



KIIT POLYTECHNIC

LECTURE NOTES

ON

ELECTRICAL MEASUREMENT AND INSTRUMENTATION

Compiled by

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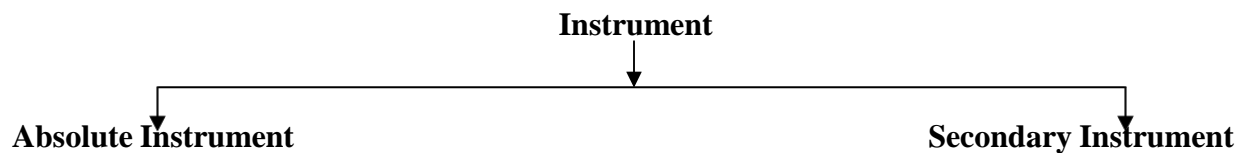
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MEASURING INSTRUMENTS

Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.



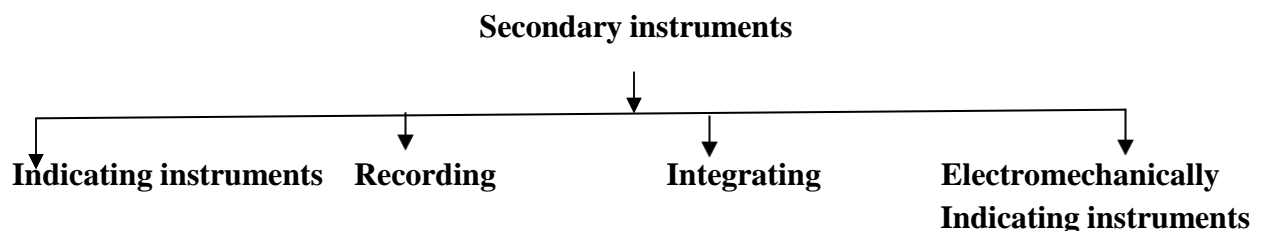
Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary.

They are

- (a) Deflecting force
- (b) Controlling force
- (c) Damping force

Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

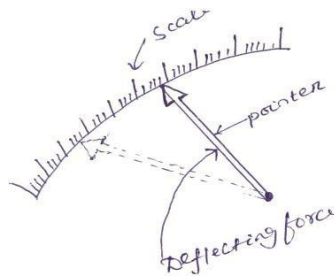


Fig. 1.1 Pointer scale

Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

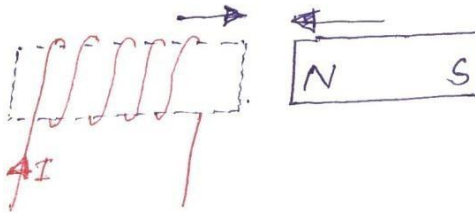


Fig. 1.2

If two soft iron pieces are placed near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

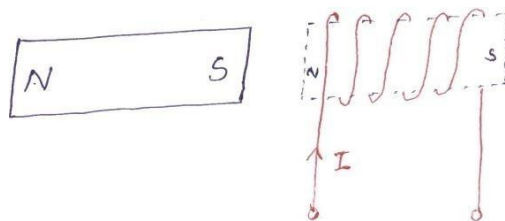


Fig. 1.3

Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electro-dynamometer type instrument.

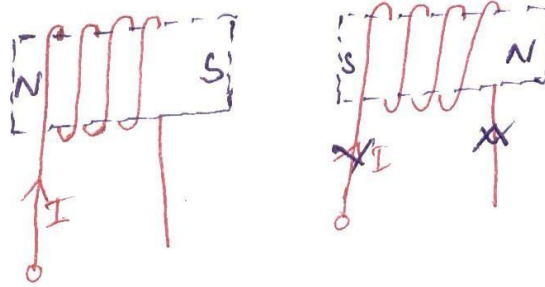


Fig. 1.4

Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \quad (1.1)$$

Spring control

Two springs are attached on either end of spindle (Fig. 1.5). The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ .

$$T_C \propto \theta \quad (1.2)$$

The deflecting torque produced T_d proportional to 'I'. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \quad (1.3)$$

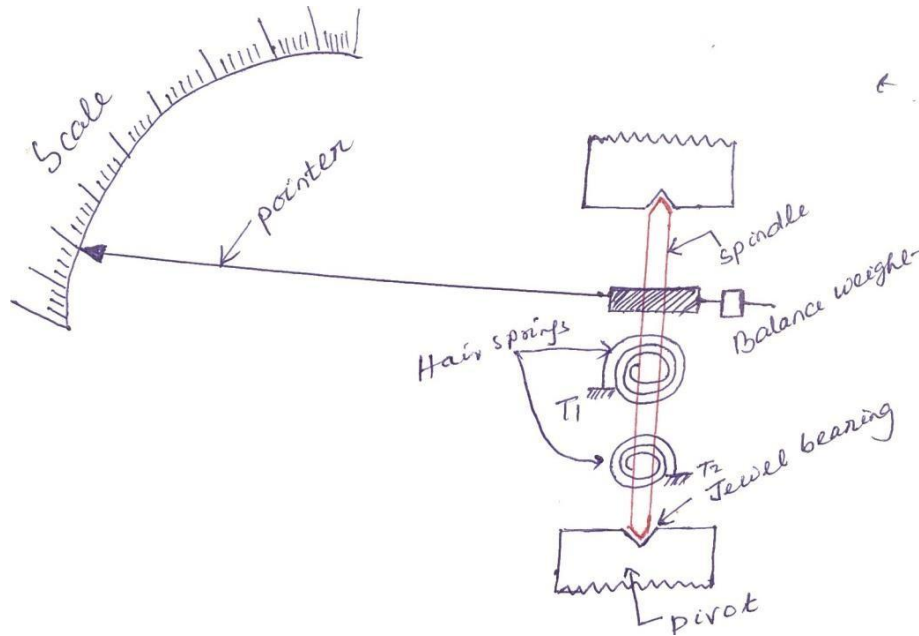


Fig. 1.5

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about its final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation quickly, a damping force is necessary. This force is produced by different systems.

- (a) Air friction damping
- (b) Fluid friction damping
- (c) Eddy current damping

Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

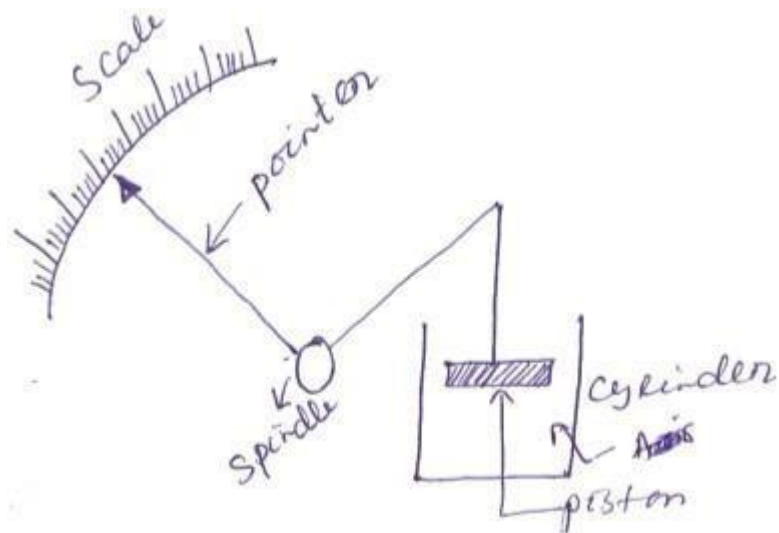


Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

Eddy current damping

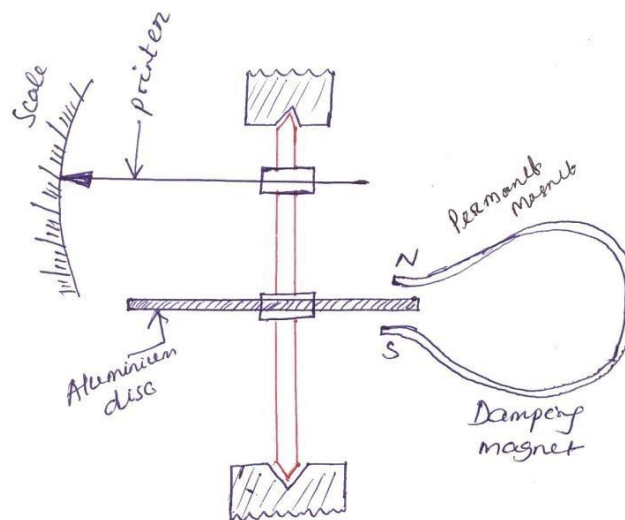


Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by Faraday's law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produces a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

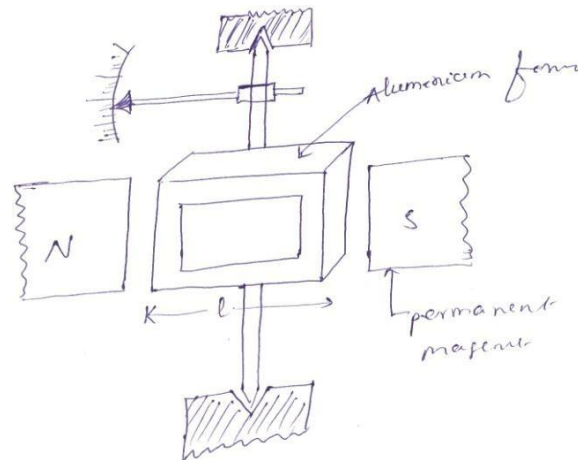


Fig. 1.6 Rectangular type

Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

Construction: A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.

Control: Spring control is used.

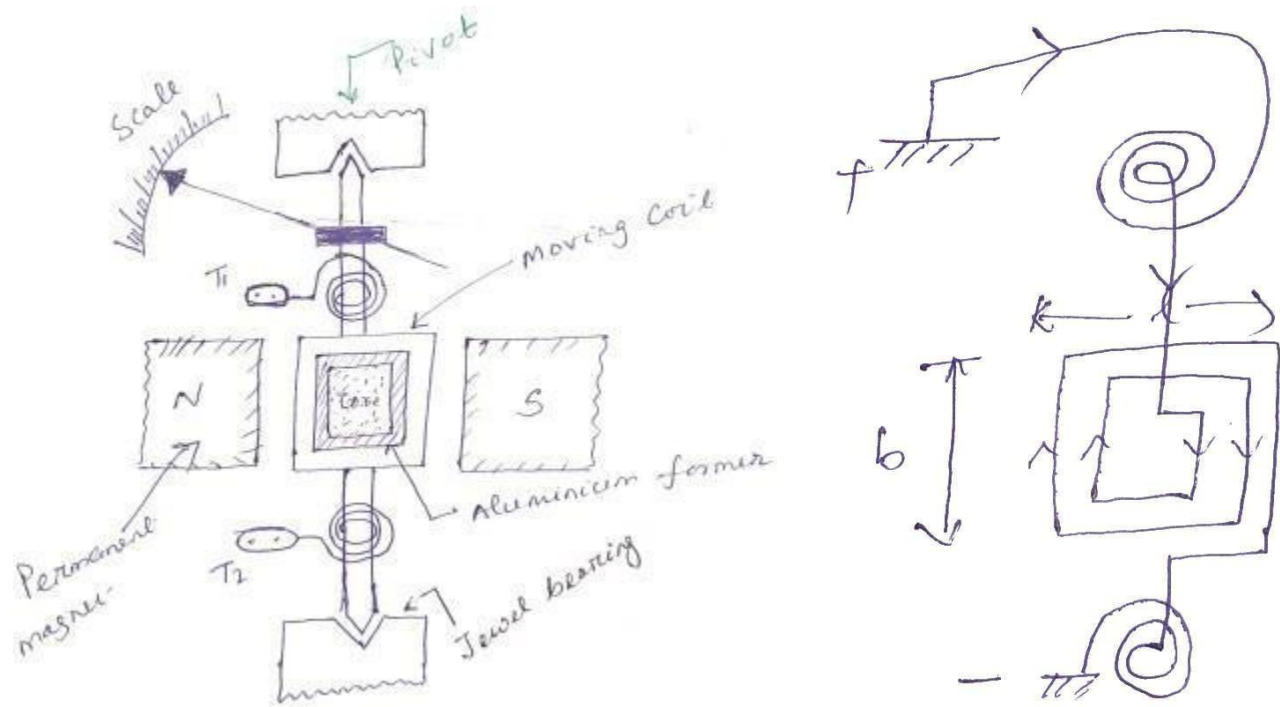


Fig. 1.7

Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.

If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC

Let T_d = deflecting torque
 T_C = controlling torque
 θ = angle of deflection
 K = spring constant
 b = width of the coil

l =height of the coil or length of coil

N =No. of turns

I =current

B =Flux density

A =area of the coil

The force produced in the coil is given by

$$F = BIL \sin\theta \quad (1.4)$$

When $\theta = 90^\circ$

$$\text{For } N \text{ turns, } F = NBIL \quad (1.5)$$

$$\text{Torque produced } T_d = F \times \perp_r \text{ distance} \quad (1.6)$$

$$T_d = NBIL \times b = BINA \quad (1.7)$$

$$T_d = BAN I \quad (1.8)$$

$$T_d \propto I \quad (1.9)$$

Advantages

- ✓ Torque/weight is high
- ✓ Power consumption is less
- ✓ Scale is uniform
- ✓ Damping is very effective
- ✓ Since operating field is very strong, the effect of stray field is negligible
- ✓ Range of instrument can be extended

Disadvantages

- ✓ Use only for D.C.
- ✓ Cost is high
- ✓ Error is produced due to ageing effect of PMMC
- ✓ Friction and temperature error are present

Extension of range of PMMC instrument Case-**I: Shunt**

A low shunt resistance connected in parallel with the ammeter to extend the range of current. Large current can be measured using low current rated ammeter by using a shunt.

