


Fig 3.2

$$\phi = \tan^{-1} \frac{\tau}{\sigma}$$

$$\sigma_r = \sqrt{\sigma^2 + \tau^2}$$

Stress on oblique plane

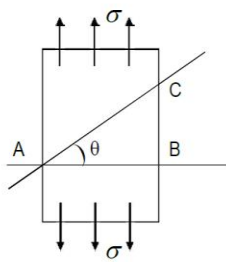


Fig 3.3

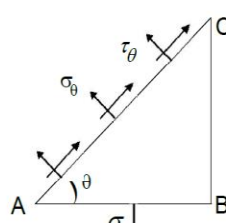


Fig 3.4

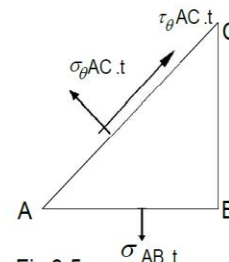


Fig 3.5

The problem is to find the stress acting on any plane AC at an angle θ to AB. This stress will not be normal to the plane, and may be resolved into two components σ_θ and τ_θ .

As per Figure 3.4 show the stresses acting on the three planes of the triangular prism. There can be no stress on the plane BC, which is a longitudinal plane of the bar, the stress τ_θ must

be up the plane for equilibrium.

Figure 3.5 shows the forces acting on the prism, taking a thickness t perpendicular the figure.

The equations of equilibrium resolve in the direction of σ_θ .

$$\begin{aligned} \sigma_\theta \cdot AC \cdot t &= \sigma_{AB} \cdot t \cos \theta \\ &= \sigma_\theta = \sigma \left(\frac{AB}{AC} \right) \cos \theta \\ &= \sigma \cos^2 \theta \end{aligned}$$

Resolve in the direction τ_θ

$$\tau_\theta \cdot AC \cdot t = \sigma AB \cdot t \sin \theta$$

$$\Rightarrow \tau_\theta = \sigma \left(\frac{AB}{AC} \right) \sin \theta$$

$$\Rightarrow \tau_\theta = \sigma \cos^2 \theta \sin \theta$$

$$\Rightarrow \tau_\theta = \frac{1}{2} \sigma \sin 2\theta$$

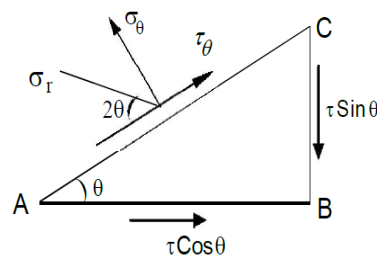
$$\Rightarrow \sigma_r = \sqrt{(\sigma_\theta^2 + \tau_\theta^2)}$$

$$\Rightarrow \sigma_r = \sqrt{\cos^4 \theta + \cos^2 \theta \cdot \sin^2 \theta}$$

$$\therefore \sigma_r = \sigma \cos \theta$$

Pure Shear

As the figures will always be right-angled triangles there will be no loss of generality by assuming the hypotenuse to be of unit length. By making use of these specification it will be found that the area on which the stresses act are proportional to 1 (for AC), $\sin \theta$ (for BC) and $\cos \theta$ (for AB) and future figures will show the forces acting on such an element.



Let τ act on a plane AB and there is an equal complementary shear stress on plane BC. The aim is to find σ_θ & τ_θ acting on AC at an angle θ to AB.

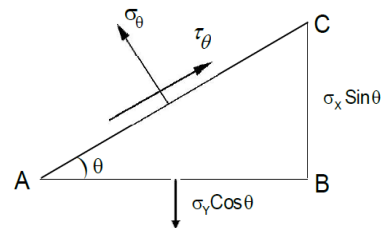
Resolving in the direction of σ_θ

$$\begin{aligned}\sigma_\theta \times 1 &= (\tau \cos \theta) \sin \theta + (\tau \sin \theta) \cos \theta \\ &= \tau \sin 2\theta\end{aligned}$$

Resolving in the direction of τ_θ

$$\begin{aligned}\tau_\theta \times 1 &= (\tau \sin \theta) \sin \theta - (\tau \cos \theta) \cos \theta \\ &= -\tau \cos 2\theta \quad (\theta < 45^\circ) \text{ down to plane} \\ \sigma_r &= \sqrt{\sigma_\theta^2 + \tau_\theta^2} = \tau \text{ at } 2\theta \text{ to } \tau_\theta\end{aligned}$$

Pure Normal stresses on give planes



Let the known stresses be σ_x on BC and σ_y on AB, then the forces on the element are proportional to those shown.

