FLUID FLOW

1.1 Definition of fluids: - Fluid mechanics that branch of science which deals with the behavior of the fluids (liquids or gases) at rest as well as in motion. Thus, this branch of science deals with the static, kinematics and dynamic aspects of fluids.

Definition of fluid: - Fluid is a substance having particles which readily change their relative motion.

Properties of fluid:-

(1)Density or mass density: - It is the mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho).

Unit: - The unit of mass density in S.I. Unit = kg/m^3

Mathematically, mass density is written as $\rho = \frac{mass\ of\ fluid}{volume\ of\ fluid}$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3

(2)Specific weight or weight density:-

Specific weight or weight density of a fluid is the ratio between the weights of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w or y.

Thus, mathematically,

$$W = \frac{weight\ of\ fluid}{volume\ of\ fluid} = \frac{mass\ of\ fluid \times acceleration\ due\ to\ gravity}{volume\ of\ fluid} = \rho \times g$$

Y or w = ρ × g

We know that 1 liter = $\frac{1}{1000}$ m^3, 1liter = $1000cm^3$

The value of specific weight or weight density (w) for water is y = 9.81 m/ sec^2 × 1000 kg/m^3 =9810 N/m^3 in S.I. units

(3) Specific gravity or relative density (G) or (S):- Specific gravity is defined as the ratio of the weight of a given liquid to the specific mass (specific weight) of standard liquid (water at 4^0c)

Mathematically,

G or S (for liquids) = weight density (density) of liquid/weight density (density) of water

$G = \rho_{given}/\rho_{standard}$

S (for gases) = $\frac{weight\ density\ (density) of\ gas}{weight\ density\ (density) of\ air}$

Thus, weight density of a liquid = S× weight density of water, = S× 1000× 9.81 N/m^3

Density of a liquid = S × density of water = S × 1000 kg/m^3

G = **γ** given/**γ** standard ($\rho_{standard} = 1000\ kg/m^3$)

($Y_{standard} = 9810\ N/m^3$)

(4)Viscosity: - Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Units of viscosity: - μ = shear stress/change of velocity/change of distance $= \frac{force/area}{length/time} \times$ 1/length

= force time/ $(length)^2$

MKS Unit of viscosity = kgf-sec/m^2

CGS Unit of viscosity = dyne- sec/cm^2

SI unit of viscosity = N.sec/m^2

N/m^2 is also known as Pascal

Hence, N/m^2 = Pascal

SI unit of viscosity = N sec/m^2 = pa.sec

⟹ 10 poise = 1N sec/m^2

It is of two types

(i) Kinematic viscosity , (ii) Newton's law of viscosity

(i) **Kinematic viscosity: -** It is defined as the ratio between the dynamic viscosity and density of fluid.

It is denoted by √ (nu)

$$\sqrt{} = \frac{viscosity}{density} = \frac{\sqrt{}}{\rho}$$

Unit of kinematic viscosity

$$\surd = \frac{unit\ of\ \mu}{unit\ \ of\ \rho} = \frac{force\ \times time}{(length)2\ \times mass/length}$$

$$= \frac{(length)2}{time}$$

In MKS and SI, the unit of kinematic viscosity is m^2/sec.

CGS unit is cm^2 /sec.

Thus, one stoke = cm^2/sec= $(\frac{1}{100})^2$ m^2/sec

Centistokes means = $\frac{1}{100}$ =stoke

(II) **Newton's law of viscosity**: - It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

Mathematically

$$\tau = \mu\ \frac{du}{dy}$$

τ = shear stress, μ = co-efficient of friction or dynamic viscosity or absolute viscosity

u = velocity of fluid, y = distance covered, $\frac{du}{dy}$ = velocity gradient

Variation of viscosity with temperature: - Temperature affects the viscosity. The viscosity of liquids decreases with the increases of temperature.

Capillarity: - It is defined as a phenomenon of rise or fall of a liquid surface in a capillary tube relative to an adjacent general level of liquid .Hence the tube is held vertically in the liquid .The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.

SURFACE TENSION (σ) OR (T):-

Which offers tensile resistance at the surface is called surface tension between the molecules at the surface of a liquid.

S. I .Unit = N/M OR MKS Unit = kgf/m.

Compressibility (β)-: The compressibility of liquid is the variation in its volume with variation of pressure i.e. it is the reciprocal of bulk modular. (β = 1/k)

Unit = m^2 /N (.I.e. volumetric strain per unit compressive stress.)

Problem.

(1)A liquid occupying 1.6 m^3 volume weights 12.8 KN. What is the specific mass and specific weight of the liquid?

Ans: - Specific weight = weight/volume = 12.8/1.6 = 8 KN/m^3

Specific mass $\Longrightarrow \nu = \rho g \Longrightarrow \rho = \nu/g$ = 8000/9.81 = 815.49 kg/ m^3

(2) What is the specific gravity of a liquid whose specific weight is 7.36 KN/m^3

Ans: - G = sp.wt.of given fluid / sp. Wt. of standard fluid= 7.36 × 1000/9810 = 0.75

<u>Types of fluid.</u>

Ideal fluid or (perfect fluid) & Real fluid.

Ideal fluid of two types - (I) Newtonian fluid (ii) Non- Newtonian fluid.

Newton fluid is of two types – (I) compressible fluid, (ii) Incompressible fluid.

IDEAL FLUID OR PERFECT FLUID

Ideal fluids are only imaginary fluids which are incompressible, non-viscous, no surface tension.

In nature ideal fluid does not exist.

Real fluid -: The fluid's actually available in nature causing property such as viscosity, surface tension and compressibility.

Newtonian fluid -: The real fluid that obeys Newton's law of viscosity is called as Newtonian fluid.

NEWTON'S LAW OF VISCOSITY-: As per Newton's law of viscosity the shear stress between various layers of a fluid is proportional to the rate of shear strain or velocity gradient.

$\Rightarrow Z \propto \frac{du}{dy}$

$Z = N \frac{du}{dy}$

'N" is constant for Newtonian fluid.

Non Newtonian fluid -: The real fluid that does not obey the Newton's law of viscosity is called as Non-Newtonian fluid.