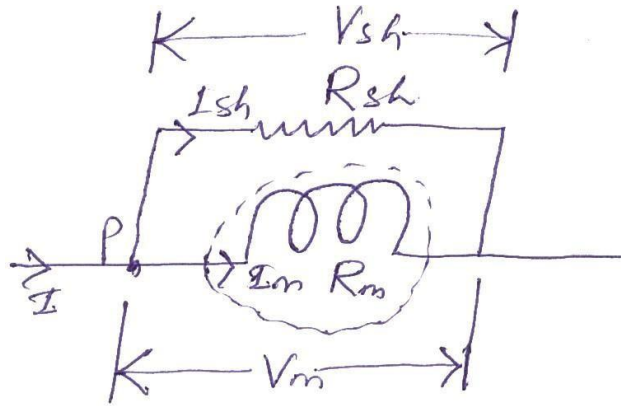


Fig. 1.8

Let R_m = Resistance of meter

R_{sh} = Resistance of shunt

I_m = Current through meter

I_{sh} = current through shunt

I = current to be measure

$$\therefore V_m = V_{sh} \quad (1.10)$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m} \quad (1.11)$$

Apply KCL at 'P' $I = I_m + I_{sh}$ (1.12)

Eqⁿ (1.12) \div by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m} \quad (1.13)$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \quad (1.14)$$

$$\therefore I = I_m \left(1 + \frac{R_m}{R_{sh}} \right) \quad (1.15)$$

$\left(1 + \frac{R_m}{R_{sh}} \right)$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermoelectric emf. The change in resistance, due to change in temperature is negligible.

Case (II): Multiplier

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter of small rating with a multiplier.

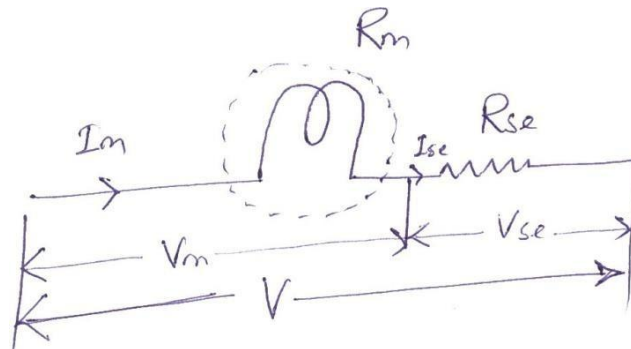


Fig. 1.9

Let R_m = resistance of meter

R_{se} = resistance of multiplier

V_m = Voltage across meter

V_{se} = Voltage across series resistance

V = voltage to be measured

$$I_m = I_{se} \quad (1.16)$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}} \quad (1.17)$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m} \quad (1.18)$$

Apply KVL, $V = V_m + V_{se}$ (1.19)

Eqⁿ (1.19) $\div V_m$

$$\frac{V}{m} = 1 + \frac{V_{se}}{m} = \left(1 + \frac{R_{se}}{m} \right) \frac{V}{m}$$
 (1.20)

$$\therefore V = V_m \left(1 + \frac{R_{se}}{m} \right)$$
 (1.21)

$$\left(1 + \frac{R_{se}}{m} \right) \rightarrow \text{Multiplication factor}$$

Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let ‘ θ ’ be the deflection corresponding to a current of ‘ i ’ amp

Let the current increases by di , the corresponding deflection is ‘ $\theta + d\theta$ ’

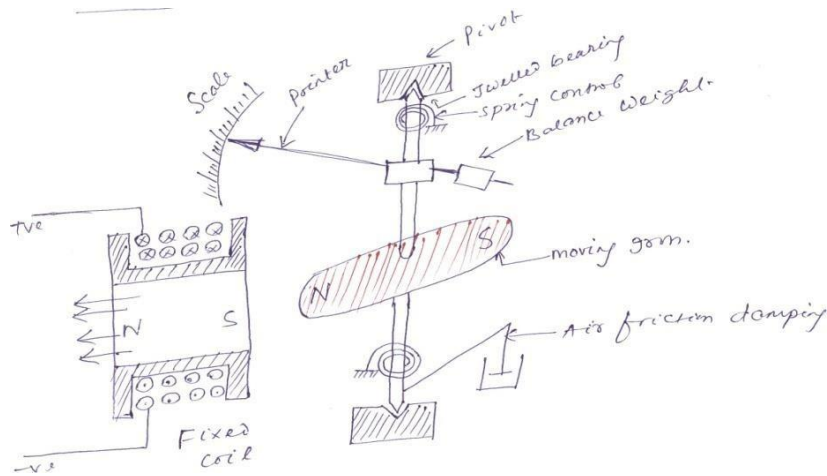


Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ' $L+dL$ '. The current change by ' di ' is dt seconds.

Let the emf induced in the coil be ' e ' volt.

$$e = \frac{d}{dt} (Li) = L \frac{di}{dt} + i \frac{dL}{dt} \tag{1.22}$$

Multiplying by ' idt ' in equation (1.22)

$$e \times idt = L \frac{di}{dt} \times idt + i \frac{dL}{dt} \times idt \tag{1.23}$$

$$e \times idt = Lidi + i^2 dL \tag{1.24}$$

Eqⁿ (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.

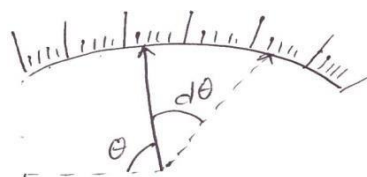
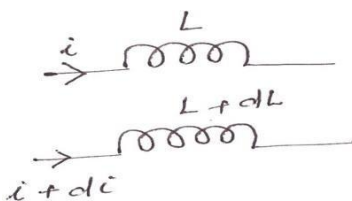


Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$\begin{aligned}
 &= \frac{1}{2} (L + dL)(i + di)^2 - \frac{1}{2} Li^2 \\
 &= \frac{1}{2} \{ (L + dL)(i^2 + di^2 + 2idi) - Li^2 \} \\
 &= \frac{1}{2} \{ (L + dL)(i^2 + 2idi) - Li^2 \} \\
 &= \frac{1}{2} \{ Li^2 + 2Lidi + i^2dL + 2ididL - Li^2 \} \\
 &= \frac{1}{2} \{ 2Lidi + i^2dL \} \\
 &= Lidi + \frac{1}{2} i^2dL
 \end{aligned} \tag{1.25}$$

Mechanical work to move the pointer by $d\theta$

$$= T_d d\theta \tag{1.26}$$

By law of conservation of energy,

Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi + i^2dL = Lidi + \frac{1}{2} i^2dL + T_d d\theta \tag{1.27}$$

$$\frac{1}{2} i^2dL = T_d d\theta \tag{1.28}$$

$$T_d = \frac{1}{2} i^2 \frac{dL}{d\theta} \tag{1.29}$$

At steady state condition $T_d = T_C$

$$\frac{1}{2} i^2 \frac{dL}{d\theta} = K\theta \tag{1.30}$$

$$\theta = \frac{1}{2K} i^2 \frac{dL}{d\theta} \tag{1.31}$$

$$\theta \propto i^2 \tag{1.32}$$

When the instruments measure AC, $\theta \propto i_{rms}^2$

Scale of the instrument is non uniform.

Advantages

- ✓ MI can be used in AC and DC
- ✓ It is cheap
- ✓ Supply is given to a fixed coil, not in moving coil.
- ✓ Simple construction
- ✓ Less friction error.

Disadvantages

- ✓ It suffers from eddy current and hysteresis error
- ✓ Scale is not uniform
- ✓ It consumed more power
- ✓ Calibration is different for AC and DC operation

Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

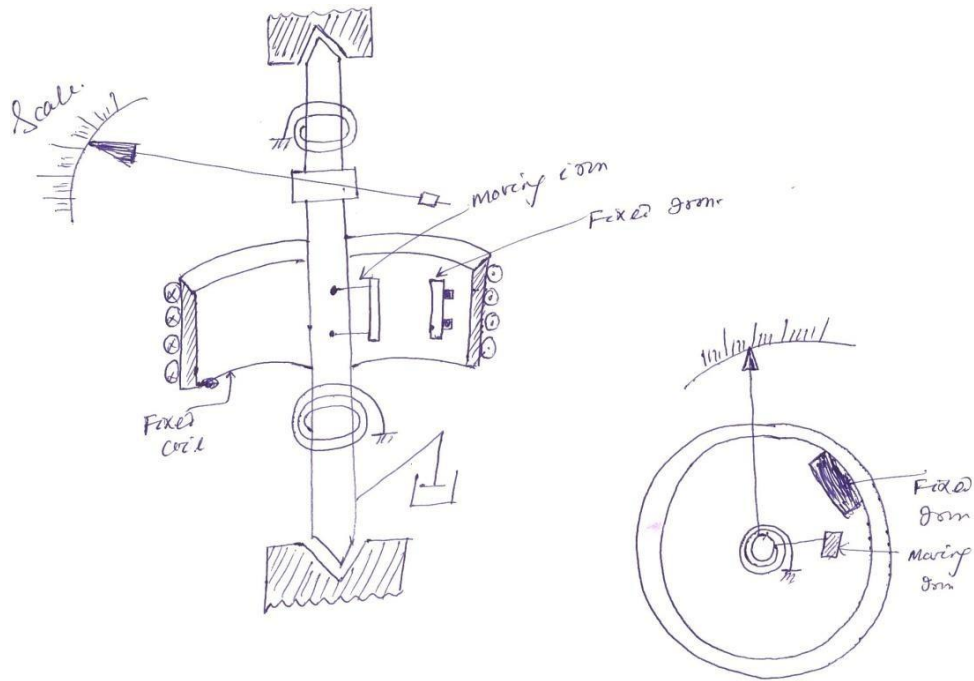


Fig. 1.12

Dynamometer (or) Electromagnetic moving coil instrument (EMMC)

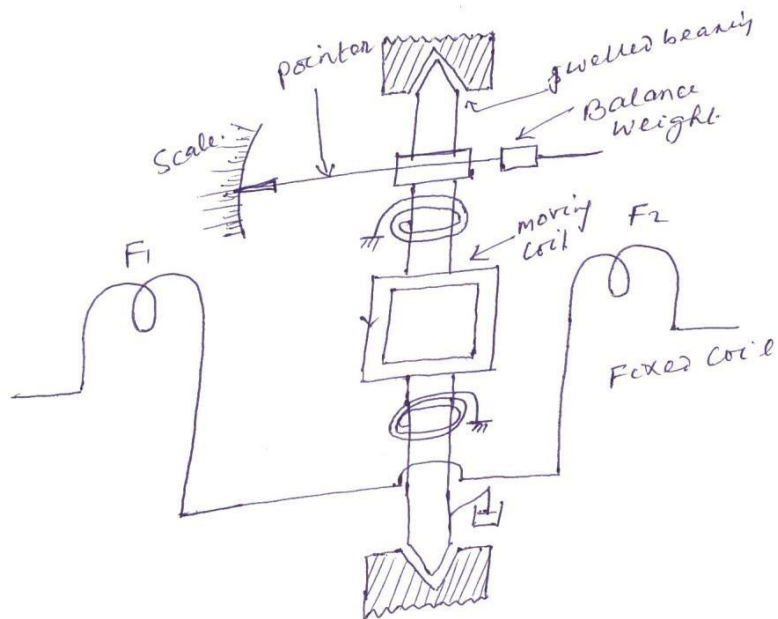


Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction: A fixed coil is divided into two equal halves. The moving coil is placed between the two halves of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non-metallic former the moving coil is wound.

Control: Spring control is used.

Damping: Air friction damping is used.

Principle of operation:

When the current flows through the fixed coil, it produces a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC

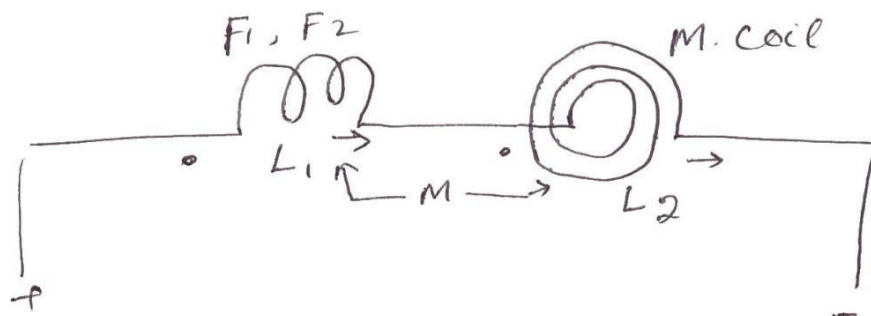


Fig. 1.14

Let

L_1 =Self inductance of fixed coil

L_2 = Self inductance of moving coil

M =mutual inductance between fixed coil and moving coil

i_1 =current through fixed coil

i_2 =current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \quad (1.33)$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \quad (1.34)$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta} (L_1 + L_2 + 2M) \quad (1.35)$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \quad (1.36)$$

$$T_d = i^2 \frac{dM}{d\theta} \quad (1.37)$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \quad (1.38)$$

$$T_C = T_d \quad (1.39)$$

$$\therefore \theta = \frac{i_1 i_2 dM}{K d\theta} \quad (1.40)$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

Extension of EMMC instrument

Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same.