Resolving in the direction of σ_θ

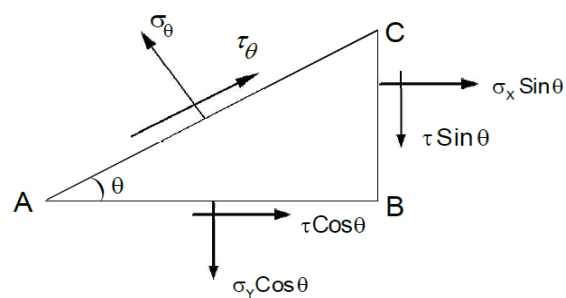
$$\therefore \sigma_\theta = \sigma_y \cos^2 \theta + \sigma_x \sin^2 \theta$$

Resolving in the direction of τ_θ

$$\tau_\theta = \sigma_y \cos \theta \sin \theta - \sigma_x \sin \theta \cos \theta$$

$$\therefore \tau_\theta = \frac{1}{2} (\sigma_y - \sigma_x) \sin 2\theta$$

General two dimensional Stress system



Resolving in the direction of σ_θ

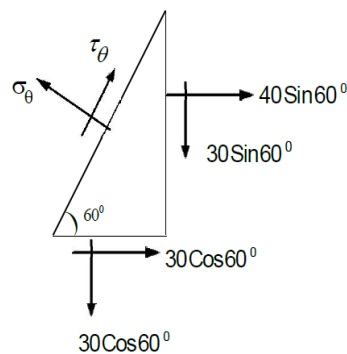
$$\begin{aligned}\sigma_\theta &= \sigma_y \cos\theta \cos\theta + \sigma_x \sin\theta \sin\theta + \tau \cos\theta \sin\theta + \tau \sin\theta \cos\theta \\ &= \sigma_y \left(\frac{1 + \cos^2\theta}{2} \right) + \sigma_x \left(\frac{1 - \cos^2\theta}{2} \right) + \tau \sin^2\theta \\ &= \frac{1}{2}(\sigma_y + \sigma_x) + \frac{1}{2}(\sigma_y - \sigma_x) \cos^2\theta + \tau \sin^2\theta\end{aligned}$$

Resolving in the direction of τ_θ

$$\begin{aligned}\tau_\theta &= \sigma_y \cos\theta \sin\theta - \sigma_x \sin\theta \cos\theta \\ &\quad - \tau \cos\theta \cos\theta + \tau \sin\theta \sin\theta \\ \therefore \tau_\theta &= \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta - \tau \cos 2\theta\end{aligned}$$

Example – 1

If the stress on two perpendicular planes through a point are 60 N/mm² tension, 40 N/mm² compression and 30 N/mm² shear find the stress components and resultant stress on a plane at 60° to that of the tensile stresses.



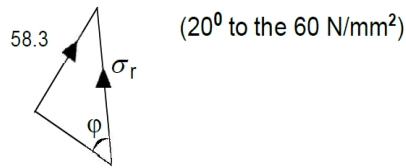
$$\begin{aligned}\sigma_\theta &= 60 \cos 60^\circ \cdot \cos 60^\circ - 40 \sin 60^\circ \cdot \sin 60^\circ + 30 \cos 60^\circ \sin 60^\circ + 30 \sin 60^\circ \cos 60^\circ \\ &= 60 \times \frac{1}{2} \times \frac{1}{2} - 40 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 30 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + 30 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ &= 15 - 30 + 7.5\sqrt{3} + 7.5\sqrt{3} \\ &= \sigma_\theta = 11 \text{ N/mm}^2\end{aligned}$$

and

$$\begin{aligned}\tau_\theta &= 60 \cos 60^\circ \cdot \sin 60^\circ + 40 \sin 60^\circ \cdot \cos 60^\circ - 30 \cos 60^\circ \cos 60^\circ + 30 \sin 60^\circ \sin 60^\circ \\ &= 15\sqrt{3} + 10\sqrt{3} - 7.5 + 22.5 \\ &= 58.3 \text{ N/mm}^2 \\ &= \sigma_r = \sqrt{(11^2 + 58.3^2)} = 59.3 \text{ N/mm}^2\end{aligned}$$

at angle to the

$$\gamma = \tan^{-1} \frac{58.3}{11} = 80^\circ 15'$$



Principal Planes

From equation

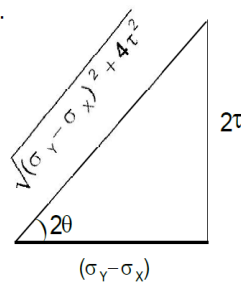
$$\tau_{\theta} = \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta - \tau \cos 2\theta$$

There are values of θ for which τ_{θ} is zero and the plane on which the shear component is zero are called principal planes.

From equation above.

$$\tan 2\theta = \frac{2\tau}{(\sigma_y - \sigma_x)} \quad (\text{when } \tau_{\theta} = 0)$$

This gives two values of 2θ differing by 180° and hence two values of θ differing by 90° i.e. the principle planes are two planes at right angles.



$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\cos 2\theta = \pm \frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

CHAPTER-3

Stresses in Beams

Bending Stress in Beams

Bending Stress:

When a beam is loaded with external loads, all the sections of the beam will experience bending moments and shear forces. The shear forces and bending moments at various sections of the beam can be evaluated as discussed in the earlier chapter. In this chapter, the bending and bending stress distribution across a section will be dealt with.

Some practical applications of bending stress shall also be dealt with. These are

1. Moment carrying capacity of a section
2. Evaluation of extreme normal stresses due to bending
3. Design of beam for bending
4. Evaluation of load bearing capacity of the beam

The major stresses induced due to bending are normal stresses of tension and compression. But the state of stress within the beam includes shear stresses due to the shear force in addition to the major normal stresses due to bending although the former are generally of smaller order when compared to the latter.