In such fluid shear stress is not proportional to rate of shear strain (N) is not constant.

Compressible fluid -: Fluid's whose volume and density change with pressure is called compressible fluids.

In compressible fluid -: Fluid's whose volume and density does not change with pressure is called incompressible fluid. i.e. -: all liquids are incompressible liquid.

1.2 FLUID KINEMATICS & HYDROKINEMATICS

Kinematics is defined as that branch of science which deals with motion particles without considering the forces causing the motion or energy causing it.

It describes fluid in velocity, acceleration, pressure, temperature, density etc. in motion condition.

Types of fluid flow:-

In terms of space and time the fluid flow is classified as:-

(1) Steady and unsteady flows (factor of time)
(2) Uniform & non uniform flow(factor space)
(3) Laminar & turbulent flow
(4) Compressible and incompressible flow
(5) Rotational and irrotational flow
(6) One, two and three dimensional flow.

Here we only discuss about the following two types of fluid flow:-

(1) Laminar flow or streamline flow:-

When the various fluid particles move in laminar or layer, gliding or sliding smoothly over one layer to the other layer is called laminar flow or stream line flow

⟹ for a pipe flow, the type of flow is determined by non- dimensional number VD/ν called the Reynolds number, where D= diameter of pipe

V= mean velocity of flow in pipe
ν = Kinematic viscosity of fluid

⟹ If the Reynolds number is less than 2000, the flow is called laminar

(2)Turbulent flow: - The flow in which the liquid particle has not a definite path & moving randomly or turbulently is called turbulent flow.

OR/

Turbulent flow is that type of flow in which the fluid particles move in a Zigzag way. Due to movement of fluid particles in a zigzag, the eddies formation takes place which are responsible for high energy loss.

⟹If the Reynolds number is more than 4000, it is called turbulent flow.

⟹ If the Reynolds number lies between 2000 and 4000, the flow may be laminar or turbulent.

TOTAL HEAD OF A LIQUID PARTICLE IN MOTION: - The total head of a liquid particle, in motion, is the sum of potential head, kinetic head and pressure head.

Mathematically, Total Head, $H = Z + V^2/2g + P/W$ m of liquid.

Continuity Equation: - The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per section is constant.

Consider two cross-section of a pipe as shown in figure.

Let V_1 = Average velocity at cross-section1-1

ρ_1= Density at section 1-1

A_1 = Area of pipe at section 1-1

And V_2, ρ_2, A_2 are corresponding values at section, 2-2

Then, rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass,

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This equation is applicable to the compressible as well as incompressible fluids and is called continuity equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation $A_1 V_1 = A_2 V_2$

Bernoulli's Equation: -

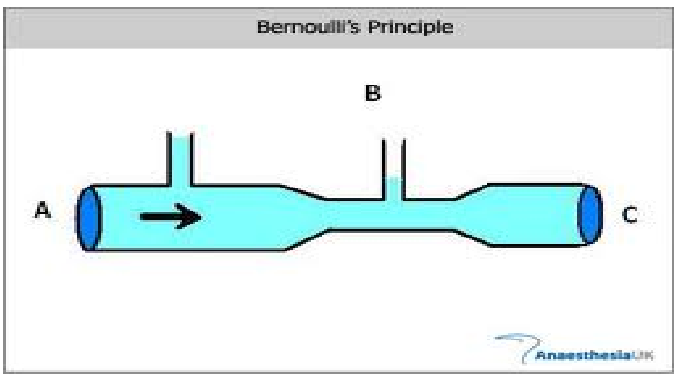
It states " ***for a perfect incompressible liquid, flowing in a continuous stream, the total energy of a particle moves from one point to another.*** " This statement is based on the assumption that there are no losses due to friction in the pipe.

Mathematically, $Z + V^2/2g + p/w$ =constant.

Where, Z = potential energy, $V^2/2g$ =Kinetic energy, P/W= pressure energy

PROVE: -

Consider a perfect incompressible liquid, flowing through a non-uniform pipe as shown in the figure.



Let us consider two sections AA & BB of the pipe. Now let us assume that the pipe is running full and there is a continuity of flow between the two sections.

Let Z_1 = Height of AA above the datum.

P_1 = Pressure at AA.

V_1 = Velocity of liquid at AA,

a_1= cross-sectional area of the pipe at AA, and Z_2, P_2, V_2, a_2 = corresponding values at BB.

Let the liquid between the two sections AA and BB moves to A^1A^1 and B^1B^1 through very small lengths $d\,l_1$ and $d\,l_2$ as shown in fig. This movement of the liquid between AA & BB is equivalent to the movement of the liquid between AA and A^1A^1 to BB and B^1B^1 the remaining liquid between A^1A^1 and BB being unaffected.

Let W be the weight of the liquid between AA and A^1A^1. Since the flow is continuous,

Therefore $W = wa_1 d\,l_1 = w\,a_2\,d\,l_2$

$\Longrightarrow a_1\,dl_1 = a_2\,dl_2 = W/w$ -------- (1)

Or $a_1\,d\,l_1 = a_2\,d\,l_2$ ---------------- (ii).

We know that work done by pressure at AA in moving the liquid to A^1A^1 = force× distance

$= p_1 a_1 d\,l_1$

Similarly, work done by pressure at BB, in moving the liquid to $B^1B^1 = -P_2 a_2 d\,l_2$(minus sign is taken as the direction of p_2 is opposite to that of p_1)

Total work done by the pressure $= P_1a_1dl_1 — P_2a_2dl_2$

$= P_1a_1dl_1 – P_2a_1dl_1$ $(a_1dl_1 = a_2\,dl_2)$

$= a_1dl_1\,(P_1 - P_2) = W/w\,(P_1 - P_2)$ $(a_1dl_1 = W/w)$

Loss of potential energy $= w\,(z_1 - z_2)$ & again in kinetic energy $= w\,(V_2^2/2g - V_1^2/2g) = w/2g\,(V_2^2 - V_1^2)$

We know that loss of potential energy + work done by pressure

$W\,(Z_1 - Z_2) + W/w\,(P_1 - P_2) = W/2g\,(V_2^2 – V^2_1)$

$(Z_1 – Z_2) + P_1/w - P_2/w = V_2^2/2g – V_1^2/2g$

Or $Z_1 + V_1^2/2g + P_1/w = Z_2 + V_2^2/2g + P_2/w$

This proves the Bernoulli’s equation.