Therefore

$$I_1 = I_2 = I$$

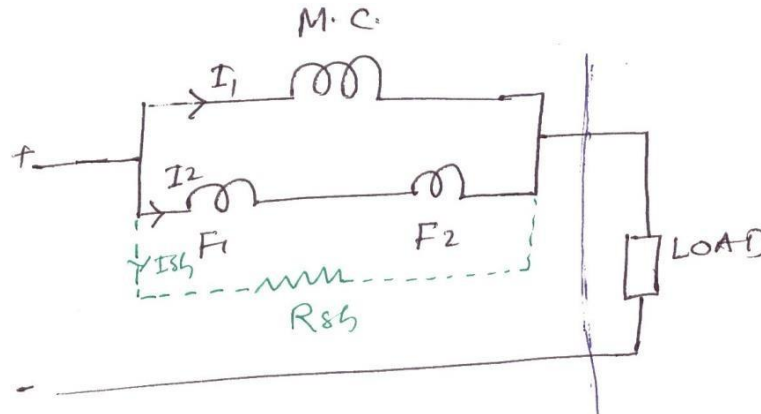


Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value R_{sh} is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \tag{1.41}$$

$$\text{Or } \therefore T_d = I^2 \frac{dM}{d\theta} \tag{1.42}$$

$$T_C = K\theta \tag{1.43}$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \tag{1.44}$$

$$\therefore \theta \propto I^2 \text{ (Scale is not uniform)} \tag{1.45}$$

Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extent the range of voltmeter.

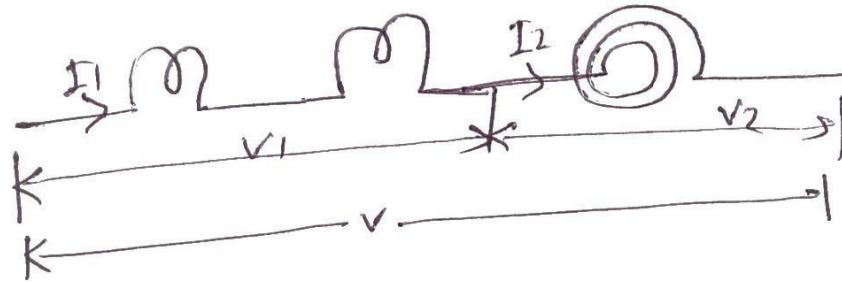


Fig. 1.16

$$I = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \tag{1.46}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \tag{1.47}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \tag{1.48}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \tag{1.49}$$

$$T_d \propto V^2 \tag{1.50}$$

$$\therefore \theta \propto V^2 \text{ (Scale is not uniform)} \tag{1.51}$$

Case-III As wattmeter

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

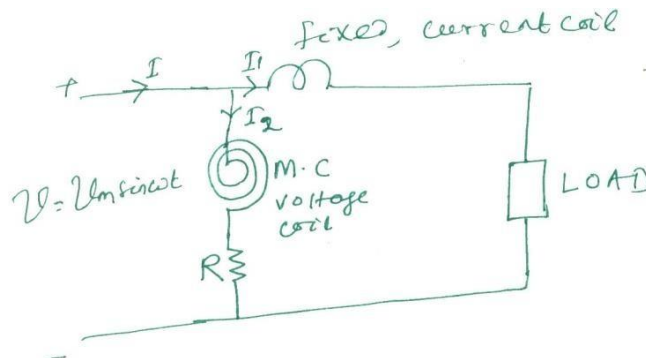


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R'. The current,

$$I_2 = \frac{V_m \sin wt}{R} \quad (1.52)$$

Let the phase difference between the currents I_1 and I_2 is ϕ

$$I_1 = I_m \sin(wt - \phi) \quad (1.53)$$

$$T_d = I_1 I_2 \frac{dM}{d\theta} \quad (1.54)$$

$$T_d = I_m \sin(wt - \phi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta} \quad (1.55)$$

$$T_d = \frac{1}{R} (I_m V_m \sin wt \sin(wt - \phi)) \frac{dM}{d\theta} \quad (1.56)$$

$$T_d = \frac{1}{R} I_m V_m \sin wt \sin(wt - \phi) \frac{dM}{d\theta} \quad (1.57)$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} T_d dwt \quad (1.58)$$

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{R} \times I_m V_m \sin wt \sin(wt - \phi) \frac{dM}{d\theta} dwt \quad (1.59)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\pi} \times \frac{1}{R} \times \frac{dM}{d\theta} \left[\int_0^{2\pi} \{\cos\phi - \cos(2wt - \phi)\} dwt \right] \quad (1.60)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} \left[\int_0^{2\pi} \cos\phi \cdot dwt - \int_0^{2\pi} \cos(2wt - \phi) \cdot dwt \right] \quad (1.61)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} \left[\cos\phi [wt]_0^{2\pi} \right] \quad (1.62)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} [\cos\phi(2\pi - 0)] \quad (1.63)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{1}{2} \times \frac{dM}{d\theta} \times \cos\phi \quad (1.64)$$

$$(T_d)_{avg} = \frac{2}{R} \times I_{rms} \times \cos\phi \times \frac{1}{2} \times \frac{dM}{d\theta} \quad (1.65)$$

$$(T_d)_{avg} \propto KVI \cos\phi \quad (1.66)$$

$$T_C \propto \theta \quad (1.67)$$

$$\theta \propto KVI \cos\phi \quad (1.68)$$

$$\theta \propto VI \cos\phi \quad (1.69)$$

Advantages

- ✓ It can be used for voltmeter, ammeter and wattmeter
- ✓ Hysteresis error is nill
- ✓ Eddy current error is nill
- ✓ Damping is effective
- ✓ It can be measure correctively and accurately the rms value of the voltage

Disadvantages

- ✓ Scale is not uniform
- ✓ Power consumption is high(because of high resistance)
- ✓ Cost is more
- ✓ Error is produced due to frequency, temperature and stray field.
- ✓ Torque/weight is low.(Because field strength is very low)

Errors in PMMC

- ✓ The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- ✓ The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- ✓ When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

Difference between attraction and repulsion type instrument

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

Characteristics of meter**Full scale deflection current(I_{FSD})**

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between $2 \mu A$ to $30mA$.

Resistance of the coil(R_m)

This is ohmic resistance of the moving coil. It is due to ρ , L and A. For an ammeter this should be as small as possible.

Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}} (\Lambda / volt), \uparrow S = \frac{Z \uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in $\Omega/volt$. When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

Error in M.I instrument**Temperature error**

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_S)^2 + X_L^2} \tag{1.70}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_S)^2 + X_L^2}} \tag{1.71}$$

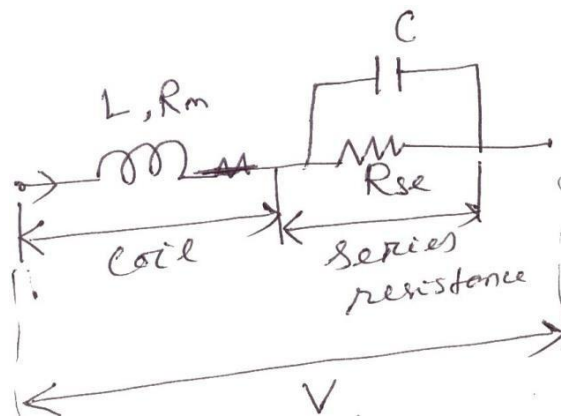


Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, $(X_L - X_C)$ is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_S)^2} \tag{1.72}$$

Electrostatic instrument

In multi cellular construction several vans and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes

and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

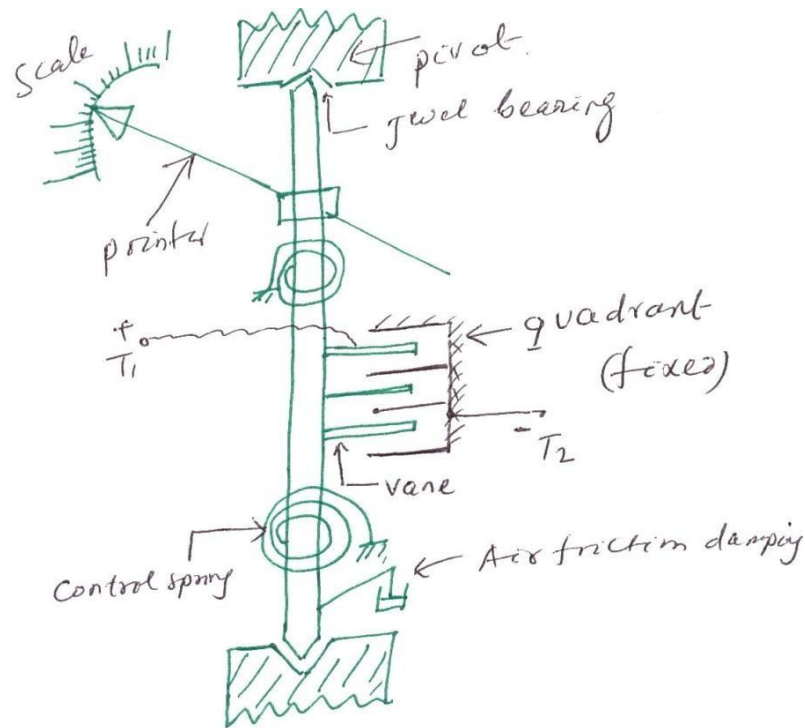


Fig. 1.19

Torque develop by electrostatic instrument

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant

$$\text{Energy stored} = \frac{1}{2} CV^2 \tag{1.73}$$

Let 'θ' be the deflection corresponding to a voltage V.

Let the voltage increases by dv, the corresponding deflection is 'θ + dθ'

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt}V + C \frac{dV}{dt} \tag{1.74}$$

V × dt multiply on both side of equation (1.74)

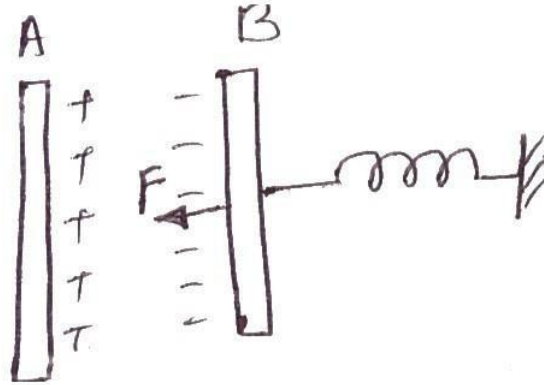


Fig. 1.20

$$Vidt = \frac{dC}{dt} V^2 dt + CV \frac{dV}{dt} dt \quad (1.75)$$

$$Vidt = V^2 dC + CVdV \quad (1.76)$$

$$\text{Change in stored energy} = \frac{1}{2} (C + dC)(V + dV)^2 - \frac{1}{2} CV^2 \quad (1.77)$$

$$\begin{aligned} &= \frac{1}{2} [(C + dC)V^2 + dV^2 + 2VdV] - \frac{1}{2} CV^2 \\ &= \frac{1}{2} [CV^2 + CdV^2 + 2CVdV + V^2 dC + dCdV^2 + 2VdVdC] - \frac{1}{2} CV^2 \\ &= \frac{1}{2} V^2 dC + CVdV \\ V^2 dC + CVdV &= \frac{1}{2} V^2 dC + CVdV + F \times rd\theta \end{aligned} \quad (1.78)$$

$$T \times d\theta = \frac{1}{2} V^2 dC \quad (1.79)$$

$$T_d = \frac{1}{2} V^2 \left(\frac{dC}{d\theta} \right) \quad (1.80)$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2} V^2 \left(\frac{dC}{d\theta} \right) \quad (1.81)$$

$$\theta = \frac{1}{2K} V^2 \left(\frac{dC}{d\theta} \right) \quad (1.82)$$

Advantages

- ✓ It is used in both AC and DC.
- ✓ There is no frequency error.
- ✓ There is no hysteresis error.
- ✓ There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- ✓ It is used for high voltage
- ✓ Power consumption is negligible.

Disadvantages

- ✓ Scale is not uniform
- ✓ Large in size
- ✓ Cost is more

Multi range Ammeter

When the switch is connected to position (1), the supplied current I_1

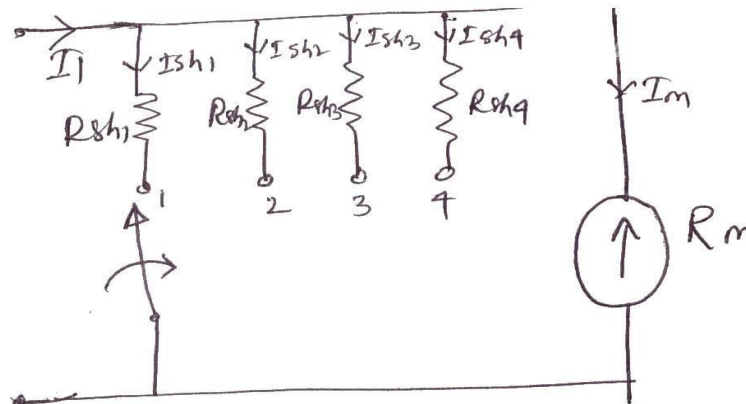


Fig. 1.21

$$I_{sh1}R_{sh1} = I_m R_m \quad (1.83)$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \quad (1.84)$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m} \tag{1.85}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m} \tag{1.86}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m} \tag{1.87}$$

Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \tag{1.88}$$

$$R_2 = R_{sh2} - R_{sh3} \tag{1.89}$$

$$R_3 = R_{sh3} - R_{sh4} \tag{1.90}$$

$$R_4 = R_{sh4} \tag{1.91}$$

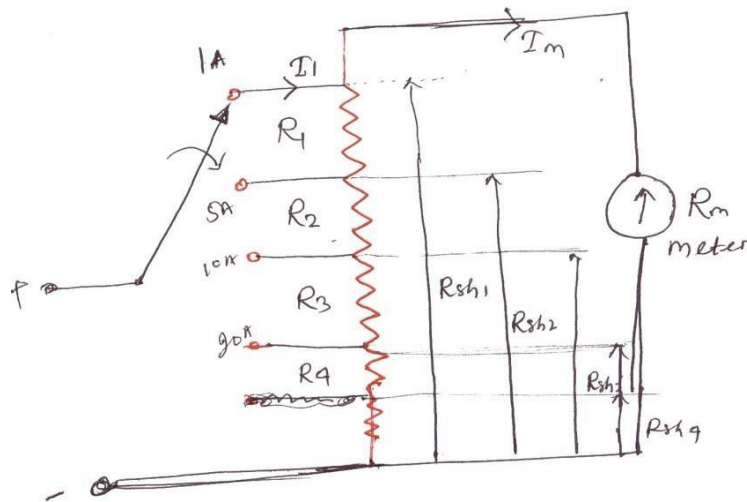


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Ayrton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

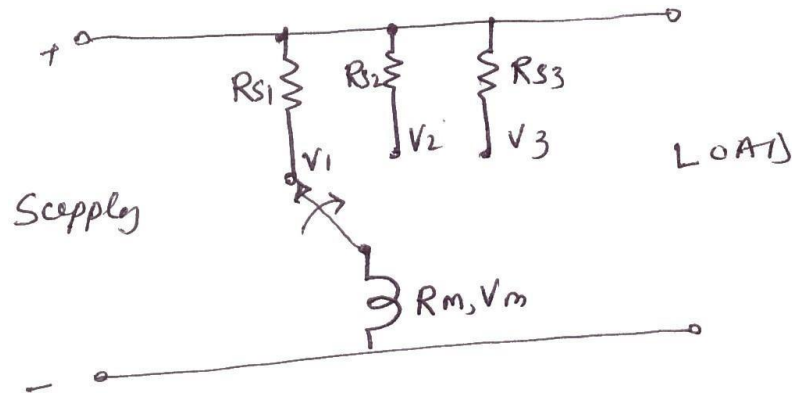
Multi range D.C. voltmeter

Fig. 1.23

$$R_{s1} = R_m (m_1 - 1)$$

$$R_{s2} = R_m (m_2 - 1) \quad (1.92)$$

$$R_{s3} = R_m (m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m}, m_2 = \frac{V_2}{V_m}, m_3 = \frac{V_3}{V_m} \quad (1.93)$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required.