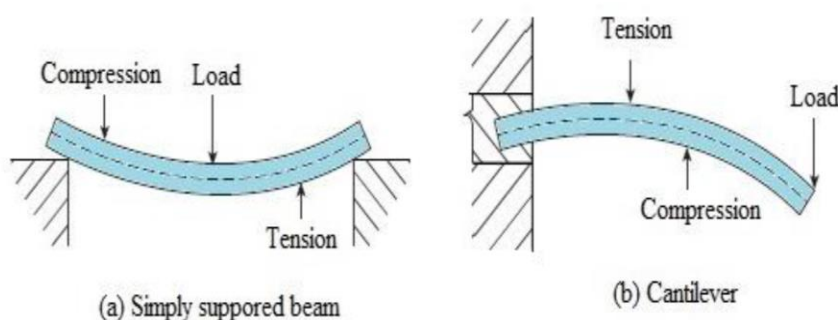


Fig. 1

Simple Bending or Pure Bending

A beam or a part of it is said to be in a state of pure bending when it bends under the action of uniform/constant bending moment, without any shear force.

Alternatively, a portion of a beam is said to be in a state of simple bending or pure bending when the shear force over that portion is zero. In that case there is no chance of shear stress in the beam. But, the stress that will propagate in the beam as a result will be known as normal stress.

Examples of pure bending are –

1. Bending of simple supported beam due to end coupling (Uniform pure bending)
2. Bending of cantilever beam with end moment (Uniform pure bending)
3. Bending of the portion between two equal point loads in a simple supported beam with two-point loading (Non-uniform pure bending)

Theory of Simple Bending

The theory which deals with the determination of stresses at a section of a beam due to pure bending is called theory of simple bending. In this chapter, bending of straight homogeneous beams of uniform cross sectional area with vertical axis of symmetry shall be considered. The application of this theory can be extended to beams with two or more different materials as well as curved beams.

A beam subjected to sagging moment is shown in the Fig. 8. The beam is imagined to be consisting of a number of longitudinal fibres; one such fibre is shown in colour. It is obvious that the fibres near the upper side of the beam are compressed; hence an element in the upper part is under compression. The fibres at the bottom side of the beam get stretched and, hence, the elements on the lower side are subjected to tension. Somewhere in between, there will be a plane where the fibres are subjected to neither tension nor compression. Such a plane is termed as neutral surface or neutral plane.

In the conventional coordinate system attached to the beam in Fig. 8, x axis is the longitudinal axis of the beam, the y axis is in the transverse direction and the longitudinal plane of symmetry is in the x-y plane, also called the plane of bending.

Assumptions for theory of pure bending:

The assumptions made in the theory of simple bending are as follows:

1. The material of the beam is perfectly homogeneous (i.e. of the same kind throughout) and isotropic (i.e. of same elastic properties in all directions).
2. The material is stressed within elastic limit and obeys Hooke's law.
3. The value of modulus of elasticity for the material is same in tension and compression.
4. The beam is subjected to pure bending and therefore bends in the form of an arc of a circle.
5. The transverse sections, which are plane and normal to the longitudinal axis before bending, remain plane and normal to the longitudinal axis of the beam after bending.
6. The radius of curvature of the bent axis of the beam is large compared to the dimensions of the section of beam.
7. Each layer of the beam is free to expand or contract independently.

8. The cross-sectional area is symmetric about an axis perpendicular to the neutral axis.

Flexural rigidity:

EI is known as flexural rigidity. Flexural rigidity is the measure of flexural strength of a beam section. Higher is the flexural rigidity better is the flexural strength. It depends upon the material as well as the geometric property of the section. Elastic modulus, E reflects the material character and moment of inertia, I reflects the geometric characteristic,

Moment carrying capacity of a section:

From equation of flexure, we have

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M}{I} y$$

It is obvious that bending stress is maximum on the extreme fibre at the top and bottom of the beam where y is maximum. In design of beam, the extreme fibre stress should not be allowed to exceed the allowable or permissible stress of the material. If σ_{allow} is the allowable stress for bending, then for safe design

$$\sigma_{max} \leq \sigma_{allow}$$

$$\frac{M}{I} y_{max} \leq \sigma_{allow}$$

If M is taken as the maximum moment carrying capacity of the section,

$$\frac{M}{I} y_{max} \leq \sigma_{allow}$$

$$M \leq \frac{I}{y_{\max}} \sigma_{\text{allow}}$$

The moment of inertia I and the extreme fibre distance y_{\max} are the geometrical properties of the section. The ratio of the moment of inertia and the extreme fibre distance (I/y_{\max}) for a given cross-section of beam is constant and is known as **section modulus** (Z). Thus the moment carrying capacity of a beam is given by

$$M = \sigma_{\text{allow}} Z$$

If σ_{allow} in tension and compression are same, doubly symmetric section is selected. Doubly symmetric section means a section which is symmetric about the vertical as well as neutral axis. If σ_{allow} in tension and compression are different, un-symmetric cross-section is selected such that the distance to the extreme fibers are nearly the same ratio as the respective allowable stresses. In the latter case, the moment carrying capacity in tension and compression are found separately and the smaller one is taken as the moment carrying capacity of the section.

Section Modulus of Sections of Standard Geometry

1. Rectangular section

Let us consider a rectangular section of width b and depth d as shown in the Fig. The neutral axis coincides with the centroidal axis of the beam.

