KIIT POLYTECHNIC

LECTURE NOTES

ON

ENGG. MATH -I

PART-1

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CONTENTS

CHAPTER -1

DETERMINANTS

Determinant will be used for solving the system of linear equations.

$like\ 2x + y = 0\ and\ x - y = 3$

Determinant of order 2

A determinant of order 2 can be written in the form of $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$which\ is\ defined\ as \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21} \qquad OR \qquad \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - b_1a_2$

Note:

1.A determinant of order 2 contains two rows and two columns.

2.a_{11}, a_{12} *are the elements of* R_1 *and* a_{21}, a_{22} *are the elements of* R_2.

3.a_{11}, a_{21} *are the elements of* C_1 *and* a_{12}, a_{22} *are the elements of* C_2.

3.a_{11}, a_{22} *are the elements of principal diagonal and* a_{12}, a_{21} *are the elements of* sec*ondary diagonal*.

4.*It contains* $2 \times 2 = 4$ *elements.*

Determinant of order 3

A determinant of order 3 can be written in the form of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$Which\ is\ defined\ as \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Note :

1.A determinant of order 3 can be expanded by using 6 ways (any one row or any one column).

2.A determinant of order 3 can be expanded by using the respective sign of the element in different rows or columns.i.e $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

3.*The sign of all the elements in a 2nd order* determinant *is* $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$

General element

If an element occurring in the ith row and jth column of a determinant *then it is called* (i, j)*th element. It is denoted by* a_{ij}. $(i \rightarrow ith\ row,\ j \rightarrow jth\ column)$

Ex: Construct a determinant of order 3×3 by using general element a_{ij}

$$Let\ A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor of an element a_{ij}

The minor *of an element* a_{ij} *is defined as the value of a* determinant *will be obtained after deleting all the elements in the ith row and jth column. It is denoted by* M_{ij}.

Cofactor of an element a_{ij}

The cofactor of an element a_{ij} *is denoted by* C_{ij}, *which is defined as* $C_{ij} = (-1)^{i+j} M_{ij}$.

Ex: *Find the Minor and Cofactor of* a_{11}, a_{12} *and* a_{13} *in* $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$Ans: Minor\ of\ a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$Minor\ of\ a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$Minor\ of\ a_{13} = M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$cofactor\ of\ a_{11} = C_{11} = (-1)^{1+1} M_{11} = M_{11}$$

$$cofactor\ of\ a_{12} = C_{12} = (-1)^{1+2} M_{12} = -M_{12}$$

$$cofactor\ of\ a_{13} = C_{13} = (-1)^{1+3} M_{13} = M_{13}$$

Similarly find the minor *and cofactor of other elements.*

Properties of determinants

$P-2$: If any two adjacent rows or columns of a determinant are interchanged then the numerical value is same but the sign is changed.

$$Ex: A=\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37$$

$$Let\ B=\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & -2 \\ -2 & 1 & -3 \end{vmatrix}=37$$

$P-3$: If any two rows or columns a determinant identical or same then the value of the determinant vanishes or zero.

$$Ex: A=\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

Note : *I*f all the elements of a row or column of a determinant are zero then the value of the determinant is zero.

$$\text{Ex:}A=\begin{vmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

Note : *I*f two rows or columns of a determinant are proportional then the value of the determinant is zero.

$$Ex: A=\begin{vmatrix} -4 & 2 & -6 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

$P-4$: If each element of a row or column of a determinant be multiplied by a constant k then the determinant is also multiplied by the same constant k.

$$Ex: A=\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37$$

$$Let\ B=\begin{vmatrix} 2k & 3k & -2k \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37k=k\left(A\right)$$

$P-5$: If each element of a row or column of a determinant be the sum or difference of two or more terms then the determinant can be expressed as the sum or difference of two or more determinant.