ASSUMPTIONS:-

The following are the assumptions made in the derivation of Bernoulli’s equation.

(i)The fluid is ideal, i.e., viscosity is zero.

(ii) The flow is steady

(iii)The flow is incompressible

(iv)The flow is irrotational.

<u>Limitations of Bernoulli's equation:</u> - The Bernoulli's theorem or Bernoulli's equation has been derived on certain assumptions, which are rarely possible. Thus the Bernoulli's theorem has the following limitations.

(1)The Bernoulli's equation has been derived under the assumption that the velocity of every liquid particle across any cross-section of a pipe is uniform. But in actual practice, It is not so. The velocity of liquid particle in the center of a pipe is maximum & gradually decreases towards the walls of the pipe due to the pipe friction. Thus, while using the Bernoulli's equation, only the mean velocity of the liquid should be taken into account.

(2)The Bernoulli's equation has been derived under the assumption that no external force, except the gravity force is acting on the liquid. But in actual practice, it is not so. There is always some external force such as pipe friction etc. acting the liquid, which affect the flow of the liquid.

Thus, while using the Bernoulli's equation, all such external force should be neglected. But if some energy is supplied to, or, extracted from the flow the same should also be taken into account.

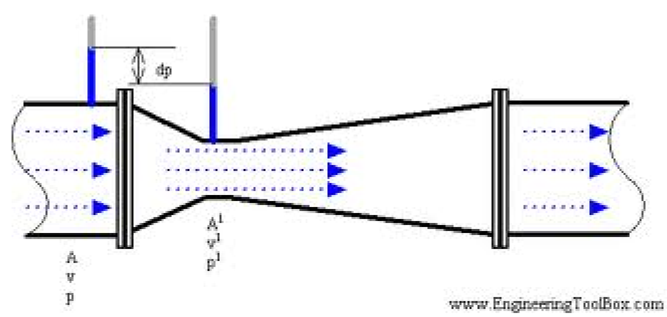
(3)The Bernoulli's equation has been derived, under the assumption that there is no loss of energy of the liquid particle while flowing. But in actual practice, it is rarely so. In a turbulent flow, some kinetic energy is converted into heat energy. And in a viscous flow, some energy is lost due to shear forces. Thus, while using Bernoulli's equation, all such losses should be neglected.

(4)If the liquid is flowing in a curved path, the energy due to centrifugal force should also be taken into account.

PRACTICAL APPLICATIONS OF BERNOULL'S EQUATION

There are three types :-(1) Venturi meter (2) Orifice meter, (3) Pitot tube.

(1) Venturi meter: -



Venturi meter is an apparatus for finding out the discharge of a liquid flowing in a pipe. A Venturi meter in its simplest form consists of the following three parts.

(A)Convergent cone (B) Throat (C) Divergent cone.

(A) It is a short pipe which converges from a diameter d_1 (diameter of the pipe in which the Venturi meter is fitted) to a smaller cone is also known as inlet of the Venturi meter.

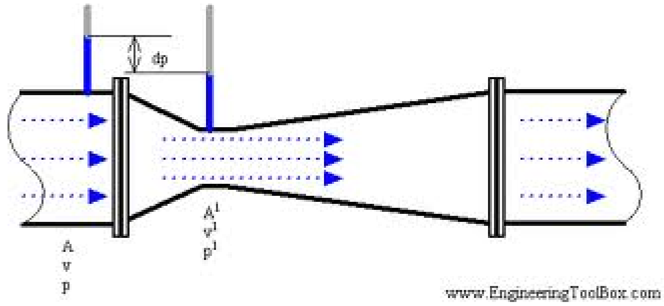
(B)It is a small portion of circular pipe in which the diameter d_2 is kept constant.

(C)Divergent cone: - It is a pipe which diverges from a diameter d_2 to a large diameter d_1. The divergent cone is also known as outlet of the Venturi meter. The length of the divergent cone is larger.

Divergent cone is longer than convergent type because liquid flowing through the Venturi meter is accelerated between the sections 1& 2 (i.e., while flowing through the convergent cone). As a result of the acceleration, the velocity of liquid at section-2(i.e. at the throat) becomes higher than that at section-1. This increase in velocity results in considerably decreasing the pressure at section2.

⟹The liquid while flowing through the Venturi meter is decelerated between the sections 2 and 3(i.e., while flowing through the divergent cone). As a result of this retardation, the velocity of liquid decreases, this consequently increases the pressure. If the pressure is rapidly recovered, then there is every possibility for the stream of liquid to break away from the walls of the meter due to boundary layer effects. In order to avoid the tendency of breaking away the stream of liquid, the divergent cone is made sufficiently longer. Another reason for making the divergent cone longer is to minimize the frictional losses. Due to these reasons, the divergent cone is 3 to 4 times longer than convergent cone.

Discharge through a Venturi meter: - Consider a Venturi meter through which some liquid is flowing as shown in figure.



(VENTURIMETER)

Let P_1 = Pressure at section-1, V_1 = Velocity of water at section-1, Z_1 = Datum head at section-1, a_1= area of the venturi meter at section-1 and P_2, V_2, Z_2, a_2 are corresponding values at section-2

Applying Bernoulli's equation at section-1 and section-2, i.e.

$Z_1 + V_1^2/2g + P_1/W = Z_2 + V_2^2/2g + P_2/W$ ------------ (1)

Let us pass our datum line through the axis of the Venturi meter as shown in the figure.