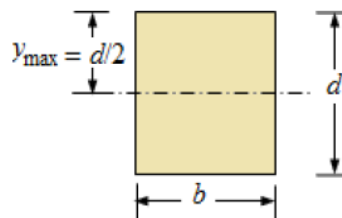


Fig. 16

Moment of inertia about the neutral axis, $I = \frac{bd^3}{12}$

Distance of outermost fibre from the neutral axis, $y_{\max} = \frac{d}{2}$

Section modulus, $Z = \frac{I}{y_{\max}} = \frac{bd^3}{12} \times \frac{2}{d}$

$$= \frac{1}{6} bd^2$$

Let σ is the maximum bending stress developed at the outermost layer.

Moment of resistance, $M = \sigma Z = \frac{1}{6} \sigma bd^2$

2. Hollow Rectangular section

Let us consider a hollow rectangular section of size $B \times D$ with a symmetrical opening $b \times d$ as shown in the Fig. 17.

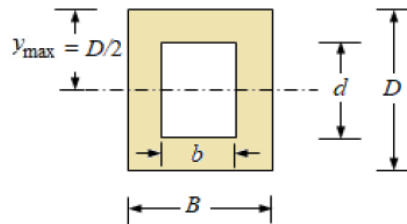


Fig. 17

Moment of inertia about the neutral axis, $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

Distance of outermost fibre from the neutral axis, $y_{\max} = \frac{D}{2}$

Section modulus,

$$Z = \frac{I}{y_{\max}} = \frac{BD^3 - bd^3}{12} \times \frac{2}{D}$$

$$= \frac{1}{6} \frac{(BD^3 - bd^3)}{D}$$

Let σ is the maximum bending stress developed at the outermost layer.

Moment of resistance, $M = \sigma Z = \frac{1}{6} \sigma \frac{(BD^3 - bd^3)}{D}$

3. Circular section

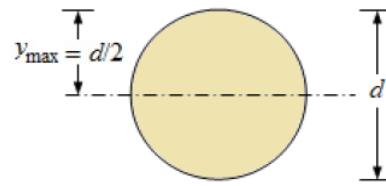


Fig. 18

Moment of inertia about the neutral axis, $I = \frac{\pi d^4}{64}$

Distance of outermost fibre from the neutral axis, $y_{\max} = \frac{d}{2}$

Section modulus,

$$Z = \frac{I}{y_{\max}} = \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$= \frac{\pi d^3}{32}$$

Let σ is the maximum bending stress developed at the outermost layer.

Moment of resistance, $M = \sigma Z = \sigma \frac{\pi d^3}{32}$

CHAPTER-4

Columns and Struts

Introduction

A structural member, subjected to an axial compressive force, is called a strut. As per definition, a strut may be horizontal, inclined or even vertical. But a vertical strut, used in buildings or frames, is called a column.

Failure of a Column or Strut

It has been observed, that when a column or a strut is subjected to some compressive force, then the compressive stress induced,

$$\sigma = P/A$$

where P = Compressive force and

A = Cross-sectional area of the column.

Euler's Column Theory

The first rational attempt, to study the stability of long columns, was made by Mr. Euler. He derived an equation, for the buckling load of long columns based on the bending stress. While deriving this equation, the effect of direct stress is neglected. This may be justified with the statement that the direct stress induced in a long column is negligible as compared to the bending stress. It may be noted that the Euler's formula cannot be used in the case of short columns, because the direct stress is considerable and hence cannot be neglected.

Assumptions in the Euler's Column Theory

The following simplifying assumptions are made in the Euler's column theory:

1. Initially the column is perfectly straight and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.

Types of End Conditions of Columns

In actual practice, there are a number of end conditions, for columns. But, we shall study the Euler's column theory on the following four types of end conditions, which are important from the subject point of view:

1. Both ends hinged,
2. Both ends fixed,
3. One end is fixed and the other hinged, and
4. One end is fixed and the other free.

Now we shall discuss the value of critical load for all the above mentioned type of and conditions of columns one by one.

Euler's Formula and Equivalent length of a Column

In the previous articles, we have derived the relations for the crippling load under various end conditions. Sometimes, all these cases are represented by a general equation called Euler's formula,

$$P_E = 2 \pi^2 EI / l_e^2$$

where l_e is the equivalent or effective length of column.

There is another way of representing the equation, for the crippling load by an equivalent length of effective length of a column. The equivalent length of a given column with given end conditions, is the length of an equivalent column of the same material and cross-section with both ends hinged and having the value of the crippling load equal to that of the given column.

The equivalent lengths (l_e) for the given end conditions are given below:

S.No.	End conditions	Relation between equivalent length (L_e) and actual length (l)	Crippling load (P)
1.	Both ends hinged	$L_e = l$	$P = \frac{\pi^2 EI}{(l)^2} = \frac{\pi^2 EI}{l^2}$
2.	One end fixed and the other free	$L_e = 2 l$	$P = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}$
3.	Both ends fixed	$L_e = \frac{l}{2}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}$
4.	One end fixed and the other hinged	$L_e = \frac{l}{\sqrt{2}}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI}{l^2}$

Slenderness Ratio

We have already discussed in Art. 34.11 that the Euler's formula for the crippling load,

$$P_E = 2 \pi^2 EI / L_e^2$$

We know that the buckling of a column under the crippling load will take place about the axis of least resistance. Now substituting $I = Ak^2$ (where A is the area and k is the least radius of gyration of the section) in the above equation,

$$P_E = \pi^2 E(Ak^2)/L_e^2 = \pi^2 EA/(L_e/k)^2$$

where L_e/k is known as slenderness ratio.