$$Ex: A=\begin{vmatrix} a_1+\alpha_1 & a_2+\alpha_2 & a_3+\alpha_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}+\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$P-6$: If each element of a row or column of a determinant be increased or decreased by a constant multiple of the corresponding elements of another row or column then the value of the determinant remains unchanged.

$$Ex: A=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$B=\begin{vmatrix} a_1+kc_1 & a_2+kc_2 & a_3+kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}+\begin{vmatrix} kc_1 & kc_2 & kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=A+k\begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=A+0=A$$

$$Note: If\ A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} then |A|=a_{11}c_{11}+a_{12}c_{12}+a_{13}c_{13}\,(\text{using row})\ \ or\ \ |A|=a_{11}c_{11}+a_{21}c_{21}+a_{31}c_{31}\,(\text{using column})$$

$$Note: If\ A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\ then\ a_{11}c_{21}+a_{12}c_{22}+a_{13}c_{23}=0$$

Method for evaluating a determinant

We should always try to make maximum number of possible zeros in any one row or column.
It is possible by using

(1) properties of determinant.

(2) Row to Row addition or subtraction.

(3) column to column addition or subtraction.

(4) Any other method

Cramer's Rule :

Solution for two unknowns: The solution of two variables x and y of two linear equations $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ then $x=\frac{D_1}{D}$ and $y=\frac{D_2}{D}$.

Solution for three unknowns:

The solution of two variables x and y of two linear equations $a_1x + b_1y + c_1z = d_1$,$a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$ then $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$.

Consistent: A system of linear equation is said to be consistent if it gives a solution.

Ex:$2x + 3y = -1$ and $x - 3y = 2$

Inconsistent: A system of linear equation is said to be inconsistent if it gives no solution.

Ex:$2x + 3y = -1$ and $4x + 6y = 2$

Rules for consistency and inconsistency:(For Three unknowns)

We have $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$.

1.If $D \neq 0$ then the system of equations are consistent and gives unique solution.

2. If $D = 0, D_1 = 0, D_2 = 0$ and $D_3 = 0$then the system of equations are consistent and gives Infinite number of solutions.

3. If $D = 0$, and at least one of D_1, D_2 and D_3 is nonzero then the system of equations are inconsistent and gives no solutions.

Note: Same rule we can apply for two unknowns

Ex: Evaluate $\begin{vmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{vmatrix}$

Ex: Evaluate (a) $\begin{vmatrix} 2 & 33 \\ 4 & -25 \end{vmatrix}$,(b) $\begin{vmatrix} 1 & w \\ w^2 & 1 \end{vmatrix}$,(c) $\begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$,(d) $\begin{vmatrix} \omega^6 & \omega^4 \\ -\omega^6 & \omega^5 \end{vmatrix}$, $\omega^3 = 1$

Ex: Find the maximum value of $\begin{vmatrix} 1 + sin\theta & cos\theta \\ -cos\theta & sin\theta \end{vmatrix}$

Ex: Solve $\begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} = 0$

Ans: $\begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} = 0$

$\Rightarrow x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Ex: Solve $\begin{vmatrix} x + 1 & -2 \\ 1 & 3 \end{vmatrix} = 3$

Ex: Evaluate (a) $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 0 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{vmatrix}$,(c) $\begin{vmatrix} -6 & 0 & 0 \\ 3 & -5 & 7 \\ 2 & 8 & 11 \end{vmatrix}$.

Ex: Prove that $\begin{vmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} = 0$

Ans:

$$\begin{vmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} (\textit{After adding the elements of } R_1, R_2, R_3 \textit{ then replace } R_1)$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} \left(\because 1+\omega+\omega^2 = 0, \textit{ you know in complex number}\right)$$

$= 0$

Ex: Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} = 1\begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix} - 0 + 0$$

$$= (b-a)(c^2-a^2) - (c-a)(b^2-a^2) = (b-a)(c-a)(c+a) - (c-a)(b-a)(b+a)$$

$$= (b-a)(c-a)\{(c+a)-(b+a)\} = (b-a)(c-a)(c-b) = (a-b)(b-c)(c-a)$$

(Taking common -sign from (b-a) and (c-b) then you will get the answer)