Potential divider arrangement

The resistance \$R_1, R_2, R_3\$ and \$R_4\$ is connected in series to obtained the ranges \$V_1, V_2, V_3\$ and \$V_4\$

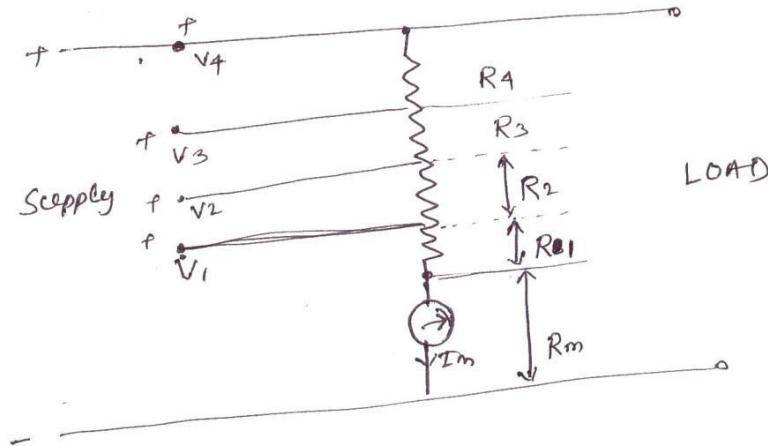


Fig. 1.24

Consider for voltage V_1 , $(R_1 + R_m)I_m = V_1$

$$\therefore R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_1}{R_m}\right)} - R_m = \left(\frac{V_1}{V_1}\right) R_m - R_m \quad (1.94)$$

$$R_1 = (m_1 - 1)R_m \quad (1.95)$$

$$\text{For } V_2, (R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m \quad (1.96)$$

$$R_2 = \frac{V_2}{\left(\frac{V_2}{R_m}\right)} - (m_1 - 1)R_m - R_m \quad (1.97)$$

$$\begin{aligned} R_2 &= m_2 R_m - R_m - (m_1 - 1)R_m \\ &= R_m(m_2 - 1 - m_1 + 1) \end{aligned} \quad (1.98)$$

$$R_2 = (m_2 - m_1)R_m \quad (1.99)$$

For V_3 $(R_3 + R_2 + R_1 + R_m)I_m = V_3$

$$\begin{aligned} R_3 &= \frac{V_3}{I_m} - R_2 - R_1 - R_m \\ &= \frac{V_3}{\frac{V_3}{R_m}} - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ &= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ R_3 &= (m_3 - m_2)R_m \end{aligned}$$

$$\text{For } V_4 \quad (R_4 + R_3 + R_2 + R_1 + R_m)I_m = V_4$$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$= \left(\frac{V_4}{m} \right) R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_4 = R_m [m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1]$$

$$R_4 = (m_4 - m_3)R_m$$

Example: 1.1

A PMMC ammeter has the following specification

Coil dimension are 1cm×1cm. Spring constant is $0.15 \times 10^{-6} \text{ N-m/rad}$, Flux density is $1.5 \times 10^{-3} \text{ wb/m}^2$. Determine the no. of turns required to produce a deflection of 90° when a current 2mA flows through the coil.

Solution:

At steady state **condition** $T_d = T_C$

$$BANl = K\theta$$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

$$A = 1 \times 10^{-4} \text{ m}^2$$

$$K = 0.15 \times 10^{-6} \frac{\text{N-m}}{\text{rad}}$$

$$B = 1.5 \times 10^{-3} \text{ wb/m}^2$$

$$I = 2 \times 10^{-3} \text{ A}$$

$$\theta = 90^\circ = \frac{\pi}{2} \text{ rad}$$

$$N = 785 \text{ ans.}$$

Example: 1.2

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV. The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Solution:

Case-1

$$V_m = 400\text{mV}$$

$$I_m = 20\text{mA}$$

$$I = 200\text{A}$$

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right)$$

$$200 = 20 \times 10^{-3} \left[1 + \frac{20}{R_{sh}} \right]$$

$$R_{sh} = 2 \times 10^{-3} \Omega$$

Case-II

$$V = 1000\text{V}$$

$$V = V_m \left(1 + \frac{R_{se}}{R_m} \right)$$

$$4000 = 400 \times 10^{-3} \left(1 + \frac{R_{se}}{20} \right)$$

$$R_{se} = 49.98\text{k}\Omega$$

Example: 1.3

A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

Solution:

$$R_m = 3\text{k}\Omega, V_m = 150\text{V}, V = 300\text{V}$$

$$V = V_m \left(1 + \frac{R_{se}}{R_m} \right)$$

$$300 = 150 \left(1 + \frac{R_{se}}{3} \right) \Rightarrow R_{se} = 3k\Omega$$

Case-II $V = V_m \left(1 + \frac{R_{se}}{R_m} \right)$

$$200 = V_m \left(1 + \frac{3}{3} \right)$$

$$\therefore V_m = 100V \text{ Ans}$$

Example: 1.4

What is the value of series resistance to be used to extent ‘0’ to 200V range of 20,000Ω/volt voltmeter to 0 to 2000 volt?

Solution:

$$V_{se} = V - V = 1800$$

$$I_{FSD} = \frac{1}{20000} = \frac{1}{\text{Sensitivity}}$$

$$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega \text{ ans.}$$

Example: 1.5

A moving coil instrument whose resistance is 25Ω gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA.

Calculate the error caused by a 10⁰C rise in temperature when:

- (a) Copper moving coil is connected directly across the manganin shunt.
- (b) A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is 0.004/⁰C and that of manganin is 0.00015/⁰C.

Solution:

Case-1

$$I_m = 1mA$$

$$R_m = 25\Omega$$

$I=100\text{mA}$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 1 \left(1 + \frac{25}{R_{sh}} \right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for 10°C rise in temperature, $R_{mt} = 25(1 + 0.004 \times 10)$

$$R_t = R_o(1 + \rho_t \times t)$$

$$R_{m/t=10^\circ} = 26\Omega$$

Shunt resistance for 10°C , rise in temperature

$$R_{sh/t=10^\circ} = 0.2525(1 + 0.00015 \times 10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for 10°C rise in temperature

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right) \Big|_{t=10^\circ\text{C}}$$

$$100 = I_{mt} \left(1 + \frac{26}{0.2529} \right)$$

$$I_m|_{t=10} = 0.963\text{mA}$$

But normal meter current=1mA

Error due to rise in temperature=(0.963-1)*100=-3.7%

Case-b As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25 + 75 = 100\Omega$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 1 \left(1 + \frac{100}{R_{sh}} \right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for 10°C rise in temperature

$$R_m|_{t=10} = 25(1 + 0.004 \times 10) + 75(1 + 0.00015 \times 10) = 101.11\Omega$$

Shunt resistance for 10°C rise in temperature

$$R_{sh}|_{t=10} = 1.01(1 + 0.00015 \times 10) = 1.0115\Omega$$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = I_m \left(1 + \frac{101.11}{1.0115} \right)$$

$$I_m|_{t=10} = 0.9905\text{mA}$$

$$\text{Error} = (0.9905 - 1) \times 100 = -0.95\%$$

Example: 1.6

The coil of a 600V M.I meter has an inductance of 1 henry. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

Solution:

$$V_m = 600\text{V}, I_m = 100\text{mA}$$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\pi fL = 314\Omega$$

$$R_m = \sqrt{Z_m^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33\text{mA}$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39\text{mA}$$

$$\text{Error} = \frac{I_{DC} - I_{AC}}{I_{AC}} \times 100 = \frac{33.39 - 33.33}{33.33} \times 100 = 0.18\%$$

Example: 1.7

A 250V M.I. voltmeter has coil resistance of 500Ω, coil inductance of 1.04 H and series resistance of 2kΩ. The meter reads correctly at 250V D.C. What will be the value of capacitance to be used for shunting the series resistance to make the meter read correctly at 50HZ? What is the reading of voltmeter on A.C. without capacitance?

Solution:

$$C = 0.41 \frac{L}{(R_S)^2}$$

$$= 0.41 \times \frac{1.04}{(2 \times 10^3)^2} = 0.1 \mu F$$

For A.C $Z = \sqrt{(R_m + R_{Se})^2 + X_L^2}$

$$Z = \sqrt{(500 + 2000)^2 + (314)^2} = 2520 \Omega$$

With D.C

$$R_{total} = 2500 \Omega$$

For 2500Ω → 250V

$$1 \Omega \rightarrow \frac{250}{2500}$$

$$2520 \Omega \rightarrow \frac{250}{2500} \times 2520 = 248V$$

Example: 1.8

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance (μH)	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is 1.5A?

Solution:

For current I=1.5A, θ =55.5 degree=0.96865 rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11 \mu H / \text{deg} = 6.3 \mu H / \text{rad}$$

$$\text{Deflecting torque, } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2} (1.5)^2 \times 6.3 \times 10^{-6} = 7.09 \times 10^{-6} \text{ N - m}$$

$$\text{Spring constant, } K = \frac{T_d}{\theta} = \frac{7.09 \times 10^{-6}}{0.968} = 7.319 \times 10^{-6} \frac{\text{N - m}}{\text{rad}}$$

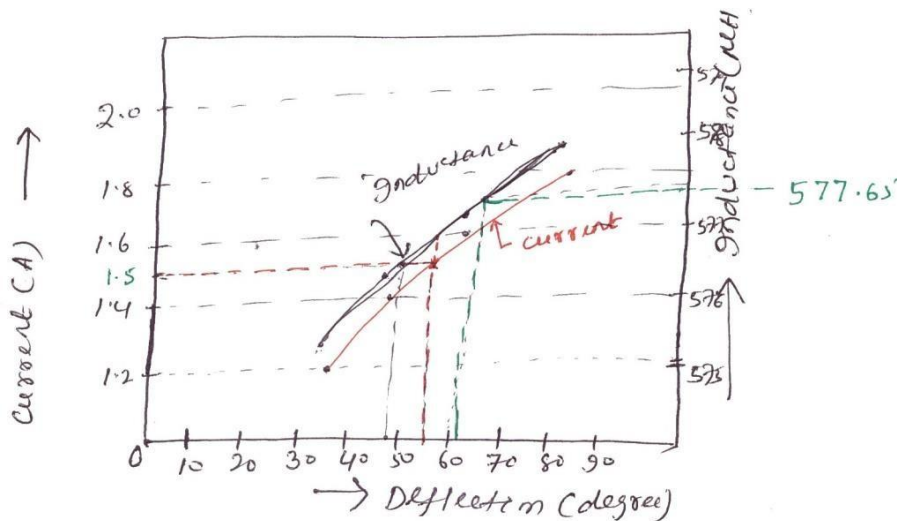


Fig. 1.25

Example: 1.9

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6 \cos(\theta + 30^\circ) \text{ mH}$. Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60° .

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6 \cos(\theta + 30^\circ)$$

$$\frac{dM}{d\theta} = 6 \sin(\theta + 30^\circ) \text{ mH}$$

$$\left. \frac{dM}{d\theta} \right|_{\theta=60} = 6 \sin 90 = 6 \text{ mH / deg}$$

$$T_d = I^2 \frac{dM}{d\theta} = (50 \times 10^{-3})^2 \times 6 \times 10^{-3} = 15 \times 10^{-6} \text{ N - m}$$

Example: 1.10

The inductance of a moving iron ammeter with a full scale deflection of 90° at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

Solution:

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\frac{dL}{d\theta} \Big|_{\theta=90^\circ} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\frac{dL}{d\theta} \Big|_{\theta=90} = 40 - 8 \times \frac{\pi}{2} - 3 \left(\frac{\pi}{2}\right)^2 \mu H / rad = 20 \mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$$

$$\frac{\pi}{2} = \frac{1}{2} \frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

$K = \text{Spring constant} = 14.32 \times 10^{-6} N - m / rad$

For $I=1A$, $\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}} (40 - 8\theta - 3\theta^2)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

Example: 1.11

The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12 \times 10^{-6} N - m / rad$. Estimate the deflection for a current of 5A.