Thus slenderness ratio is defined as ratio of equivalent (or unsupported) length of column to the least radius of gyration of the section.

Slenderness ratio does not have any units.

NOTE. It may be noted that the formula for crippling load, in the pervious articles, have been derived on the assumption the the slenderness ratio $e L_e/k$ is so large, that the failure of the column occurs only due to bending, the effect of direct stress (i.e., P/A) being negligible.

Limitation of Euler's Formula

We have discussed in Art. 32.12 that the Euler's formula for the crippling load,

$$P_E = \pi^2 EA/(L_e/K)^2$$

\therefore Euler's crippling stress,

$$\sigma_E = P/A = \pi^2 E/(L_e/K)^2$$

A little consideration will show that the crippling stress will be high, when the slenderness ratio is small. We know that the crippling stress for a column cannot be more than the crushing stress of the column material. It is thus obvious that the Euler's formula will give the value of crippling stress of the column (equal to the crushing stress of the column material) corresponding to the slenderness ratio. Now consider a mild steel column. We know that the crushing stress for the mild steel is 320 MPa or 320 N/mm² and Young's modulus for the mild steel is 200 GPa or 200×10^3 N/mm². Now equating the crippling stress to the crushing stress,

$$320 = \pi^2 E/(L_e/K)^2 = \pi^2 \times (200 \times 10^3)/(L_e/K)^2$$

CHAPTER-5

Shear Force and Bending Moment

Types of beam and load

Beam

A structural member which is acted upon by a system of external loads at right angles to its axis is known as beam.

Types of Beam

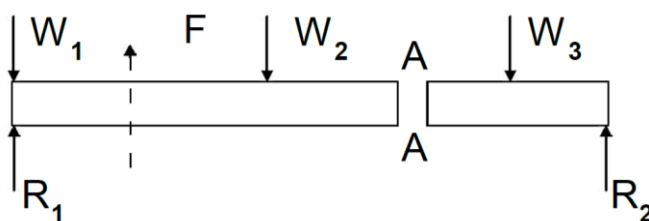
1. Cantilever beam
2. Simply supported beam
3. Over hanging beam
4. Rigidity fixed or built in beams
5. Continuous beam

Types of load

1. Concentrated or point load
2. Uniformly distributed load
3. Uniformly varying load

Shear force

The shearing force at any section of beam represents the tendency for the portion of beam to one side of the section to slide or shear laterally relative to the other portion.



The resultant of the loads and reactions to the left of A is vertically upwards and the since the whole became is in equilibrium, the resultant of the forces to the right of AA must also be F acting downward. F is called the shearing force.

Definition

The shearing force at any section of a beam is the algebraic sum of the lateral component of the forces on either side of the section. Shearing force will be considered positive when the resultant of the forces to the left is upwards or to the right in downward.

A shear force diagram is one which shows the variation of shearing force along the length of the beam.

Concepts of Bending Moment

In a small manner it can be argued that if the moment about the section AA of the forces to the left is M clockwise then the moment of the forces to the right of AA must be anticlockwise. M is called the bending moment.

Definition

The algebraic sum of the moments about the section of all the forces acting on other side of the section. Bending moment will be considered positive when the moment on the left of section is clockwise and on the right portion anticlockwise. This is referred as sagging the beam because concave upwards. Negative B.M is termed as hogging. A BMD is one which shows the variation of bending moment along the length of the beam.

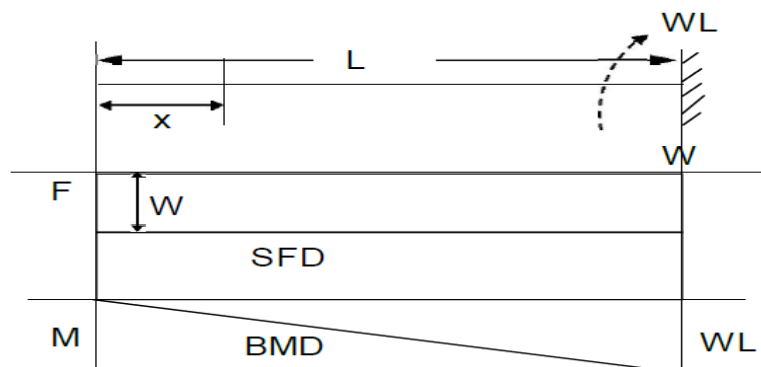
Shear force and bending moment diagram and its silent features

- i. Illustration in cantilever beam
- ii. Illustration in simply supported beam
- iii. Illustration in overhang beam Carrying point load and u.d.L.

Concentrated loads

Example -1

A cantilever of length L carries a concentrated load W at its free end, draw the SF & BM diagram.



Solution

At a section a distance x from the free end, consider the forces to the left.

Then $F = -W$, and is constant along the whole beam for all values of x . Taking moments about the section given $M = -Wx$

At $x = 0$, $M = 0$, At $x = L$, $M = -WL$

At end from equilibrium condition the fixing moment is WL and reactions W .

BENDING MOMENT & SHEAR FORCE

When any structure is loaded, stresses are induced in the various parts of the structure and in order to calculate the stresses, where the structure is supported at a number of points, the bending moments and shearing forces acting must also be calculated.