Ex: Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Ans:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$Multiply\, a\, in\, C_1, b\, in\, C_2\, , c\, in\, C_3\, and\, divide\, abc$

$$= \frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & cba & abc \end{vmatrix} taking\, common\, abc\;\, from\, C_3$$

$$= \frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

$$= \begin{vmatrix} a^2 & b^2 - a^2 & c^2 - a^2 \\ a^3 & b^3 - a^3 & c^3 - a^3 \\ 1 & 0 & 0 \end{vmatrix} = 1\begin{vmatrix} b^2 - a^2 & c^2 - a^2 \\ b^3 - a^3 & c^3 - a^3 \end{vmatrix} - 0 + 0 \;\; \text{(expand it by using row3)}$$

$$= (b^2 - a^2)(c^3 - a^3) - (c^2 - a^2)(b^3 - a^3)$$

$$= (b-a)(b+a)(c-a)(c^2 + ca + a^2) - (c-a)(c+a)(b-a)(b^2 + ab + a^2)$$

$$= (b-a)(c-a)\left\{(b+a)(c^2 + ca + a^2) - (c+a)(b^2 + ab + a^2)\right\}$$

$$= (b-a)(c-a)(bc^2 + abc + a^2b + ac^2 + a^2c + a^3 - cb^2 - abc - ca^2 - ab^2 - a^2b - a^3)$$

$$= (b-a)(c-a)(bc^2 + ac^2 - cb^2 - ab^2) = (b-a)(c-a)\left\{bc(c-b) + a(c^2 - b^2)\right\}$$

$$= (b-a)(c-a)(c-b)(bc + ac + ab) = (a-b)(c-a)(b-c)(bc + ac + ab)$$

Ex: solve $\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$

Ans:

$$\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} x+3 & x+3 & x+3 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

Taking common x+3 from row1(R_1)

$$\Rightarrow (x+3)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

either $x+3=0$ or $\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$

since $x+3=0 \Rightarrow x=-3$

$$Again \begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & x \end{vmatrix} = 0 \Rightarrow 1\begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} - 0 + 0 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

$Hence\ x=0\ and\ x=-3$

Ex: Solve by Cramer's rule $2x+3y=-1$ and $x-2y=3$

Ans: Given equations are $2x+3y=-1$ and $x-2y=3$

Here $a_1=2, b_1=3, c_1=-1, a_2=1, b_2=-2, c_2=3$

Let $D=\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$

$D_1=\begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} = -7$

$D_2 = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7$

$x = \frac{D_1}{D} = \frac{-7}{-7} = 1 \; and \; y = \frac{D_2}{D} = \frac{7}{-7} = -1$

Ex: Solve by cramer's rule $2x + y + 2z = 2, 3x + 2y + z = 2 \; and - x + y + 3z = 6$

Ans:

Let $D = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} = 2(5) - 1(10) + 2(5) = 10$

$D_1 = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 6 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 6 & 1 \end{vmatrix} = 2(5) - 1(0) + 2(-10) = -10$

$D_2 = \begin{vmatrix} 2 & 2 & 2 \\ 3 & 2 & 1 \\ -1 & 6 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} = 2(0) - 2(10) + 2(20) = 20$

$D_3 = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ -1 & 1 & 6 \end{vmatrix} = 2\begin{vmatrix} 2 & 2 \\ 1 & 6 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} = 2(10) - 1(20) + 2(5) = 10$

$x = \frac{D_1}{D} = \frac{-10}{10} = -1, y = x = \frac{D_2}{D} = \frac{20}{10} = 2, z = x = \frac{D_3}{D} = \frac{10}{10} = 1$