Now Z_1=0 and Z_2=0

$V_1^2/2g + P_1/W = V_2^2/2g + P_2/W$

Or, $P1/W – P_2/W = V_2^2/2g – V_1^2/2g$ ----------- (2)

Since the discharge at section 1 & 2 is continuous, therefore

$$V_1 = a_2v_2/a_1 \ (a_1v_1 = a_2v_2)$$
$$V_1^2 = a_2^2 \ V_2^2/a_1^2$$

Substituting the above values of V_1^2 in equation (3)

$$P_1/W – P_2/W = V_2^2/2g - (a_2^2/a_1^2 \times V_2^2/2g)$$
$$= V_2^2/2g \ (1- a_2^2/a_1^2)$$
$$= V_2^2/2g \ (a_1^2 – a_2^2)/a_1^2)$$

We know that $P_1/W – P_2/W$ is the difference between the pressure heads at sections 1 and 2. When the pipe is horizontal, this difference represents the venture head and is denoted by h.

$$\text{Or, } h = V_2^2/2g \ (a_1^2- a_2^2/a_1^2)$$
$$V_2^2 = 2gh \ (a_1^2/ (a_1^2- a_2^2))$$

$V_2 = \sqrt{2gh}\ (a_1/\sqrt{a}_1{}^2 - a_2{}^2)$

We know that the discharge through a venturi meter.

Q = Coefficient of Venturi meter $\times a_2 v_2$

$= C \times a_2 v_2 = c\ a_1 a_2 \sqrt{2gh}/\sqrt{a}_1{}^2 - a_2{}^2$

<u>Orifice meter: -</u> As orifice meter is used to measure the discharge in pipe. An orifice meter, in its simplest form, consists of a plate having sharp edged circular hole known as orifice. This plate is fixed inside a pipe.

A mercury manometer is inserted to know the difference of pressures between the pipe and the throat (i.e. orifice)

Let h = Reading of the mercury manometer

P_1= Pressure at inlet, V_1= velocity of liquid at inlet, a_1= Area of pipe at inlet and P_2, V_2, a_2 is corresponding values at the throat. (i.e., orifice)

Let h= reading of mercury manometer.

P_1=pressure at inlet.
V_1=velocity of liquid at inlet.
A_1= area of pipe at inlet and
$P_2 V_2 a_2$ =corresponding values at the throat.
Now applying Bernoulli's equation for inlet of the pope and the throat

$Z_1 + V_1{}^2/2g + P_1/W = Z_2 + V_2{}^2/2g + P_2/W$ --------- (1)

$P_1/W - P_2/W = V_2{}^2/2g - V_1{}^2/2g \quad (Z_1 - Z_2)$

$h = V_2{}^2/2g - V_1{}^2/2g = 1/2g\ (V_2{}^2 - V_1{}^2)$ ------------ (2)

Since the discharge is continuous, therefore $a_1 v_1 = a_2 v_2 \Rightarrow v_1 = a_2\ v_2/a_1 \Rightarrow v_1{}^2 = a_2{}^2 \times v_2{}^2/a_1{}^2$

Substituting the above values of $v_1{}^2$ in equ^{ine} (ii)

$h = 1/2g\ (v_2{}^2 - a_2 \times v_2{}^2/a_1{}^2)$

$= v_2{}^2/2g\ (1 - a_2{}^2/a_1{}^2)$

$= v_2{}^2/2g\ (a_1{}^2 - a_2{}^2/a_1{}^2)$

$V_2{}^2 = 2gh\ (a_1{}^2/a_1{}^2 - a_2{}^2)$

Or $V_2 = \sqrt{2gh}(a_1/\sqrt{a}_1{}^2 - a_2{}^2$

We know that the discharge,

Q = Coefficient of orifice meter $\times a_2 v_2 = C\ a_1 a_2\ \sqrt{2gh}/\sqrt{a}_1{}^2 - a_2{}^2$

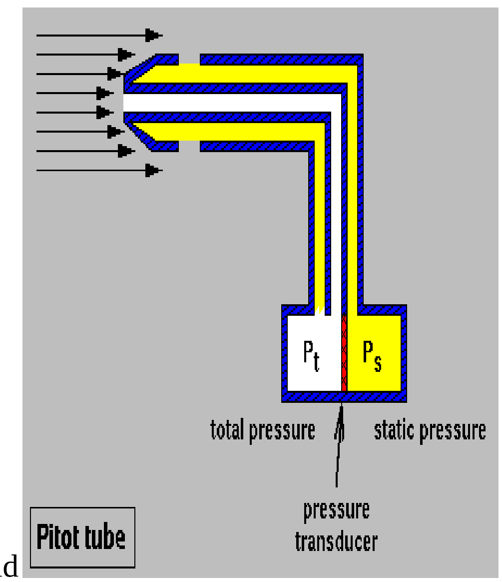
PITOT TUBE:-

A pitot tube an instrument to determine the velocity of flow at the required point in a pipe or a stream in its simplest form a pitot tube consists of a glass tube bent through 90^0 as shown in figure.

The lower end of the tube poses the direction of the flow as shown in figure. The liquid rises up in the tube due to pressure exerted by the flowing liquid by measuring the rise of liquid in the tube, we can find out the velocity of the liquid flow.

Let h- height of the liquid in the Pitot tube above the surface.

H- Depth of tube in the liquid and



V- Velocity of the liquid

Applying Bernoulli's equation for the section 1 and 2.

$H+V_2/2g = Hoh \ (Z_1=Z_2)$

$H= v_2 /2g$

$V=\sqrt{2gh}$

FLOW THROUGH ORIFICES:-

Orifice in a small opening of any cross section such as circular, triangular rectangular etc. on the side or at the bottom of a tank through which a fluid is flowing.

- Mouth piece is a short length of a pipe which is two or three times its diameter in length, fitted in a tank or vessel containing the fluid.

- Orifices as well as mouth piece are used for measuring the rate of flow of fluid.

TYPES OF ORIFICES

1. According to size.
a. Small orifice.
b. Large orifice

2. According to shape.
   a. Circular orifice
   b. Rectangular orifice
   c. Triangular orifice

3. According to shape of the edge
   a. sharp – edged orifice
   b. Bell– mouthed orifice

4. according to nature of discharge-:
a. fully submerged orifice
b. Partially sub-merged orifice.

JET OF WATER.

The continues stream of a liquid, that comes out or flows out of an orifice is known as the jet of water.

VENNA CONTRACTA-:

Consider a tank fitted with a circular orifice is one of its sides as shown in fig.