Solution:

$$\frac{dL}{d\theta} = (5 - 2\theta) \frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5 - 2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 \text{ rad}, 96.8^\circ$$

Example: 1.12

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance (μH)	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a) 30° (b) 80° . Given that control spring constant is $0.4 \times 10^{-6} N - m / \text{deg ree}$

Solution:

$$\theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta} \right)$$

(a) For $\theta = 30^\circ$

The curve is linear

$$\therefore \left(\frac{dL}{d\theta} \right)_{\theta=30} = \frac{355.5 - 335}{40 - 20} = 1.075 \mu H / \text{deg ree} = 58.7 \mu H / \text{rad}$$



Fig. 1.26

Example: 1.13

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through 90° . The torsional constant is $10 \times 10^{-6} \text{ N - m / rad}$. The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is 90° ?

Solution:

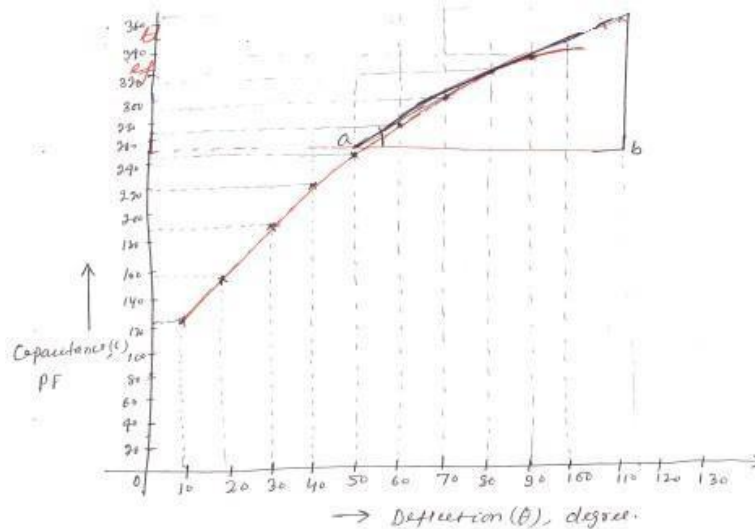


Fig. 1.27

$$\frac{dC}{d\theta} = \tan\theta = \frac{bc}{ab} = \frac{370-250}{110-44} = 1.82PF / \text{deg ree} = 104.2PF / \text{rad}$$

Spring constant $K = 10 \times 10^{-6} \frac{N-m}{rad} = 0.1745 \times 10^{-6} N-m / \text{deg ree}$

$$\theta = \frac{1}{2K} V^2 \left(\frac{dC}{d\theta} \right) \Rightarrow V = \sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V = \sqrt{\frac{2 \times 0.1745 \times 10^{-6} \times 90}{104.2 \times 10^{-12}}} = 549 \text{ volt}$$

Example: 1.14

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1\text{mA}$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

\therefore Shunt resistance $R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

Case-II 0-50mA

$m = \frac{50}{1} = 50$

$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$

Case-IV 0-500mA, $m = \frac{500}{1} = 500\Omega$

$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$

Example: 1.15

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120° , when a potential difference of 100mV is applied across it. The moving coil has dimension of $30\text{mm} \times 25\text{mm}$ and is wound with 100 turns. The control spring constant is

$0.375 \times 10^{-6} \text{ N - m / deg ree}$. Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper = $1.7 \times 10^{-8} \Lambda \text{m}$.

Solution:

Data given

$$V_m = 100\text{mV}$$

$$R_m = 20\Lambda$$

$$\theta = 120^\circ$$

$$N = 100$$

$$K = 0.375 \times 10^{-6} \text{ N - m / deg ree}$$

$$R_C = 30\% \text{ of } R_m$$

$$\rho = 1.7 \times 10^{-8} \Lambda \text{m}$$

$$I_m = \frac{V_m}{R_m} = 5 \times 10^{-3} \text{ A}$$

$$T_d = BAN I, T_C = K\theta = 0.375 \times 10^{-6} \times 120 = 45 \times 10^{-6} \text{ N - m}$$

$$B = \frac{T_d}{AN I} = \frac{45 \times 10^{-6}}{30 \times 25 \times 10^{-6} \times 100 \times 5 \times 10^{-3}} = 0.12 \text{ wb / m}^2$$

$$R_C = 0.3 \times 20 = 6\Lambda$$

Length of mean turn path = $2(a+b) = 2(55) = 110\text{mm}$

$$R_C = N \left(\frac{\rho l}{A} \right)$$

$$A = \frac{N \times \rho \times (l_t)}{R_C} = \frac{100 \times 1.7 \times 10^{-8} \times 110 \times 10^{-3}}{6}$$

$$= 3.116 \times 10^{-8} m^2$$

$$= 31.16 \times 10^{-3} mm^2$$

$$A = \frac{\Pi}{4} d^2 \Rightarrow d = 0.2mm$$

Example: 1.16

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

- (1) The shunt resistance for a full scale deflection corresponding to 100A
- (2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

Solution:

Data given

$$I_m = 10mA$$

$$V_m = 100mV$$

$$I = 100A$$

$$I = I_m \left(1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 10 \times 10^{-3} \left(1 + \frac{10}{R_{sh}} \right)$$

$$R_{sh} = 1.001 \times 10^{-3} \Omega$$

$$R_{se} = ??, V = 1000V$$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10 \Omega$$

$$V = V_m \left(1 + \frac{R_{se}}{R_m} \right)$$

$$1000 = 100 \times 10^{-3} \left(1 + \frac{R_{se}}{10} \right)$$

$$\therefore R_{se} = 99.99K \Omega$$

Example: 1.17

Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of 50w and a full scale deflection current of 1mA is to be used.

Solution: Data given

$$I_m = 1 \times 10^{-3} \text{ A} \quad R_m = 50 \Omega \quad \left| \begin{array}{l} I_1 = 1 \text{ A} \\ I_2 = 5 \text{ A} \\ I_3 = 10 \text{ A} \\ I_4 = 20 \text{ A} \end{array} \right. \quad \left| \begin{array}{l} m_1 = \frac{I_1}{I_m} = 1000 \text{ A} \\ m_2 = \frac{I_2}{I_m} = 5000 \text{ A} \\ m_3 = \frac{I_3}{I_m} = 10000 \text{ A} \\ m_4 = \frac{I_4}{I_m} = 20000 \text{ A} \end{array} \right.$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000 - 1} = 0.05 \Omega$$

$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000 - 1} = 0.01 \Omega$$

$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000 - 1} = 0.005 \Omega$$

$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000 - 1} = 0.0025 \Omega$$

∴ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04 \Omega$$

$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005 \Omega$$

$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.0025 = 0.0025 \Omega$$

$$R_4 = R_{sh4} = 0.0025 \Omega$$

Example: 1.18

A basic d’ Arsonval meter movement with an internal resistance $R_m = 100 \Omega$ and a full scale current of $I_m = 1 \text{ mA}$ is to be converted in to a multi range d.c. voltmeter with ranges of 0-10V, 0-50V, 0-250V, 0-500V. Find the values of various resistances using the potential divider arrangement.

Solution:

Data given

$$R_m = 100\Omega$$

$$I_m = 1mA$$

$$V_m = I_m \times R_m$$

$$V_m = 100 \times 1 \times 10^{-3}$$

$$V_m = 100mV$$

$$m = \frac{V_1}{V_m} = \frac{10}{100 \times 10^{-3}} = 100$$

$$m = \frac{V_2}{V_m} = \frac{50}{100 \times 10^{-3}} = 500$$

$$m = \frac{V_3}{V_m} = \frac{250}{100 \times 10^{-3}} = 2500$$

$$m = \frac{V_4}{V_m} = \frac{500}{100 \times 10^{-3}} = 5000$$

$$R_1 = (m_1 - 1)R_m = (100 - 1) \times 100 = 9900\Omega$$

$$R_2 = (m_2 - m_1)R_m = (500 - 100) \times 100 = 40K\Omega$$

$$R_3 = (m_3 - m_2)R_m = (2500 - 500) \times 100 = 200K\Omega$$

$$R_4 = (m_4 - m_3)R_m = (5000 - 2500) \times 100 = 250K\Omega$$

AC BRIDGES

General form of A.C. bridge

AC bridge are similar to D.C. bridge in topology (way of connecting). It consists of four arm AB, BC, CD and DA. Generally the impedance to be measured is connected between 'A' and 'B'. A detector is connected between 'B' and 'D'. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at 'B' and 'D' can be made equal. This is called balancing of the bridge.

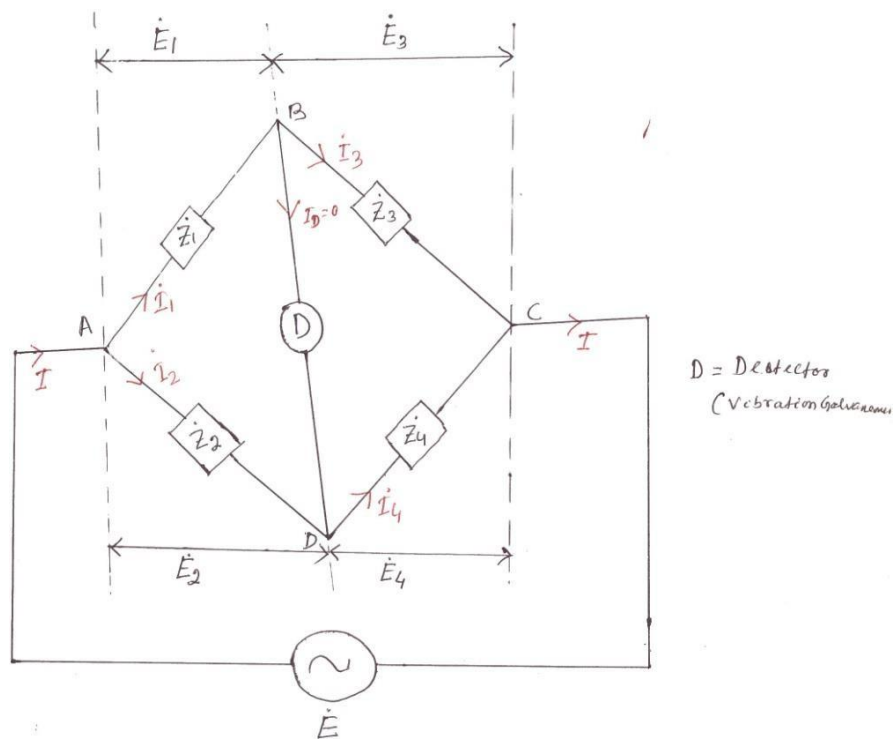


Fig. 2.1 General form of A.C. bridge

At the balance condition, the current through detector is zero.

$$\therefore \dot{I}_1 = \dot{I}_3$$

$$\dot{I}_2 = \dot{I}_4$$

$$\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{I}_3}{\dot{I}_4} \tag{2.1}$$

At balance condition,

Voltage drop across 'AB'=voltage drop across 'AD'.

$$\dot{E}_1 = \dot{E}_2$$

$$\therefore \dot{I}_1 \dot{Z}_1 = \dot{I}_2 \dot{Z}_2 \quad (2.2)$$

Similarly, Voltage drop across 'BC'=voltage drop across 'DC'

$$\dot{E}_3 = \dot{E}_4$$

$$\therefore \dot{I}_3 \dot{Z}_3 = \dot{I}_4 \dot{Z}_4 \quad (2.3)$$

From Eqn. (2.2), we have $\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{Z}_2}{\dot{Z}_1}$ (2.4)

From Eqn. (2.3), we have $\therefore \frac{\dot{I}_3}{\dot{I}_4} = \frac{\dot{Z}_4}{\dot{Z}_3}$ (2.5)

From equation -2.1, it can be seen that, equation -2.4 and equation-2.5 are equal.

$$\therefore \frac{\dot{Z}_2}{\dot{Z}_1} = \frac{\dot{Z}_4}{\dot{Z}_3}$$

$$\therefore \dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$$

Products of impedances of opposite arms are equal.

$$\therefore |Z_1| \angle \theta_1 |Z_4| \angle \theta_4 = |Z_2| \angle \theta_2 |Z_3| \angle \theta_3$$

$$\Rightarrow |Z_1| |Z_4| \angle \theta_1 + \theta_4 = |Z_2| |Z_3| \angle \theta_2 + \theta_3$$

$$|Z_1| |Z_4| = |Z_2| |Z_3|$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

- * For balance condition, magnitude on either side must be equal.
- * Angle on either side must be equal.

Summary

For balance condition,

- $I_1 = I_3, I_2 = I_4$
- $|Z_1||Z_4| = |Z_2||Z_3|$
- $\theta_1 + \theta_4 = \theta_2 + \theta_3$
- $E_1 = E_2 \quad \& \quad E_3 = E_4$

Types of detector

The following types of instruments are used as detector in A.C. bridge.

- Vibration galvanometer
- Head phones (speaker)
- Tuned amplifier

Vibration galvanometer

Between the point 'B' and 'D' a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a dot, if the bridge is unbalanced.

Head phones

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

Tuned amplifier

If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero.