Questions carrying 2 marks

1. Find the value of $\begin{vmatrix} 5 & -2 & 1 \\ 3 & 0 & 2 \\ 8 & 1 & 3 \end{vmatrix}$

Ans :7

2. Find the value of $\begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$

Ans : 1

3. If $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ x & 0 & 0 \end{vmatrix}$, then find the value of x

Ans : x = -1

4. Find the minimum value of $\begin{vmatrix} sinx & cosx \\ -cosx & 1 + sinx \end{vmatrix}$ where $x \in R$

Ans: 0

5. Find the maximum value of $\begin{vmatrix} sin^2x & sinx.cosx \\ -cosx & sinx \end{vmatrix}$ where $x \in R$

Ans : 1

6. If ω is the cube root of unity, find the value of the determinan t $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$

Ans : 0

7. Find the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

Ans : 0

8. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$, then find x

Ans : x=a

9. Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$

Ans : x=8

10. Solve $\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix}$=$\begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$

Ans : ± 4

<u>Questions carrying 5 marks</u>

1. Solve $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

Ans: x=0 or x= -3

2. Solve $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+c \end{vmatrix} = 0$

Ans: x= $-(a+b+c)$ or $\sqrt{a^2+b^2+c^2-ab-bc-ca}$

3. Solve $\begin{vmatrix} x & a & a \\ m & m & m \\ b & x & b \end{vmatrix} = 0$

Ans: $x = a$ or $x = b$

4. Solve $\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$ where ω is cube root of unity.

Ans: x=0

5. Prove without expanding that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

6. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

7. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

8. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

9. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

10. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

11. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (b-c)(c-a)(a-b)$

12. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

13. Solve by Cramer’s rule:

(i) $4x - y = 9, 5x + 2y = 8$

Ans: $x = 2, y = -1$

(ii) $2x - y = 2, 3x + y = 13$

Ans: $x = 3, y = 4$

(iii) $x - y + z = 1, 2x + 3y - 5z = 7, 3x - 4y - 2z = -1$

Ans : $x = \frac{35}{16}, y = \frac{53}{32}, z = \frac{15}{32}$

(iv) $x + y + z = 3, 2x + 3y + 4z = 9, x + 2y - 4z = -1$

Ans: $x = 1, y = 1, z = 1$

CHAPTER-2

MATRICES

A rectangular array of mn numbers with m horizontal lines (rows) and n vertical lines (columns) is known as a matrix of order $m \times n$.

Ex: A= $\begin{bmatrix} 2 & 3 \\ a & b \\ -1 & 3 \end{bmatrix}$ it is a matrix of order 3×2 and contains 6 elements. i.e $3 \times 2 = 6$

General element of a matrix: If an element occurs in the ith row and jth column of a matrix then it is called (i,j)th element of the matrix.It is denoted by a_{ij}.

A matrix of order $m \times n$, $generally\ written\ as\ A = [a_{ij}]_{m\times n}$

Let us consider a matrix of order 2×3 by general element.

A= $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

Ex: construct a matrix of order 3×4 , whose elements are in the form of $a_{ij} = 2i - j$.

Types of a matrices:

Row matrix: A matrix having only one row is known as row matrix or Row vector.

A= $[2 \quad 3 \quad -1]$

Column matrix: A matrix having only one column is known as column matrix.

A= $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

Zero matrix or Null matrix: If all the elements of a matrix are zero then it is called null matrix.

Ex: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Square matrix: If the number of rows and columns of a matrix are equal then it is called square matrix.i.e m=n

Ex: A = $\begin{bmatrix} 2 & -1 \\ 9 & 3 \end{bmatrix}$

Rectangular matrix: If the number of rows and columns of a matrix are not equal then it is called rectangular matrix. i.e $m \neq n$.

Ex:A= $\begin{bmatrix} -1 & 2 & 4 \\ 2 & 3 & -9 \end{bmatrix}$

Diagonal elements: The elements $a_{ij}, i = j$ in a square matrix A are called diagonal elements.