Let H be the head of the liquid above the center of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross section is less than that of orifice the area of jet of fluid goes on decreasing and at a section c-c, the area is minimum. Thus section is approximately at a distance of half of diameter of the orifice at this section, the stream lines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called vena contract a beyond this section the jet diverges and is attracted in the down ward direction by the gravity.

Consider two points 1 & 2 as shown in figure; point-1 is inside the tank and point-2 at the Vena-contract. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 & 2,

$P_1/\rho g + V_1^2/2g + Z_1 = P_2/\rho g + V_2^2/2g + Z_2$

But $Z_1 = Z_2$

$P_1/go + V_1^2/2g = P_2/go + V_2^2/2g$

Now, $P_1/\rho g = H$, $P_2/\rho g = 0$ (atmospheric pressure)

V_1 is very small in comparison to V_2 as area of tank is very large as compared to the area of jet of liquid.

$H + O = O + V_2^2/2g \Longrightarrow V_2 = \sqrt{2gH}$

This is theoretical value.

Hydraulic Co-efficient: **-** The hydraulic coefficients are (1) Coefficient of velocity (C_V)

(2)Coefficient of contraction (C_C)

(3)Coefficient of discharge (C_d)

(1) Coefficient of velocity (C_v): - It is defined as the ratio between the actual velocity of a jet of liquid at Vena contracta and the theoretical velocity of jet. It is denoted by C_V

Mathematically, C_V = Actual velocity of jet at vena-contracta/ Theoretical Velocity

$= \frac{V}{\sqrt{2gH}}$ Where V= Actual velocity, $\sqrt{2gH}$ =Theoretical velocity

The value of C_V varies from 0.95 to 0.99 for different orifices. Generally the value of C_V =0.98 is taken for sharp edged orifices.

(2)Co-efficient of contraction (C_C):-

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_C.

Let a = area of orifice and a_c = area of jet at vena-contracta

C_C = area of jet at vena-contracta/area of orifice

$C_C = \frac{\text{area of jet at ven-contracta}}{\text{area of orifice}} = a_c/a$

The value of C_C varies from 0.61 to 0.69.

It general the value of C_C may be taken as 0.64

(3)Co-efficient of Discharge (Cd):- It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. Let Q is actual discharge and Q_{th} is the theoretical discharge.

Mathematically, C_d = Q/Q_{th}= Actual velocity× actual area/theoretical velocity× theoretical velocity

The value of C_d is taken as 0.62. The value C_d varies from 0.61 to 0.65.

Relation between C_c, C_d, C_v

C_d = Q/Q_{th}= Actual velocity × Actual area/Theoretical velocity× Theoretical area

$\Longrightarrow C_d = C_V \times C_C$

Problem:-

Q: - The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take C_d=0.6 and C_V =0.98.

ANS: - Given: H=10cm, Dia. Of orifice, d=40mm=0.04m, Area, a =$\prod/4\times (0.04)^2$ = 0.001256m^2, Cd=0.6, C_V=0.98

(1)$\frac{\textit{Actual discharge}}{\textit{Theoretical discharge}}$=0.6

But Theoretical discharge= $V_{th}\times$ Area of orifice

V_{th} = theoretical velocity, where $V_{th}=\sqrt{2gH}=\sqrt{2\times 9.8\times 10}$= 14m/sec.

Theoretical discharge =14×0.001256 = 0.01758m^2/sec

Actual discharge = 0.6 ×theoretical discharge

= 0.6 ×0.1758 = 0.01054 m^3/sec

$\frac{\textit{Actual velocity}}{\textit{Theoretical velocity}}$ = C_V =0.98$\Longrightarrow$ Actual velocity =0.98×14= 13.72 m/sec. (Ans.)

1.3 FLOWS THROUGH PIPES

When the Reynolds number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynolds number is more than 4000, the flow is known as turbulent flow.

LOSS OF ENERGY IN PIPES:-

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

There are two types of energy losses :-(1) Major energy losses (2) Minor energy losses.

(1)Major energy losses: - This is due to friction. It is of two types :-(a) Dracy-Weisbac, (b) Chevy's formula

(2)Minor energy losses: - This is due to (a) sudden expansion of pipe
(b) Sudden contraction of pipe
(c)Bend in pipe.
(d)Pipe fitting
(e) An obstruction in pipe.

LOSS OF ENERGY OR HEAD DUE TO FRICTION

(a)Darcy- Weisbach Formula:-The loss of head (or energy) in pipes due to friction is calculated Darcy-Weisbach equation

$$h_f = 4. f.L. V^2/d \times 2g$$

Where h_f = loss of head due to friction, f = Co-efficient of friction which is a function of Reynolds number

$= 16/R_e$ for $R_e < 2000$(viscous flow)

$=0.079/R_e^{1/4}$ for R_e varying from 4000 to 10^6

Where L = length of pipe, V=mean velocity of flow, d= diameter of pipe.

(b) Chezy's formula for loss of head due to friction in pies:-

The expression for loss of head due to friction in pipes is derived the equation

$$H_f = \frac{f1}{\rho g} \times \frac{P}{A} \times L \times V^2$$

Where h_f =loss of head due to friction, p= wetted perimeter of pipe, A= area of cross section of pipe,

L= length of pipe, and V= mean velocity of flow.

Hydraulic mean depth, m= $\frac{A}{P}$ = $\frac{\frac{\pi}{4}d2}{\pi d}$= $\frac{d}{4}$

Substituting $\frac{A}{P}$ = m or $\frac{P}{A}$ =$\frac{1}{m}$ in the equation, we get,

$H_f = \frac{f1}{\rho g} \times \frac{L \times V2}{m}$, or $V^2 = h_f \times \frac{\rho g}{f1} \times m \times \frac{1}{L} = \frac{\rho g}{f1} \times m \times \frac{hf}{L}$

$V = \sqrt{\frac{\rho g}{f1} X m x \frac{hf}{L}} = \sqrt{\rho g/f1}\sqrt{mhf/L}$

Let $\sqrt{\rho g/f1}$ =C, where C is a constant known as Chezy' s constant and h_f/L= i, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\rho g/f1}$ and $\sqrt{h}_f/L$

$$V = C\sqrt{mi}$$

This equation is known as Chezy's formula.