Measurements of inductance

Maxwell's inductance bridge

The choke for which R_1 and L_1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

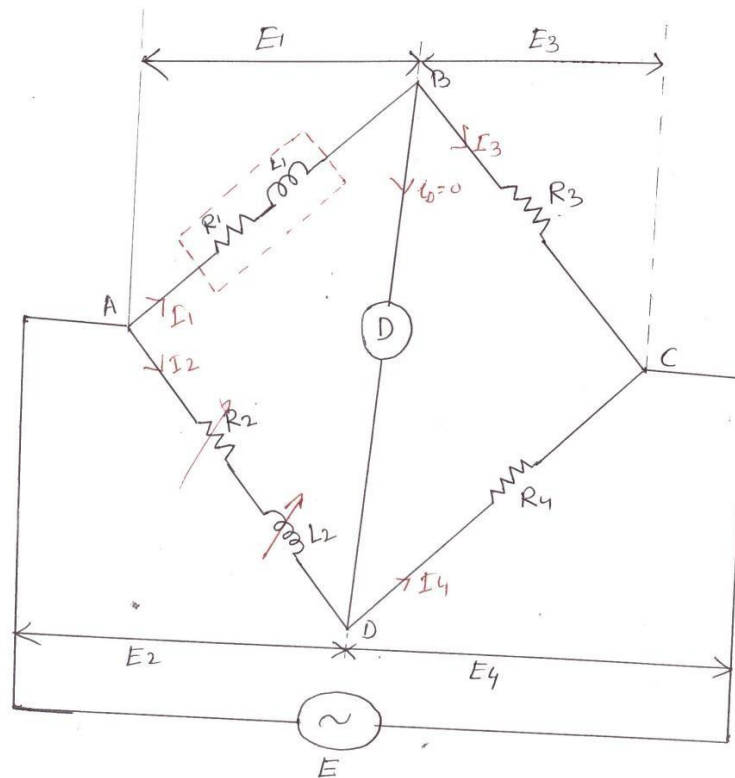


Fig. 2.2 Maxwell's inductance bridge

L_2 is adjusted, until the detector indicates zero current.

Let R_1 = unknown resistance

L_1 = unknown inductance of the choke.

L_2 = known standard inductance

R_1, R_2, R_4 = known resistances.

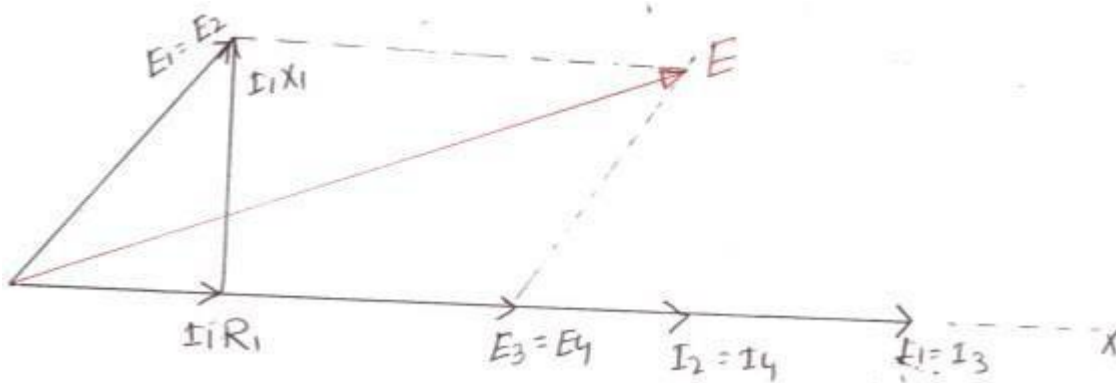


Fig 2.3 Phasor diagram of Maxwell's inductance bridge

At balance condition, $Z_1 Z_4 = Z_2 Z_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega L_2 R_3$$

Comparing real part,

$$R_1 R_4 = R_2 R_3$$

$$\therefore R = \frac{R_2 R_3}{R_4} \quad (2.6)$$

Comparing the imaginary parts,

$$\omega L_1 R_4 = \omega L_2 R_3$$

$$L_1 = \frac{L_2 R_3}{R_4} \quad (2.7)$$

$$\text{Q-factor of choke, } Q = \frac{\omega L_1}{R_1} = \frac{\omega L_2 R_3 R_4}{R_4 R_2 R_3}$$

$$Q = \frac{\omega L_2}{R_2} \quad (2.8)$$

Advantages

- ✓ Expression for R_1 and L_1 are simple.
- ✓ Equations are simple
- ✓ They do not depend on the frequency (as ω is cancelled)
- ✓ R_1 and L_1 are independent of each other.

Disadvantages

- ✓ Variable inductor is costly.
- ✓ Variable inductor is bulky.

Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying ' C_4 '.

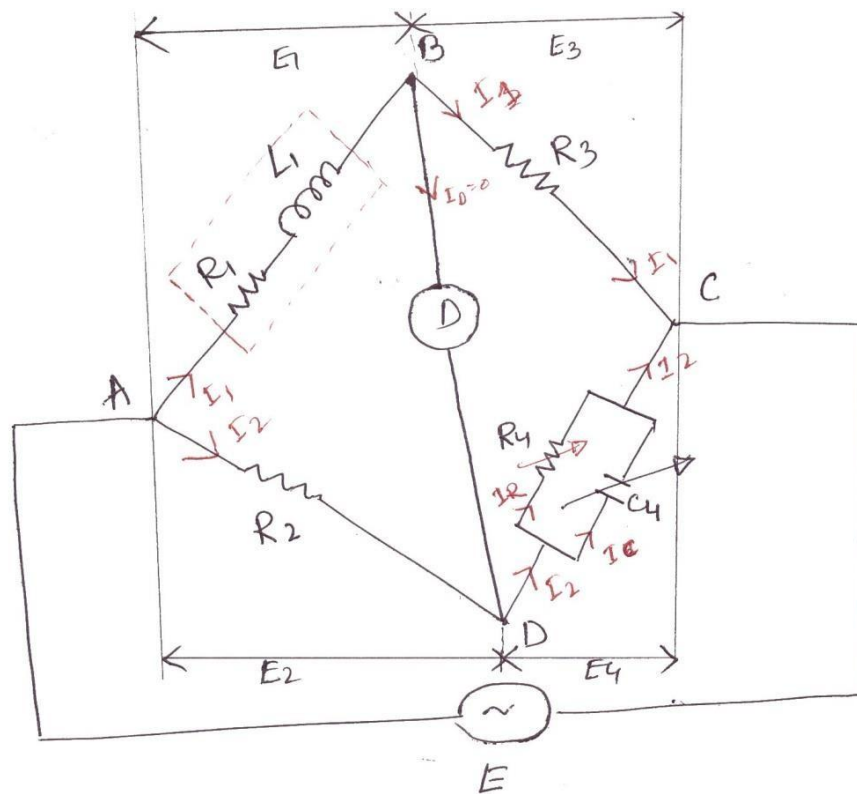


Fig 2.4 Maxwell's inductance capacitance bridge

At balance condition, $Z_1 Z_4 = Z_3 Z_2$ (2.9)

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}$$

$$Z = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{R_4}{1 + j\omega R_4 C_4}$$
(2.10)

∴ Substituting the value of Z_4 from eqn. (2.10) in eqn. (2.9) we get

$$(R_1 + j\omega L_1) \times \frac{R_4}{1 + j\omega R_4 C_4} = R_2 R_3$$

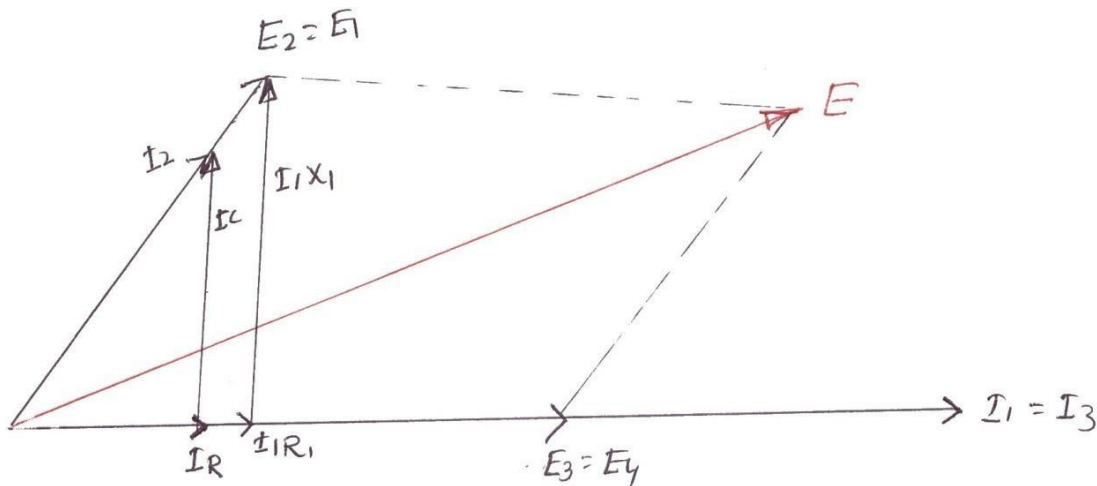


Fig 2.5 Phasor diagram of Maxwell's inductance capacitance bridge

$$(R_1 + j\omega L_1)R_4 = R_2 R_3 (1 + j\omega R_4 C_4)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_4 R_2 R_3$$

Comparing real parts,

$$R_1 R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4} \quad (2.11)$$

Comparing imaginary part,

$$\omega L_1 R_4 = \omega C_4 R_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3 \quad (2.12)$$

Q-factor of choke,

$$Q = \frac{\omega L_1}{R_1} = \omega \times C_4 R_2 R_3 \times \frac{R_4}{R_1}$$

$$Q = \omega C_4 R_4 \quad (2.13)$$

Advantages

- ✓ Equation of L_1 and R_1 are simple.
- ✓ They are independent of frequency.
- ✓ They are independent of each other.
- ✓ Standard capacitor is much smaller in size than standard inductor.

Disadvantages

- ✓ Standard variable capacitance is costly.
- ✓ It can be used for measurements of Q-factor in the ranges of 1 to 10.
- ✓ It cannot be used for measurements of choke with Q-factors more than 10.

We know that $Q = \omega C_4 R_4$

For measuring chokes with higher value of Q-factor, the value of C_4 and R_4 should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

Hay's bridge

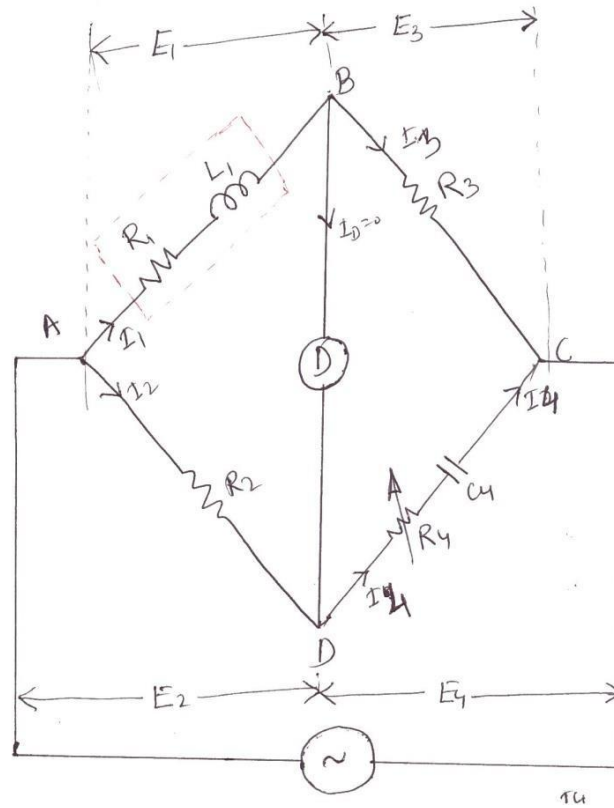


Fig 2.6 Hay's bridge

➤ $\dot{E}_1 = I_1 R_1 + j I_1 X_1$

➤ $\dot{E} = \dot{E}_1 + \dot{E}_3$

➤ $\dot{E}_4 = I_4 R_4 + \frac{I_4}{j \omega C_4}$

➤ $\dot{E}_3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{j \omega C_4} = \frac{1 + j \omega R_4 C_4}{j \omega C_4}$$

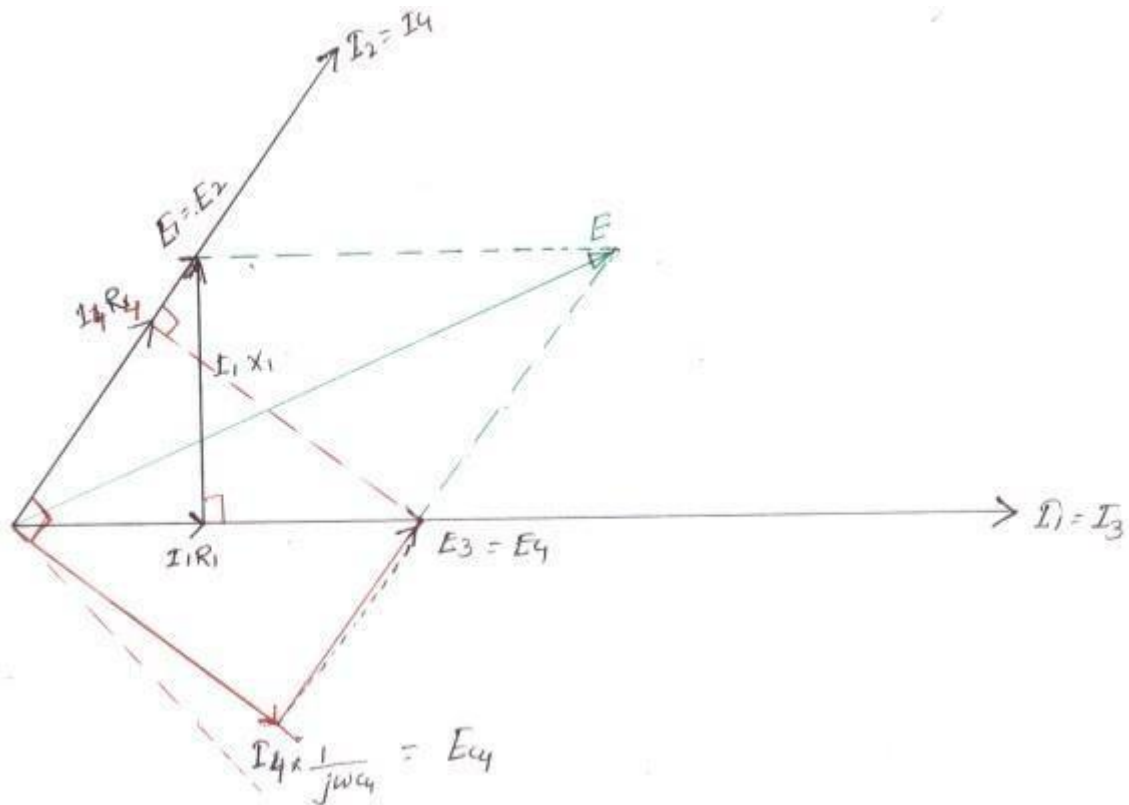


Fig 2.7 Phasor diagram of Hay's bridge

At balance condition, $Z_1 Z_4 = Z_3 Z_2$

$$(R_1 + j\omega L_1) \left(\frac{1 + j\omega R_4 C_4}{j\omega C_4} \right) = R_2 R_3$$

$$(R_1 + j\omega L_1)(1 + j\omega R_4 C_4) = j\omega R_2 C_4 R_3$$

$$R_1 + j\omega C_4 R_4 R_1 + j\omega L_1 + j^2 \omega^2 L_1 C_4 R_4 = j\omega C_4 R_2 R_3$$

$$(R_1 - \omega^2 L_1 C_4 R_4) + j(\omega C_4 R_4 R_1 + \omega L_1) = j\omega C_4 R_2 R_3$$