Ex: A = $\begin{bmatrix} 2 & 3 & 0 \\ -2 & -3 & 1 \\ 4 & -6 & 4 \end{bmatrix}$ $here\ 2, -3\ and\ 4\ are\ diagonal\ elements.$

Diagonal matrix: A square matrix A is said to be a diagonal matrix if all the diagonal elements are present but non diagonal elements are zero, i.e $a_{ij} = 0, for\ all\ i \neq j.$

Ex: A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Scalar matrix: A square matrix A is said to be a scalar matrix if all the diagonal elements are equal but non diagonal elements are zero.

Ex: A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Unit matrix or Identity matrix: A square matrix is said to be an identity matrix if all the diagonal elements are unity(1) but non diagonal elements are zero. It is denoted by $I_n\ or\ I.$

i.e $a_{ij} = 0, for\ all\ i \neq j$ and $a_{ij} = 1, for\ all\ i = j.$

Ex: $I_3\ or\ I$= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ex: $I_2\ or\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Upper triangular matrix: A square matrix is said to be an upper triangular matrix if all the elements below the main diagonal are zero. i.e $a_{ij} = 0, for\ all\ i > j.$

Ex: A = $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & -2 \end{bmatrix}$

Lower triangular matrix: A square matrix is said to be a lower triangular matrix if all the elements above the main diagonal are zero. $i.e\ a_{ij} = 0, for\ all\ i < j$

Ex: A = $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & 4 & -2 \end{bmatrix}$

Singular matrix: A square matrix A is said to be a singular matrix if det.A=0 or $|A| = 0.$

Ex: let A=$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 0 \; hence\; A\; is\; a\; singular\; matrix.$$

Non singular matrix: A square matrix A is said to be a nonsingular matrix if or det $A \neq 0$ or $|A| \neq 0$.

Ex: let A=$\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} = 6 - 10 = -4 \; hence\; A\; is\; a\; nonsingular\; matrix.$$

Comparable Matrix: Two matrices A and B are said to be comparable if they have same order.

Ex: $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ here A and B are comparable.

Equal Matrix: Two matrices A and B are said to be equal (i.e A=B) if they have same order and their corresponding elements are equal.

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 & c_2 \\ d_2 & e_2 & f_2 \end{bmatrix}$ So A = B iff $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2$

Ex: Find the value of x and y if $\begin{bmatrix} 2x - y & -1 \\ 2 & x + y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 6 \end{bmatrix}$

Ans: Here 2x-y=3 and x+y=6 solve the above two equations x=3 and y=3.

Scalar multiplication of a matrix: If A be a matrix and k be a scalar then the scalar multiplication of a matrix kA will be obtained multiplying each element of A by k.$i.e\; A = [a_{ij}] \Rightarrow kA = [ka_{ij}]$

Ex: let A=$\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 10 & 12 \end{bmatrix}$

Matrix addition:

If A and B are two matrices of order $m \times n$ then addition A+B will be obtained by adding the corresponding elements of A and B. The order of A+B is $m \times n$.

Ex: Find A+B if $A = \begin{bmatrix} 2 & -1 & 3 \\ -3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 & 3 \\ 3 & -6 & 1 \end{bmatrix}$

Ans: $A + B = \begin{bmatrix} 2 & -1 & 3 \\ -3 & -2 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 4 & 3 \\ 3 & -6 & 1 \end{bmatrix} = \begin{bmatrix} 2+5 & -1+4 & 3+3 \\ -3+3 & -2-6 & 4+1 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 6 \\ 0 & -8 & 5 \end{bmatrix}$

Similarly $A - B = \begin{bmatrix} 2 & -1 & 3 \\ -3 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 4 & 3 \\ 3 & -6 & 1 \end{bmatrix} = \begin{bmatrix} 2-5 & -1-4 & 3-3 \\ -3-3 & -2+6 & 4-1 \end{bmatrix} = \begin{bmatrix} -3 & -5 & 0 \\ -6 & 4 & 3 \end{bmatrix}$