Q:- Find the head lost due to friction in a pipe of diameter 300mm and length 50m, through which is flowing at a velocity of 3m/sec using (i)Darcy formula, (ii)Chezy's formula for which C = 60.Take ν for water = 0.01 stoke.

Ans: - Given Data:

Dia. Of pipe, d=300mm=0.30m, Length of pipe, L=50M, Velocity of flow, V=3m/sec, Chezy's constant, C=60, kinematic viscosity, ν =0.01 stoke=0.01 $cm^2/sec = 0.01 \times 10^{-4} m^2/sec$

(i)Darcy formula:

$$h_f = 4. f.L. V^2/d \times 2g$$

Where f =co-efficient of friction is a function of Reynolds number, R_e

But $R_e = V \times d/\nu = 3.0 \times 0.30/0.01 \times 10^{-4} = 9 \times 10^5$

Value of f $=0.079/R_e^{1/4} = 0.079/ (9 \times 10^5)^{1/4} = 0.00256$

Head lost, $h_f = 4 \times 0.00256 \times 50 \times 3^2/0.3 \times 2.0 \times 9.81$ =0.7828m.

(ii)Chezy's formula:

$V = C\sqrt{mi}$, where C=60, m=d/4=0.30/4=0.075m

$\Rightarrow 3=60\sqrt{\mathbf{0.075}\times \boldsymbol{i}}$ or, $i= (3/60)^2\times 1/0.075 = 0.0333$

But $i= h_f/L=h_f/50 \Rightarrow h_f/50 =0.0333 \Rightarrow h_f= 50\times 0.0333=1.665m$.

Q:- A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300 litres per sec. Find the head lost due to friction for a length of 50m of the pipe.

Ans: - Given Data:-

Kinematic viscosity, ν =0.4 stoke = 0.4 $cm^2/sec=0.4\times 10^{-4}m^2/sec$

Dia. Of pipe, d= 300mm=0.30m

Discharge, Q=300litres/sec =0.3 m^3/sec

Discharge, Q= 300litres/sec=0.3m^3/sec

Length of pipe, L= 50m

Velocity of flow, $V=Q/Area=0.3/\prod/4\ (0.3)^2= 4.24m/sec$

Reynolds number, $R_e=V\times d/\nu=4.24\times 0.30/0.4\times 10^{-4}=3.18\times 10^4$

As R_e lies between 4000 and 100,000, the value of f is given by

$f =0.079/ (R_e)^{1/4}=0.079/ (3.18\times 10^4)^{1/4}=0.00591$

Head lost due to friction, $h_f=4.f.L.V^2/d\times 2g=4\times 0.00591\times 50\times 4.24^2/0.3\times 2\times 9.81=3.61m$.

MINOR ENERGY (HEAD) LOSSES:-

The loss of head or energy due to friction in a pipe is known as major loss whiles the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy or head includes the following cases:

(1)Loss of head due to sudden enlargement:-$h_e= (V_1-V_2)^2/2g$

(2)Loss of Head to sudden contraction:-$h_c=0.5V_2^2/2g$

(3)Loss of head at the entrance of a pipe:-$h_i=0.5V^2/2g$

(4)Loss of head at the exit of pipe:-$h_o=V^2/2g$

(5) Loss of head due to an obstruction in a pipe :-$(V_C-V^2)/2g$

(6)Loss of head due to Bend in pipe: - $h_b=Kv^2/2g$

(7)Loss of head in various pipe fitting: - $KV^2/2g$, where V=Velocity of flow, K= co-efficient of pipe fitting.

HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

Hydraulic Gradient Line: - It is defined as the line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

Total Energy Line: - It is defined as the line which gives the sum of pressure head, Datum head and Kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly T.E.L. (Total energy line).

POWER TRANSMISSION THROUGH PIPES:-

Power is transmitted through pipes by flowing water or other liquids flowing through them. The power transmitted depends upon (i) the weight of liquid flowing through the pipe and (ii) the total head available at the end of the pipe. Consider a pipe AB connected to a tank. The power available at the end B of the pipe and the condition for maximum transmission of power will be obtained.

Let L = length of the pipe,

d = diameter of the pipe,

H = total head available at the inlet of pipe,

V= velocity of flow in pipe,

h_f = loss of head due to friction, and f =co-efficient of friction

The head available at the outlet of the pipe, if minor losses are neglected

= Total head at inlet – loss of head due to friction

$$= H - h_f = H - 4flv^2/d\times 2g$$

Weight of water flowing through pipe per sec,

$$W = \rho\, g \times \text{volume of water per sec} = \rho\, g \times \prod/4 d^2 \times V$$

The power transmitted at the outlet of the pipe = weight of water per sec × head at outlet

$= (\rho\ g \times \prod/4d^2 \times V) \times (H - 4f \times L \times V^2/d \times 2g)$ Watts

Power transmitted at the outlet of the pipe= weight of water per sec × head at outlet

$= (\rho g \times \prod/4d^2 \times V) \times (H- 4f \times L \times V^2/d \times 2g)$ Watts

Power transmitted at outlet of the pipe,

$P = \rho\ g/1000 \times \prod/4d^2 \times V\ (H-4FlV^2/d \times 2g)$ Watts

Power transmitted at outlet of the pipe,

$P = \rho\ g/1000 \times \prod/4d^2 \times V\ (H-4Flv^2/d \times 2g)$ kW

Efficiency of power transmission:-

= Power available at outlet of the pipe/Power supplied at the inlet of the pipe

$= \frac{\textit{Weight of water per sec} \times \textit{Head available at outlet}}{\textit{weight of water per sec} \times \textit{Head at inlet}}$ $= W \times (H-h_f)/W \times H$

Condition for Maximum Transmission of Power: - The condition for maximum transmission of power is obtained by differentiating equation:

$$\frac{d}{dv}(\boldsymbol{P}) = O$$

Or $\frac{d}{dV}[\boldsymbol{\rho\ g/1000 \times \pi/4d2(HV - 4fLV2/2gd)}]$

$P\ g/1000 \times \pi/4d^2(H-4 \times 3 \times f \times L \times V^2/d \times 2g) = 0$

Or $H - 3 \times 4fLv^2/d \times 2g = 0$ or $h - 3 \times h_f = 0$

$H = 3\ h_f$ or $h_f = H/3$

This is the condition for maximum transmission of power. It states that power transmitted through a pipe is maximum when the loss of head due to friction is one – third of the total head at inlet.