Definitions

Beam - Beam is structural member which is acted upon by a system of external loads at right angles to the axis.

Bending - Bending implies deformation of a bar produced by loads perpendicular to its axis as well as force couples acting in a plane passing through the axis of the bar.

Plane bending - If the plane of loading passes through one of the principal centroidal axes of the cross section of the beam, the bending is said to be plane.

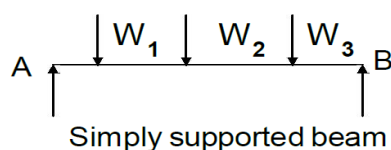
Point load - A point load or concentrated load is one which is considered to act at a point. Distributed load - A distributed load is one which is distributed or spread in some manner over the length of the beam. If the spread is uniform, it is said to be uniformly distributed load. If the spread is not at uniform rate; it is said to be non-uniformly distributed load.

CLASSIFICATION OF BEAMS

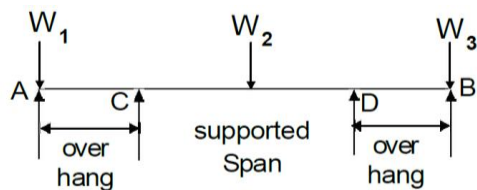
1. Cantilever – A cantilever is a beam whose one end is fixed and the other end free. Figure shows a cantilever with a rigidity fixed into its support and the other end B free. The length between A & B is known as the length of cantilever.



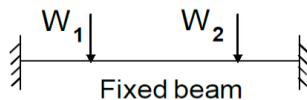
2. Simply supported beam – A simply supported beam is one whose ends freely rest on walls or columns or knife edges.



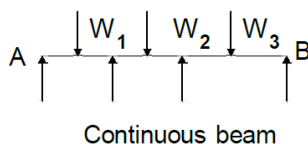
3. Over hanging beam – An overhanging beam is one in which the supports are not situated at the ends i.e. one or both the ends project beyond the supports. C & D are two supports and both the ends A and B of the beam are overhanging beyond the supports C & D respectively.



Fixed beam – A fixed beam is one whose both ends are rigidly fixed or built in into its supporting walls or columns.



Continuous beam – A continuous beam is one which has more than two supports. The supports at the extreme left and right are called the end supports and all the other supports, except the extreme, are called intermediate supports.



SHEAR FORCE

In general, if we have to calculate the shear force at a section the following procedure may be adopted.

- (i) Consider the left or the right part of the section.
- (ii) Add the forces normal to the member on one of the parts.

CANTILEVER

- (i) Cantilever of length L carrying a concentrated load W at the free end.

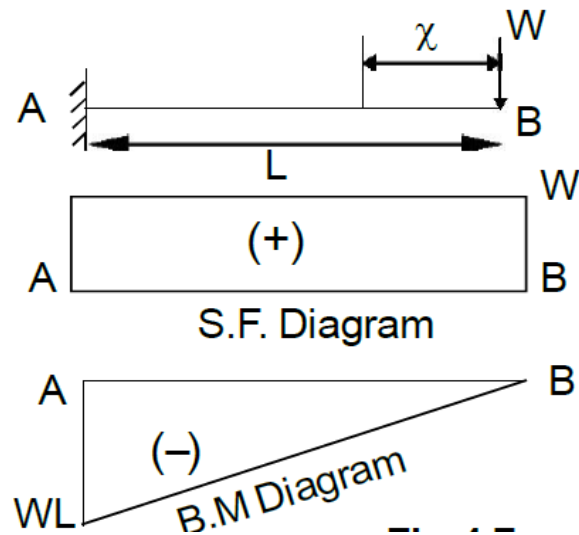


Fig shows a cantilever AB fixed at A and free at B and Carrying the load W at the free end B. Consider a section x at a distance of x from the free end. Hence, we find that the S.F. is constant at all sections of the member between A & B. But the B.M at any section is proportional to the distance of the section from the free end.

CHAPTER-6

Slope and Deflection

SLOPE OF A BEAM:

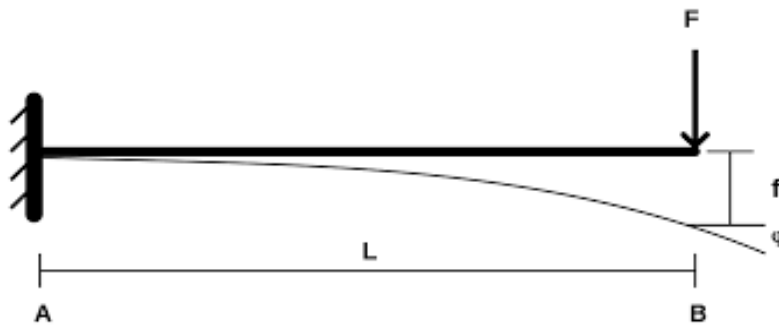
✓ slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.

✓ slope of that deflection is the angle between the initial position and the deflected position.

DEFLECTION OF A BEAM:

✓ The deflection at any point on the axis of the beam is the distance between its position before and after loading.

✓ When a structural is loaded may it be Beam or Slab, due the effect of loads acting upon it bends from its initial position that is before the load was applied. It means the beam is deflected from its original position it is called as Deflection



METHODS FOR FINDING THE SLOPE AND DEFLECTION OF BEAMS:

- Double integration method
- Moment area method
- Macaulay's method
- Conjugate beam method
- Strain energy method

DOUBLE INTEGRATION METHOD:

✓ The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

✓ This method entails obtaining the deflection of a beam by integrating the differential equation of the elastic curve of a beam twice and using boundary conditions to determine the constants of integration.