Note:

1) $A + B = B + A$

2) $A - B \neq B - A$

3) $k(A+B)=kA+kB$

4)$(\alpha+\beta)A=\alpha A+\beta A$

5)$\alpha\beta A=\alpha(\beta A)=\beta(\alpha A)$

6) A+(B+C)=(A+B)+C

7)**Existence of Additive Identity**: A+O=O+A=A(Here O be a Null matrix)

8) **Existence of Additive Inverse**: A+(-A)=O=(-A)+A

9)**Cancellation law**: A+B=A+C⇒ $B=C(left\ cancellation\ law)$

$$B+A=C+A\Rightarrow B=C(right\ cancellation\ law)$$

Transpose of a matrix:

If A be a matrix of order $m\times n$ then the transpose of the matrix will be obtained by interchanging the rows and columns. It is denoted by A^T and the order of $A^T\ is\ n\times m$.

Ex: let $A=\begin{bmatrix}2 & -3 & 4\\ 3 & 2 & 5\end{bmatrix}$ $A^T=\begin{bmatrix}2 & 3\\ -3 & 2\\ 4 & 5\end{bmatrix}$

Properties:

1)$(A^T)^T$=A

2)$(A+B)^T=A^T+B^T$

3)If A be a matrix and k is a scalar $(kA)^T=k\,A^T$

4)$(AB)^T=B^TA^T$

5) $(ABC)^T=C^TB^TA^T$

Symmetric Matrix:A square matrix A is said to be symmetric matrix if $A^T=A, i.e\ a_{ij}=a_{ji}\ for\ all\ i\ and\ j$

Ex: $=\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}.A=\begin{bmatrix}a & -1\\ -1 & a\end{bmatrix}$

Skew-symmetric matrix:A square matrix A is said to be skew-symmetric matrix if $A^T=-A, i.e\ a_{ij}=-a_{ji}\ for\ all\ i\ and\ j$

Ex: $=\begin{bmatrix}0 & h & -g\\ -h & 0 & f\\ g & -f & 0\end{bmatrix}$.

Properties:

1)$A + A^T$ $is\ a\ symmetric\ matrix\ but\ A - A^T is\ a\ skew - symmetric\ matrix.$

2)AA^T $and\ A^T A\ are\ symmetric\ matrix$

3)Every square matrix A can be expressed as the sum of symmetric and skew-symmetric matrix.

$$i.e\ A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)\ Here\ \frac{1}{2}(A + A^T)\ is\ a\ symmetric\ matrix\ and$$

$$\frac{1}{2}(A - A^T) is\ a\ skew - symmetric\ matrix$$

Note: If A and B are symmetric matrix and AB=BA then AB is a symmetric matrix.

Note:If A is a symmetric matrix then A^n $is\ a\ symmetric\ matrix\ for\ all + ve\ integer.$

Note:A matrix which is both symmetric as well as skew symmetric matrix is a null matrix.

Adjoint of a matrix:

Adjoint of a square matrix A is defined as the transpose of the cofactor matrix A.

It is denoted adj.A.

$$i.e\ adj.A = (cofactor\ matrix\ of\ A)^T = [C_{ij}]^T, Where\ C_{ij}\ is\ cofactor\ of\ a_{ij} in\ A$$

Note: If A be a square matrix of order n then A(adjA)=$|A|I_n$ $, I\ be\ an\ Identity\ matrix$

Note: If A be a non–singular matrix of order n then $|adjA| = |A|^{n-1}$

Note: If A and B are non-singular square matrix of same order then adjAB=adjBadjA

Note: If A is a non-singular matrix then adj(adjA)= $|A|^{n-2}A$

Inverse of a matrix:

A non- singular square matrix A of order n is said to be invertible (i.e A^{-1} exists) if there exist a non singular square matrix B of order n, such that $AB = BA = I, so\ A^{-1} = B\ or\ B^{-1} = A.$

Formula finding A^{-1}:

If A be non singular square matrix then A^{-1} is defined as $A^{-1} = \frac{adj.A}{|A|}$

Note: If A is invertible matrix then $(A^{-1})^{-1} = A$

Note:$(AB)^{-1} = B^{-1}A^{-1}$

Note: If A is invertible square matrix then A^T $is\ invertible\ i.e\ (A^T)^{-1} = (A^{-1})^T$

Note: If A is an invertible square matrix then adj$A^T = (adjA)^T$

Note: Adjoint of a symmetric matrix is also a symmetric matrix.

$$(adjA)^T = adjA$$

Note: If A is a non-singular matrix then $|A^{-1}| = |A|^{-1}$

Matrix Multiplication:

Existence of the product of two matrices: The product of two matrices A and B are said to be exist $(i.e\ AB\ exist)$ if the number of columns in the matrix A is equal to the number of rows in the matrix B.

Product of matrices:

$If\ \ A = [a_{ij}]_{m\times n}\ and\ B = [b_{jk}]_{n\times p}\ then\ AB\ is\ a\ matrix\ of\ order\ m\times p$, Which is defined as $AB = [c_{ik}]_{m\times p}, where\ \ c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + ------ + a_{in}b_{nk} = \sum_{j=1}^{n} a_{ij}b_{jk}$

Thus $(i,k)th\ element\ of\ AB = Sum\ of\ the\ product\ of\ the\ corresponding\ elementsof$

$ith\ row\ of\ A\ and\ kth\ column\ of\ B$

Properties of product of two matrices:

1.Matrix multiplication is not commutative in general $i.e\ AB \neq BA$

2.Matrix multiplication is associative $i.e\ A(BC) = (AB)C$

3.Matrix multiplication is distributive over addition $i.e\ A(B + C) = AB + AC$

4.$A.A = A^2$

5.$A.I = IA = A$

6.$I.I.I ------I(ntimes) = I$

Solution for system of linear equations:(Solve by matrix method)

Let us consider three linear equations

$a_1x + b_1y + c_1z = d_1, a_2x + b_2y + c_2z = d_2\ and\ a_3x + b_3y + c_3z = d_3$

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Now the above equation can be written as $AX = B \Rightarrow X = A^{-1}B$

Which is known as matrix method.

Assignment-1

1. Write the order of the matrix (a)$\begin{pmatrix} 1 & 2 \\ -1 & -3 \\ 5 & 0 \end{pmatrix}$, $(b)\begin{pmatrix} a \\ b \\ x \end{pmatrix}$, $(c)\begin{pmatrix} 3 & 2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

2. Give an example of (a) $3 \times 4\ matrix\ (b)\ 2 \times 1\ matrix$

3. Write the number of elements of a matrix whose order $is\ 4 \times 3$

4. Write the elements of $a_{12}, a_{23,} a_{31}\ and\ a_{22}\ in\ the\ given\ matrix\ \begin{pmatrix} 2 & -1 & 3 \\ 4 & 1 & 5 \\ 0 & -2 & -3 \end{pmatrix}$

5. Construct a matrix of order 2×3 $, whose\ elements\ are\ in\ the\ form\ of$

(a)$a_{ij} = 2i + j$ $,(b)a_{ij} = i - j, (c)a_{ij} = i \times j$ $,(d)a_{ij} = \frac{i}{j}, (e)a_{ij} = 2i - 3j$

6. Give an example of a square matrix .

7. Give an example of an Identity matrix of order 4.

8. Write the diagonal elements of a matrix $\begin{pmatrix} 2 & -1 & 9 \\ -1 & 1 & -7 \\ 1 & -2 & -3 \end{pmatrix}$

9. Write the number of elements of a matrix whose order $is\ p$

10. Construct a matrix of order $3 \times 2, whose\ elements\ are\ in\ the\ form\ of\ a_{ij} = 2i + j$

Assignment-2

1.Find the value of x and y if $\begin{pmatrix} x & 2y \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & 3 \end{pmatrix}$.