Comparing the real term,

$$R_1 - \omega^2 L_1 C_4 R_4 = 0$$

$$R_1 = \omega^2 L_1 C_4 R_4 \tag{2.14}$$

Comparing the imaginary terms,

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4R_4R_1 + L_1 = C_4R_2R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1 \tag{2.15}$$

Substituting the value of R_1 from eqn. 2.14 into eqn. 2.15, we have,

$$L_1 = C_4R_2R_3 - C_4R_4 \times w^2L_1C_4R_4$$

$$L_1 = C_4R_2R_3 - w^2L_1C_4^2R_4^2$$

$$L_1(1 + w^2L_1C_4^2R_4^2) = C_4R_2R_3$$

$$L_1 = \frac{C_4R_2R_3}{1 + w^2L_1C_4^2R_4^2} \tag{2.16}$$

Substituting the value of L_1 in eqn. 2.14, we have

$$R_1 = \frac{w^2C_4^2R_2R_3R_4}{1 + w^2C_4^2R_4^2} \tag{2.17}$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4R_2R_3}{1 + w^2C_4^2R_4^2} \times \frac{1 + w^2C_4^2R_4^2}{w^2C_4^2R_4R_2R_3}$$

$$Q = \frac{1}{wC_4R_4} \tag{2.18}$$

Advantages

- ✓ Fixed capacitor is cheaper than variable capacitor.
- ✓ This bridge is best suitable for measuring high value of Q-factor.

Disadvantages

- ✓ Equations of L_1 and R_1 are complicated.
- ✓ Measurements of R_1 and L_1 require the value of frequency.
- ✓ This bridge cannot be used for measuring low Q- factor.

Owen's bridge

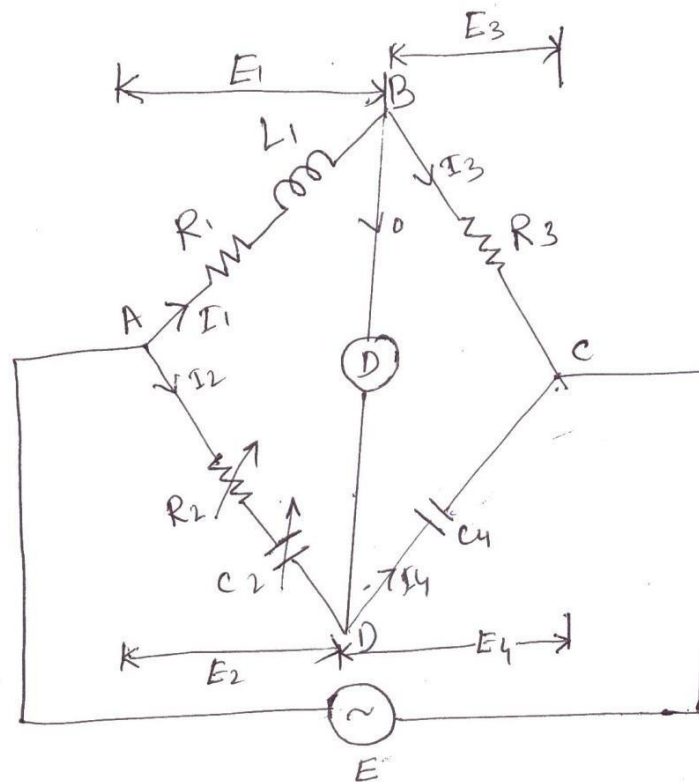


Fig 2.8 Owen's bridge

- $E_1 = I_1 R_1 + j I_1 X_1$
- I_4 leads E_4 by 90°

$$\triangleright \dot{E} = \dot{E}_1 + \dot{E}_3$$

$$\triangleright \dot{E}_2 = I_2 R_2 + \frac{I_2}{j\omega C_2}$$

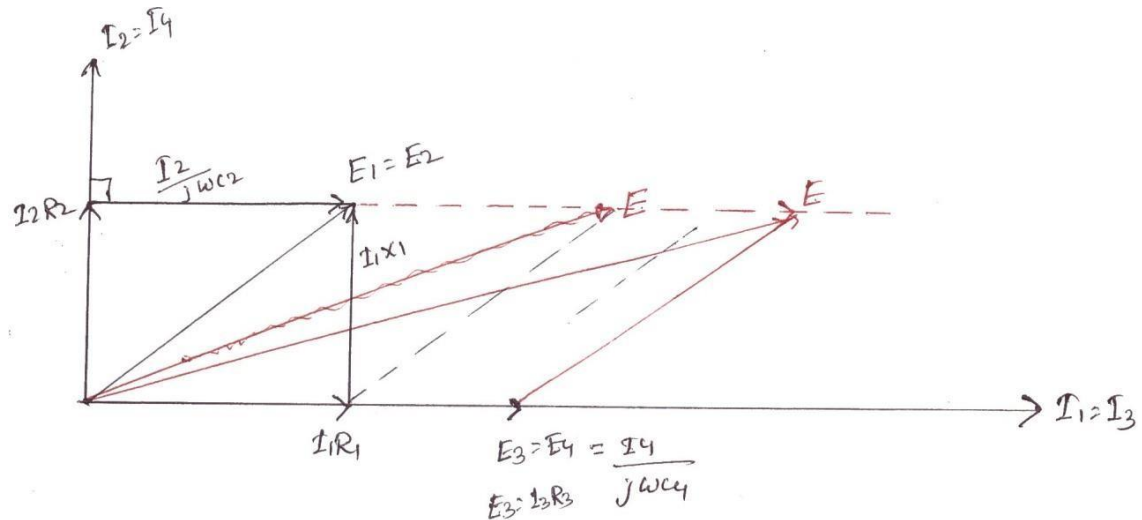


Fig 2.9 Phasor diagram of Owen's bridge

Balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$Z_2 = R_2 + \frac{1}{j\omega C_2} = \frac{j\omega C_2 R_2 + 1}{j\omega C_2}$$

$$\therefore (R_1 + j\omega L_1) \times \frac{1}{j\omega C_4} = \frac{(1 + j\omega R_2 C_2) \times R_3}{j\omega C_2}$$

$$C_2(R_1 + j\omega L_1) = R_3 C_4(1 + j\omega R_2 C_2)$$

$$R_1 C_2 + j\omega L_1 C_2 = R_3 C_4 + j\omega R_2 C_2 R_3 C_4$$

Comparing real terms,

$$R_1 C_2 = R_3 C_4$$

$$R_1 = \frac{R_3 C_4}{C_2}$$

Comparing imaginary terms,

$$\omega L_1 C_2 = \omega R_2 C_2 R_3 C_4$$

$$L_1 = R_2 R_3 C_4$$

$$Q\text{-factor} = \frac{\omega L_1}{R_1} = \frac{\omega R_2 R_3 C_4 C_2}{R_3 C_4}$$

$$Q = \omega R_2 C_2$$

Advantages

- ✓ Expression for R_1 and L_1 are simple.
- ✓ R_1 and L_1 are independent of Frequency.

Disadvantages

- ✓ The Circuits used two capacitors.
- ✓ Variable capacitor is costly.
- ✓ Q-factor range is restricted.

Anderson's bridge

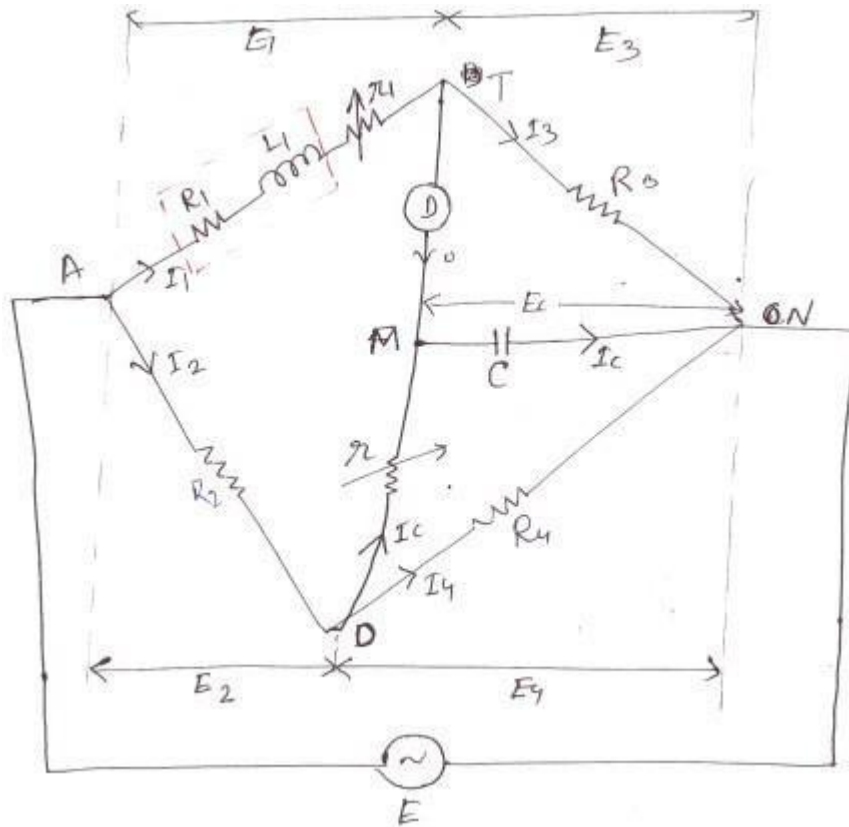


Fig 2.10 Anderson's bridge

- $\dot{E}_1 = I_1(R_1 + r_1) + jI_1X_1$
- $E_3 = E_C$
- $\dot{E}_4 = I_C r + E_C$
- $I_2 = I_4 + I_C$
- $\bar{E}_2 + \bar{E}_4 = \bar{E}$
- $\bar{E}_1 + \bar{E}_3 = \bar{E}$

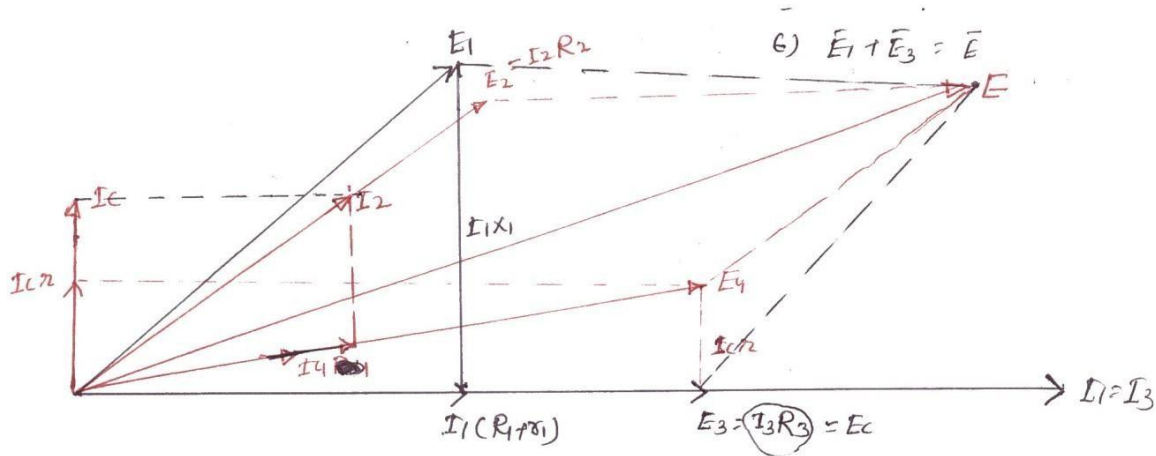


Fig 2.11 Phasor diagram of Anderson's bridge

Step-1 Take I_1 as reference vector. Draw $I_1 R_1$ in phase with I_1

$$R_1^1 = (R_1 + r_1) \quad , \quad I_1 X_1 \text{ is } \perp_r \text{ to } I_1 R_1^1$$