✓ The first integration yields the slope, and the second integration gives the deflection.

CONJUGATE BEAM:

✓ Conjugate beam is defined as the imaginary beam with the same dimensions (length) as that of the original beam but load at any point on the conjugate beam is equal to the bending moment at that point divided by EI .

✓ Slope on real beam = Shear on conjugate beam

✓ Deflection on real beam = Moment on conjugate beam

PROPERTIES OF CONJUGATE BEAM:

✓ The length of a conjugate beam is always equal to the length of the actual beam.

✓ The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.

✓ A simple support for the real beam remains simple support for the conjugate beam.

✓ A fixed end for the real beam becomes free end for the conjugate beam.

✓ The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.

✓ The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED WITH A UNIFORMLY DISTRIBUTED LOAD:

✓ A simply supported beam AB of length L and carrying a uniformly distributed load of w per unit length over the entire length is shown in fig.

✓ The reactions at A and B will be equal.

✓ Also, the maximum deflection will be at the centre of the beam.

✓ Each vertical reaction = $(w \times L)/2$

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED BEAM WITH CENTRAL POINT LOAD:

✓ A simply supported beam AB of length L carrying a point load W at the centre C.

✓ The B.M at A and B is zero and at the centre B.M will be $WL/4$.

✓ Now the conjugate beam AB can be constructed.

20

✓ The load on the conjugate beam will be obtained by dividing the B.M at that point by EI .

✓ The shape of the loading on the conjugate beam will be same as of B.M diagram.

✓ The ordinate of loading on conjugate beam will be equal to $M/EI = WL/4EI$.

CANTILEVER BEAM WITH A UDL:

- A cantilever beam AB of length L fixed at the point A and free at the point B and carrying a UDL of w per unit length over the whole length.
- Consider a section X, at a distance x from the fixed end A.
- The bending moment at this section is given by,

$$M_x = - w (L - x) (L - x) / 2$$

CHAPTER-7

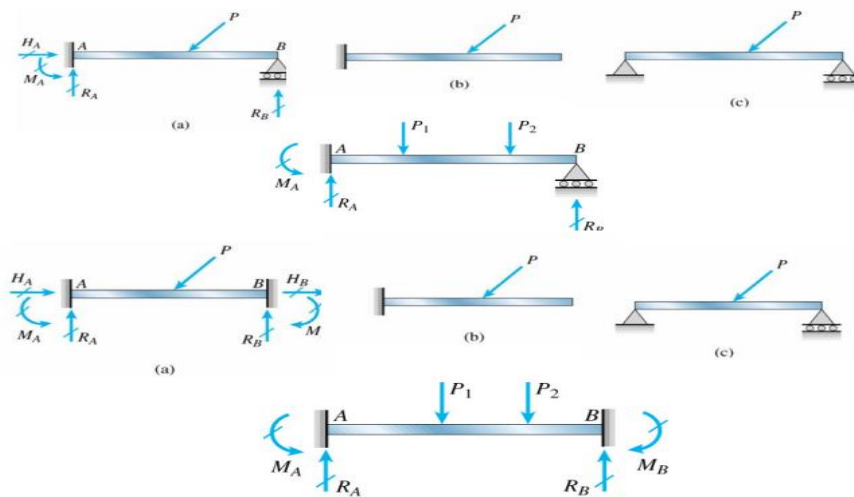
Indeterminate Beams

Introduction

In this chapter we will analyze the beam in which the number of reactions exceeds the number of independent equations of equilibrium. Integration of the differential equation, method of superposition, compatibility equation (consistence of deformation).

Types of Statically Indeterminate Beams

The number of reactions in excess of the number of equilibrium equations is called the degree of static indeterminacy.



Basic Indeterminate Beams

The most common indeterminate beam is when there is just one redundant support. For example, the simply supported beam at the left has a redundant support at B (could also state A or C is redundant, but it is generally easier to solve if the middle support is considered redundant).

The solution method is to release the redundant support and find the deflection at the support location. This can be done using beam deflection equations in handbooks (see Beam Equations Appendix) or basic integration of the moment or load-deflection equation.

Next, without the loads, place the unknown redundant reaction force on the beam and find the deflection at the support location in terms of the unknown reaction.

Second Order Indeterminate Beams

If there are two redundant supports, then the beam is indeterminate to the second degree. If there are three redundant supports, then it is a third degree indeterminate beam (and so forth).

For example, the beam at the left could be considered a cantilever beam with two redundant supports. This is an indeterminate beam to the second degree. To solve the problem, release two redundant supports. It does not matter which supports are released, but there are generally better ones than others. The key is to have a basic beam that is listed in the appendix with beam equations.

The beam without the redundant supports can be solved for the deflection at each support location. Usually, beam equation tables are used, similar to the ones listed in the Beam Equation appendix. But integration method could also be used.

Integrating Load-Deflection Equation to solve Indeterminate Structures

In addition to using superposition to solve for redundant supports, the basic load-deflection equation can be integrated to find the deflection. This method is the same technique used to determine the deflection equation for statically determinate beams.