2.Find the value of x and y $\begin{pmatrix} 1 & 2 & 3 \\ 2x & -1 & x+y \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \end{pmatrix}$

3.Find the value of x and y $\begin{pmatrix} x-y & 2 \\ 3 & 2x+y \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 3 & 0 \end{pmatrix}$

4.Find the value of x, y and z if $\begin{pmatrix} x+y & y-z \\ 5-t & 7+x \end{pmatrix} = \begin{pmatrix} t-x & z-t \\ z-y & x+z+t \end{pmatrix}$

5.Find $A + B, B + A, A - B\ and\ B - A$ if $A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \\ 5 & 6 \end{pmatrix}\ and\ B = \begin{pmatrix} 2 & 3 \\ -2 & 4 \\ 1 & 2 \end{pmatrix}$

6.Find $X\ if\ X + \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -1 & 4 \end{pmatrix}$

7.Find a matrix which when added to $\begin{pmatrix} 3 & -1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$

8.Find $A + B + C\ if\ A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & -2 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 5 & -2 \end{pmatrix}\ and\ C = \begin{pmatrix} 1 & 0 & 3 \\ 4 & 2 & 1 \end{pmatrix}$

9.Find the value of x and y if $\begin{pmatrix} x & y \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 2y & -x \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 2 & 2 \end{pmatrix}$

10.Find the order of the matrix $\begin{pmatrix} a & b & c \end{pmatrix}$

Assignment-3

1.Find the Transpose of a matrix $\begin{pmatrix} 1 & 2 \\ -1 & -3 \\ 4 & 5 \end{pmatrix}$

2.Find the transpose and order of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & -5 & -1 \\ 3 & 0 & -3 & 1 \end{pmatrix}$

3.Find the minor and co-factor of a determinant $\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix}$

4.Find the minor and co-factor of a determinant $\begin{vmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \\ -2 & 0 & 4 \end{vmatrix}$

5.Find the adjoint of a matrix $\begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$

6.Find the adjoint of a matrix $A, if\ A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$

7.Find adjA if $A = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -1 & 2 \\ 4 & -2 & 1 \end{pmatrix}$

8.Find the cofactor of 2 in the determinant $\begin{vmatrix} -1 & 2 \\ 3 & 0 \end{vmatrix}$

9.Find $2A - 3B\ if\ A = \begin{pmatrix} 2 & 0 \\ 3 & -3 \\ 4 & -1 \end{pmatrix}\ and\ B = \begin{pmatrix} 1 & 0 \\ -1 & -3 \\ 4 & 1 \end{pmatrix}$

10.Find $A - \alpha I\ if\ A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}, Where\ \ I\ be\ an\ Identity\ matrix\ of\ order\ 2\ and\ \alpha \neq 0\ is\ a\ scalar.$

Assignment-4

1.Find Inverse of a Matrix $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$.

2.Find $A^{-1}\ if\ A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

3.Find inverse of a matrix $\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

4.Find $A^{-1}\ if\ A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$.

5. Solve by Matrix Method

(a)$x + 2y = 3 \; and \; 3x + y = 4$ $\quad (b) x - 2y - 4 = 0 \; and - 3x + 5y + 7 = 0$

6. Solve by Matrix Method

(a)$x + 2y - 3z = 4$, $2x + 4y - 5z = 12 \; and \; 3x - y + z = 3$

(b)$x - 2y = 3, 3x + 4y - z = -2 \; and \; 5x - 3z = -1$

7.If A is a matrix of order 3×3 , $and \; |A| = 2 \; then \; find \; A \times adjA, where \; I \; is \; an \; Identity \; matrix.$

8.Find $AB \; if \; A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; and \; B = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$

9.Find $AB \; if \; A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{pmatrix} \; and \; B = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}$

10. Find$AB \; if \; A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix} \; and \; B = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \end{pmatrix}$