Maximum Efficiency of Transmission of power= Efficiency of power transmitted through pipe is given by equation as $n = H-h_f/H$

For maximum power transmission through pipe the condition is given by equation

$$h_f = H/3$$

Substituting the value of hf in efficiency, we get maximum n,

$$N_{max} = H- H/3/H = 1-1/3 = 2/3 \quad \text{or } 66.7\%$$

2.1 Elementary Idea on Different Modes of Heat Transfer: -

“The energy in transit is termed as heat**”**

According to the modern or dynamical theory of heat: “Heat is a form of energy. The molecules of a substance are in parallel motion. The mean kinetic energy per molecule of the substance is proportional to its absolute temperature.

Summarily heat energy given to a substance is used in increasing its internal energy. Increase in internal energy causes increase in kinetic energy or potential energy or increase in both the energies.

Importance of Heat Transfer: -

Heat transfer may be defined as:

“The transmission of energy from one region to another as a result of temperature gradient”. In heat transfer the driving potential is temperature difference. This transfer in literature is also known as ***“diffusion”***.

The study of heat transfer is carried out for the follow’s purposes:

1. To estimate the rate of flow of energy as heat through the boundary of a system under study.
2. To determine the temperature field under steady and transient condition.

In almost every branch of engineering, heat transfer problems are encountered which cannot be solved by thermodynamic reasoning alone but require an analysis based on heat transfer principles and modes of heat transfer. The areas where heat transfer analysis is required are as follows:

- Design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipment’s, catalytic converters, heat shields for space vehicles, furnaces, electronic equipment’s etc.
- Internal combustion engines.
- Refrigeration and air conditioning units.
- Design of cooling systems for electric motors, generators and transformers.
- Heating and cooling of fluids etc. in chemical operation.
- Construction of dams and structures; minimization of building –heat losses using improved insulation techniques.
- Thermal control of space vehicles.
- Heat treatment of metals.
- Dispersion of atmospheric pollutants.

Modes of Heat Transfer: -

Heat transfer as a result of temperature gradient takes place by the following three modes: -

1. **Conduction; 2. Convection; 3. Radiation;**

Heat transmission in majority of real situations, occurs as a result of combinations of these above said modes of heat transfer. *Example:* The water in a boiler shell receives its heat from the fire-bed by conducted, convicted and radiated heat from the fire to the shell, conducted heat through the shell and conducted and convicted heat from the inner shell wall, to the water. Heat always flows in the direction of lower temperature.

2.2 Define Conduction and Derive Fourier's law: -

Conduction: - *Conduction is the transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.*

In solids, the heat is conducted by the following two *mechanisms:*

(i) By lattice vibration (the faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).

(ii) By transport of free electrons (Free electrons provide an energy flux in the direction of decreasing temperature-For metals, especially good electrical conductors, the electronic mechanism is responsible for the major portion of the heat flux except at low temperature).

In case of gases, the mechanism of heat conduction is simple. The kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion exchanging energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it loses energy by collisions.

In liquids, the mechanism of heat is nearer to that of gases. However, the molecules are more closely spaced and intermolecular forces come into play.

Fourier's laws of heat Conduction: -

Fourier's law of heat conduction is an empirical law based on observation and states as follows: -

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow.

Mathematically, it can be represented by the equation: -

$$Q \propto A.\frac{dt}{dx}$$

Where, Q= Heat flow through a body per time (in watt), W

A= Surface area of heat flow (perpendicular to the direction of flow), m^2

dt= Temperature difference of the faces of block (homogeneous solid) of thickness dx through which heat flows,℃ or K, and

dx = Thickness of body in the direction of flow, m.

$$\textbf{Thus, } Q = -K.A.\frac{dt}{dx}$$

Where, k= Constant of proportionality and is known as thermal conductivity of the body.

The –ve sign of K is taken care of the decreasing temperature along with the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is always negative along positive x direction and, therefore, the value as Q becomes + ve.

Assumptions: -

The following are the assumptions on which Fourier's law is based:

1. Conduction of heat takes place under ***steady state condition.***
2. The heat flow is unidirectional.
3. The temperatures gradient is ***constant*** and the temperature profile is ***linear***.
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is constant in all direction).

Steady State heat conduction through flat walls: -

Consider a plane wall of homogeneous material through which heat is flowing only in an x- direction.

Let, L= Thickness of the plane wall,

A= Cross sectional area of the wall.
K= Thermal conductivity of the wall material, and
t_1, t_2 Temperatures maintained at the two faces 1 and 2 of the wall, respectively.

The general heat conduction equation in Cartesian coordinates is given by,

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\propto}.\frac{\partial t}{\partial \tau} \text{ --------------------------------- (1)}$$

If the heat conduction takes place under the conditions, steady state ($\frac{\partial t}{\partial \tau} = 0$), one-dimensional
($\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0$) and with no internal heat generation ($\frac{q_g}{k}$) then the above equation is reduced to,

$$\frac{\partial^2 t}{\partial x^2} = 0, \quad \text{or} \quad \frac{d^2 t}{dx^2}$$

By integrating the above differential twice, we have

$$\frac{dt}{dx} = C_1 \quad \text{and } t = C_1 x + C_2 \text{ ………………………………………….. (1)}$$

Where C_1 and C_2 are the arbitrary constants. The values of these constants may be calculated from the known boundary conditions as follows:

At x= 0, t= t_1,
At x= L, t= t_2,

Substituting the values in the equation, we get
$t_1 = 0+C_2$ and $t_2 = C_1 L+C_2$

After simplification, we have, $C_2 = t_1$ and $C_1 = \frac{t_2 - t_1}{L}$
Then, the equation (1) reduced to:

$$t = \left(\frac{t_1 - t_2}{L}\right) x + t_1 \text{ ----------------------------- (2)}$$

The equation (2) indicates that temperature distribution across a small wall is linear and independent of thermal conductivity. Now heat through the plane wall can be found by using Fourier's equation as follows:

Q= -KA $\frac{dt}{dx}$(Where$\frac{dt}{dx}$ = Temperature gradient)

But, $\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right) x + t_1\right] = \frac{t_2 - t_1}{L}$

$\therefore Q = -KA\frac{(t_2 - t_1)}{L} = KA\left(\frac{t_2 - t_1}{L}\right)$ -- (3)

Equation (3) can be written as:

$Q = \frac{(t_1 - t_2)}{\left(\frac{L}{KA}\right)} = \frac{(t_1 - t_2)}{(R_{th})conduction}$

Where, (R_{th}) cond. = Thermal resistance to heat conduction. (The equivalent thermal circuit for through the plane wall).