This method is tedious due to the large number of boundary conditions required. However, at times it can be useful when a deflection equation is needed, instead of the reaction forces or deflection at a given location.

For each span or section, four boundary conditions are needed. They can be deflection, v , slope, dv/dx or θ , shear, V , or moment, M . Each load or support change will require a new span.

Analysis of Indeterminate Beams

The analyses of indeterminate beams and frames follow the general procedure described previously. First, the primary structures and the redundant unknowns are selected, then the compatibility equations are formulated, depending on the number of the unknowns, and solved. There are several methods of computation of flexibility coefficients when analyzing indeterminate beams and frames. These methods include the use of the Mohr integral, deflection tables, and the graph multiplication method. These methods are illustrated in the solved example problems in this section.

Use of Beam-Deflection Tables for Computation of Flexibility Coefficients

This is the easiest method of computation of flexibility coefficients. It involves obtaining the constants from tabulated deflections based on the types of supports and loading configurations,

Statical determinacy

Descriptively, a statically determinate structure can be defined as a structure where, if it is possible to find internal actions in equilibrium with external loads, those internal actions are unique. The structure has no possible states of self-stress, i.e. internal forces in equilibrium with zero external loads are not possible. Statical indeterminacy, however, is the existence of a non-trivial (non-zero) solution to the homogeneous system of equilibrium equations. It indicates the possibility of self-stress (stress in the absence of an external load) that may be induced by mechanical or thermal action.

CHAPTER-8

Trusses and Frames

Introduction

A truss is a structure composed of rod members arranged to form one or more triangles. The joints are pinned (do not transmit moments) so that the members must be triangulated.

A frame, on the other hand, is a structure that consists of arbitrarily oriented beam members which are connected rigidly or by pins at joints. The members support bending as well as axial loads.

Truss is commonly used in bridges, roofs, and towers. So, it can be said that a truss allows us to create strong and durable structures while using materials efficiently and cost-effectively. frame are structures that consist of a horizontal member, called a beam, and a vertical member called a column of the frame. The vertical member of a frame provides lateral stiffness to the frame. So, we can say that the truss and frame are the structures that are used to resist the horizontal load as well as the vertical load.

A statically determinate truss is the truss which have equal number of support reactions as the available equilibrium equations. Such type of truss requires only equilibrium equations for their analysis.

A statically indeterminate truss is the truss that has more support reactions than the available equilibrium equations. Such type of truss requires compatibility equations along with equilibrium equations for their analysis.

Assumptions in Truss Analysis

As we know that assumptions are the basis of the analysis of something. An analysis of a truss can be carried out to determine the forces in any truss members, which is a complex process in large truss and frames. So, we need to simplify this complex process. So we apply some assumptions in the truss analysis.

Here are some assumptions in the truss analysis which significantly simplify their analysis process, as all these assumptions can't be satisfied in a real truss structure. Therefore, these assumptions are designed for an ideal truss.

1. All members of a truss are connected at their ends only.
2. Frictionless pins connect all members at their joints.
3. Every load must be applied only at joints.
4. Self-weight of all the members is neglected.
5. All members of a truss must be straight.
6. A pinned connection represents all the joints in the structure, i.e., all the members can rotate freely at the joints.
7. The members of a truss are rigidly connected by using a plate known as a gusset plate.
8. Loads are never applied in the middle of the member because all the joints are pinned, and members cannot carry bending Moment as they can carry only tensile or compressive loads.
9. Each joint of a truss must be in equilibrium. Therefore, the forces acting at each joint must be equal and opposite.

Types of Truss Based on Determinacy

If the number of unknown forces (reactions and internal forces) of a given structure is equal to equilibrium equations, the structure is known as a determinate structure. Determinacy is of two types; one is Internal Indeterminacy other is External Indeterminacy. Based on the determinacy, truss can be classified into two types, described below.

1. Statically Determinate Truss
2. Statically Indeterminate Truss

Statically Determinate Truss

In the statically determinate truss, all the support reactions and internal forces acting in the members of a truss are calculated by only equilibrium equations. Therefore, to design a truss, it is necessary to find the force in all the members. The main purpose of finding unknown forces is to check whether the members can resist the effect of the applied loads without fail or not. Therefore, for a planar truss to be statically determinate, the sum of the number of members and the number of support reactions must be less than twice the number of joints.

Statically Indeterminate Truss

In the statically indeterminate truss, all the support reactions and internal forces acting in the members of a truss are calculated with the help of compatibility equations and the available equilibrium equations. In planar structures, there are only three equations of equilibrium. In such structures, there is at least one more unknown force than the available equilibrium equations. Statically Indeterminate Structures can be analyzed by the Force method or Displacement method. In the Force method of analysis, redundant forces are treated as unknowns. While in the Displacement method, deformations are treated as unknown.

Rules to Find Zero Force Members in Truss

Finding the zero force members in both truss and frame is not required. It is only required in the case of truss. Some truss members don't carry any load; they are known as Zero Force Members. The purpose of Zero Force Members is to provide stability to the structure and to avoid failure because of unexpected loads. There are different ways to find these zero-force members. Some of them are explained here:

In a pin joint, if the number of members is three and two are in the same line, the force in the third member is zero. (No load, No reaction at the joint)

At the pin joint, if the number of members is 2 and they are in different lines, then the force on both members is zero. (No load, No reaction shall be present at that joint).