Thermal resistance (conduction) of the wall, (R_{th}) conduction $= \frac{L}{KA}$.

2.3 <u>CONVECTION: -</u>

In heat transfer, the exchange of heat from a wall to a fluid or from a fluid to a wall is very important process (applicable in heat exchangers and performance of engines etc.)

"*Convection is the transfer of heat within a fluid by mixing of one portion of the fluid with another*."

Convection is possible only in a fluid medium and is directly linked with the transport of medium itself. This mode of heat transfer is met with in situations where energy is transferred as heat to a flowing fluid at any surface over which flow occurs. The heat flow depends on the properties of fluid and is independent of the properties of the material of the surface. The *mechanisms* of convection in which phase changes are involved lead to the important fields of *boiling and condensation*.

The rate of heat transfer by convection, between a solid boundary and a fluid, is given by,
Q= h_aA (t_s - t_a)
Where, Q= Rate of heat transfer,

A= Area exposed to heat transfer,

i_{ts}= Surface temperature,

f_t= Fluid temperature, and

h_a= Coefficient of convective heat transfer= $\frac{Q}{A(t_s - t_f)}$ =$\frac{w}{m^2 \circ c}$ **or** $\frac{w}{m^2 k}$

The coefficient of convective heat transfer 'h' (also known as film heat transfer coefficient) may be defined as "the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time.

<u>FREE OR NATURAL CONVECTION: -</u>

When a surface is maintained in still fluid at a temperature higher or lower than that of the fluid, a layer of fluid adjacent to the surface gets heated or cooled. A density difference is created between this layer and the still fluid surrounding it. The density difference introduces a buoyant force causing flow of fluid near the surface. Heat transfer under such condition is known as free or natural convection.

Or,

"Free or natural convection is the process of heat transfer which occurs due to movement of the fluid particles by density changes associated with temperature differential in a fluid".

<u>FORCED CONVECTION: -</u>

- Forced convection is a mechanism or type of transport in which fluid motion is generated by an external source (like pump, fan, suction device etc.).
- It should be considered as one of the main methods of useful heat transfer as significant amounts of heat energy can be transported very efficiently and this mechanism is found very commonly in everyday life, including central heating, air conditioning, steam turbines and in many other machines.
- Forced convection is often encountered by engineers designing and analyzing heat exchangers, pipe flow and flow over a plate at a different temperature.

2.4 <u>RADIATION: -</u>

"Radiation is the transfer of heat through space or matter by means other than conduction or convection".

- Radiation heat is thought of as electromagnetic waves or quanta (as convenient) an emanation of the same nature as light and radio waves.
- All bodies radiate heat; so, a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits.
- Radiant energy (being electromagnetic radiation) requires no medium for propagation and will pass through vacuum.

- The wavelength of the heat radiations is longer than that of light waves, hence they are invisible to eye.

STATE THE STEFFAN BOLTZMAN'S LAW: -

"The law states that the emissive power of a black body is directly proportional to fourth power of its absolute temperature".

Q$\propto T^4$,

Q= FσA (T_1^4 -T_2^4) ------------------------------- (1)

Where, F = A factor depending on geometry and surface properties,

σ = Stefan-Boltzmann constant,

= 5.67$\times 10^{-8}$ $W/_{m^2k^4}$

A=Area,m^2

T_1, T_2= Temperatures, degrees Kelvin (k).

This equation can also be written as: Q= $\frac{T_1 - T_2}{1/[\ F\sigma A\ (T_1+T_2)(T_1^2+T_2^2)]}$

Where denominator is radiation thermal resistance, (R_{th}) rad. The values of F are available for simple configurations in the form of charts and tables.

F= 1……. (For simple cases of black surface enclosed by another surface)

F = emissivity (ε)……. For non-black surface enclosed by another surface.

Emissivity (ε) is defined as the ratio of heat radiated by a surface to that of an ideal surface.

ABSORPTIVITY, REFLECTIVITY AND TRANSMISSIVITY: -

When incident radiation (G) also called **irradiation** (defined as the total incident radiation on a surface from all directions per unit time and per unit area of surface; expressed in$W/_{m^2}$ and denoted by (G) impinges on a surface, three things happen; a part is reflected back (G_r), a part is transmitted through (G_t) and the remainder is absorbed (G_a), depending upon the characteristics of the body.)

By the conservation of energy principle, $\boldsymbol{G_r + G_a + G_t}$**= G**

Dividing both sides by G, we get $\frac{G_r + G_a + G_t}{G} = \frac{G}{G}$, **or** $\alpha + \rho + \tau$ **=1**

Where $\boldsymbol{\alpha}$**= absorptive**
$\boldsymbol{\rho}$**= reflectivity**
$\boldsymbol{\tau}$ **= transitivity**

When the incident radiation is absorbed, it is converted into internal energy.

Black Body: - For perfectly absorbing body, $\boldsymbol{\alpha}$**=1,** $\boldsymbol{\rho}$**= 0,** $\boldsymbol{\tau}$**= 0.** Such a body is called a 'black body' (i.e., a black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it.) In practice, a perfect black body ($\boldsymbol{\alpha}$**=1**) does not exist. However, its concept is very important.

Gray Body: - if the radioactive properties, $\boldsymbol{\alpha, \rho, \tau}$ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called gray body. A gray body is also defined as one who's absorptive of a surface does not vary with temperature and wavelength of the incident radiation. A colored body is one whose absorptivity of a surface varies with the wavelength of radiation.

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