$$E_1 = I_1 R_1^1 + j I_1 X_1$$

Step-2 $I_1 = I_3$, E_3 is in phase with I_3 , From the circuit ,

$$E_3 = E_C \quad , \quad I_C \text{ leads } E_C \text{ by } 90^\circ$$

Step-3 $E_4 = I_C r + E_C$

Step-4 Draw I_4 in phase with E_4 , By KCL , $I_2 = I_4 + I_C$

Step-5 Draw E_2 in phase with I_2

Step-6 By KVL , $E_1 + E_3 = E$ or $E_2 + E_4 = E$

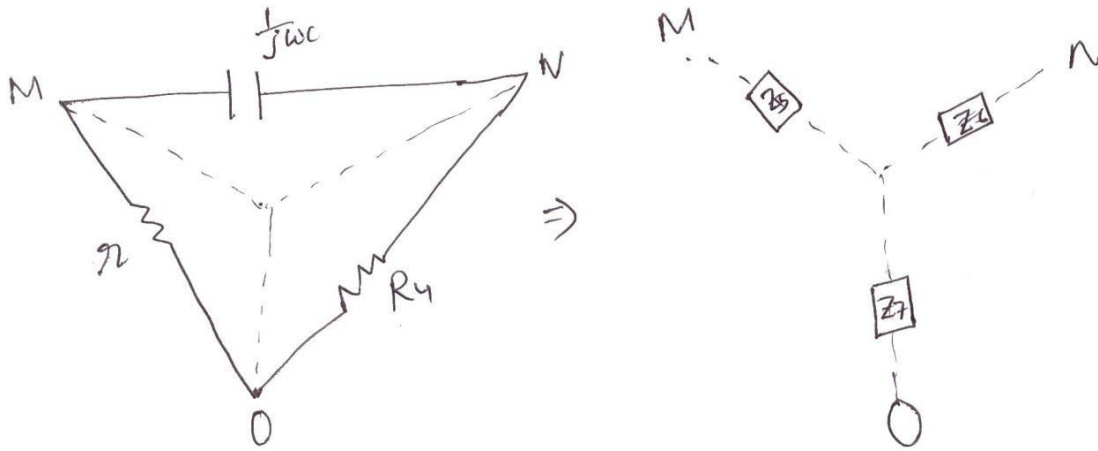


Fig 2.12 Equivalent delta to star conversion for the loop MON

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{j\omega C}} = \frac{j\omega C R_4 r}{1 + j\omega C(R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{j\omega C}}{R_4 + r + \frac{1}{j\omega C}} = \frac{R_4}{1 + j\omega C(R_4 + r)}$$

$$(R_1 + j\omega L) \times \frac{R_4}{1 + j\omega C(R_4 + r)} = R_4 \left(R_1 + \frac{j\omega C R_4 r}{1 + j\omega C(R_4 + r)} \right)$$

$$\Rightarrow \frac{(R_1 + j\omega L) R_4}{1 + j\omega C(R_4 + r)} = R_3 \left[\frac{R_4 (1 + j\omega C(R_4 + r)) + j\omega C r R_4}{1 + j\omega C(R_4 + r)} \right]$$

$$\Rightarrow R_1 R_4 + j\omega L R_4 = R_4 R_3 + j\omega C R_4 R_3 (r + R_4) + j\omega C r R_4 R_3$$

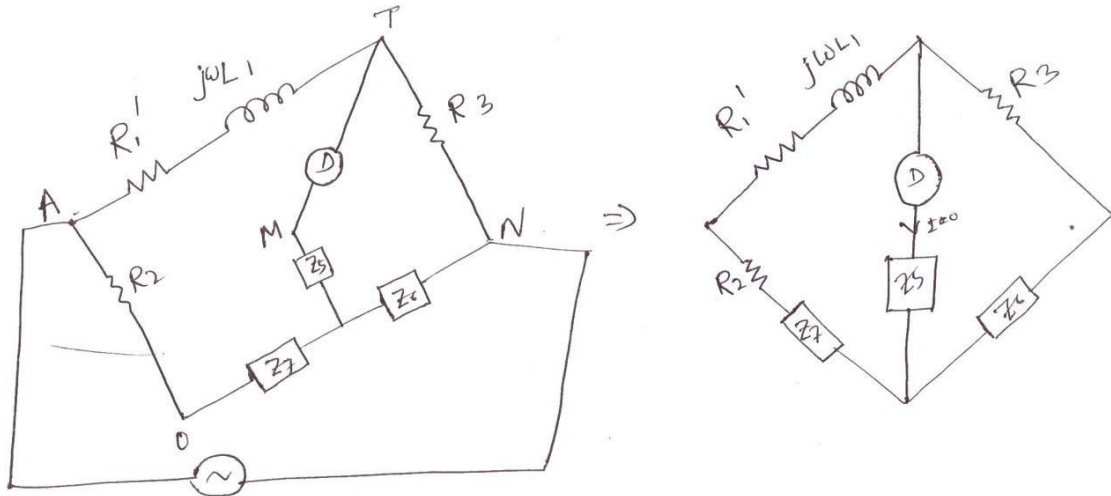


Fig 2.13 Simplified diagram of Anderson's bridge

Comparing real term,

$$R_1 R_4 = R_2 R_3$$

$$(R_1 + r_1)R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$\omega L_1 R_4 = \omega C R_2 R_3 (r_1 + R_4) + \omega C r_1 R_3 R_4$$

$$L_1 = \frac{R_2 R_3 C}{R_4} (r_1 + R_4) + r_1 C$$

$$L_1 = R_2 C \left[\frac{R_3}{R_4} (r_1 + R_4) + r_1 \right]$$

Advantages

- ✓ Variable capacitor is not required.
- ✓ Inductance can be measured accurately.
- ✓ R_1 and L_1 are independent of frequency.
- ✓ Accuracy is better than other bridges.

Disadvantages

- ✓ Expression for R_1 and L_1 are complicated.
- ✓ This is not in the standard form A.C. bridge.

Measurement of capacitance and loss angle. (Dissipation factor)

Dissipation factors (D)

A practical capacitor is represented as the series combination of small resistance and ideal capacitance.

From the vector diagram, it can be seen that the angle between voltage and current is slightly less than 90° . The angle ' δ ' is called loss angle.

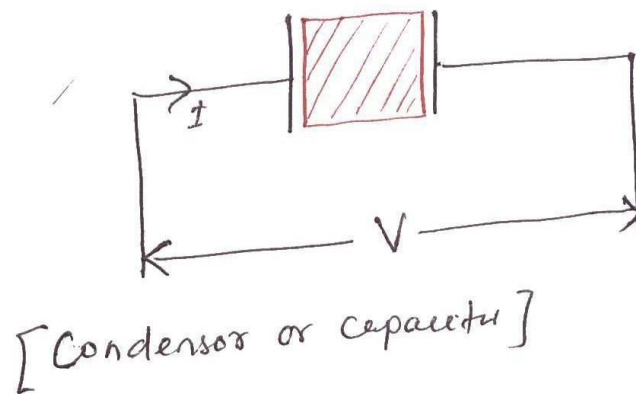


Fig 2.14 Condensor or capacitor

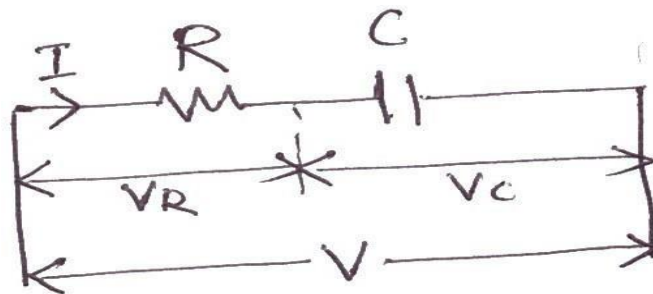


Fig 2.15 Representation of a practical capacitor

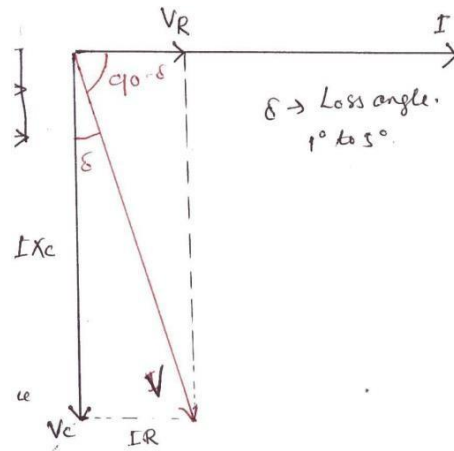


Fig 2.16 Vector diagram for a practical capacitor

A dissipation factor is defined as 'tanδ'.

$$\therefore \tan \delta = \frac{IR}{IX_C} = \frac{R}{X_C} = \omega CR$$

$$D = \omega CR$$

$$D = \frac{1}{Q}$$

$$D = \tan \delta = \frac{\sin \delta}{\cos \delta} \cong \frac{\delta}{1} \quad \text{For small value of 'δ' in radians}$$

$D \cong \delta \cong \text{Loss Angle}$ ('δ' must be in radian)

Desauty's Bridge

$C_1 = \text{Unknown capacitance}$

At balance condition,

$$\frac{1}{j\omega C_1} \times R_4 = \frac{1}{j\omega C_2} \times R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C = \frac{R_4 C_2}{R_3}$$

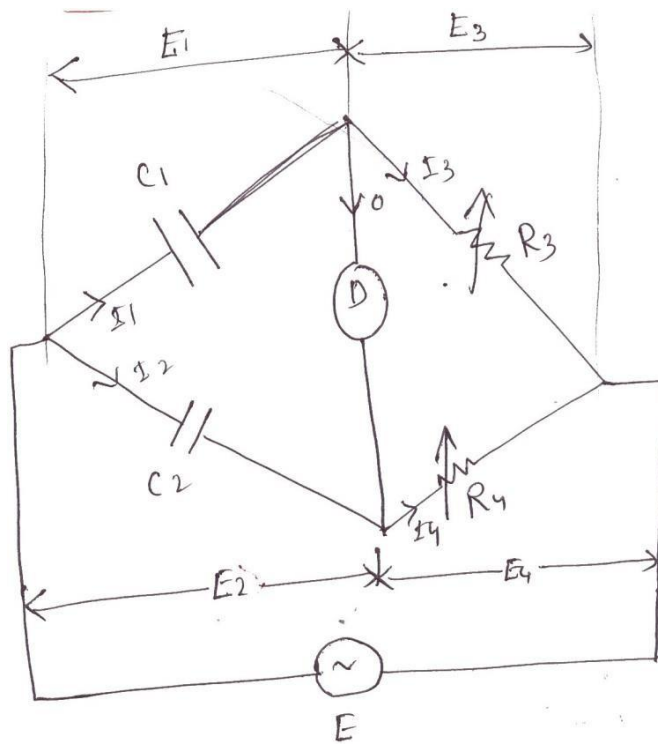


Fig 2.17 Desauty's bridge

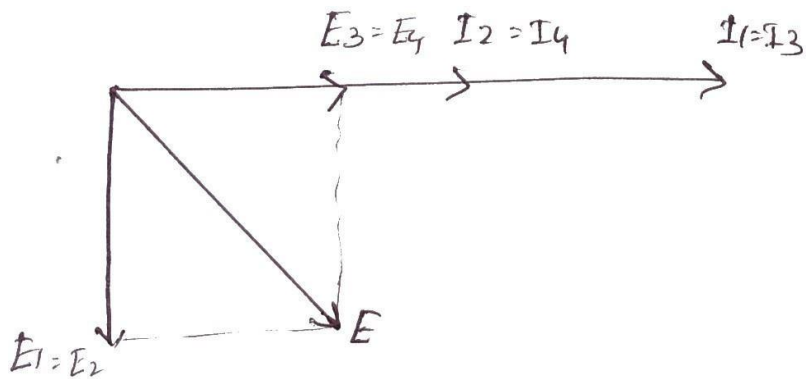


Fig 2.18 Phasor diagram of Desauty's bridge

Modified desauty's bridge

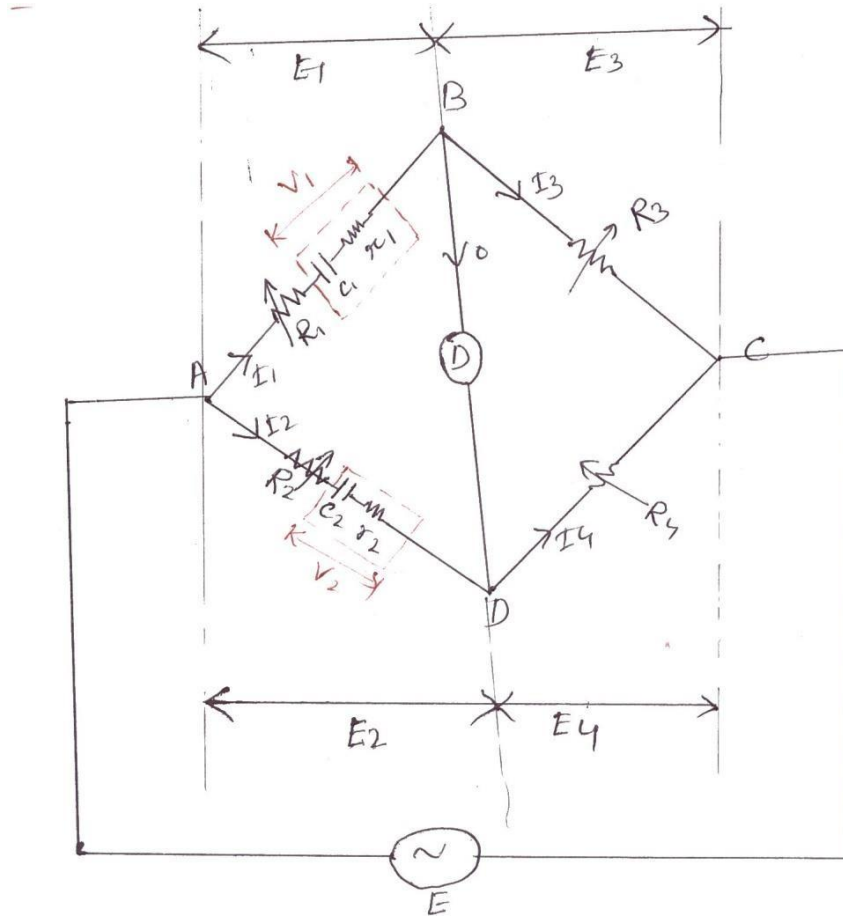


Fig 2.19 Modified Desauty's bridge

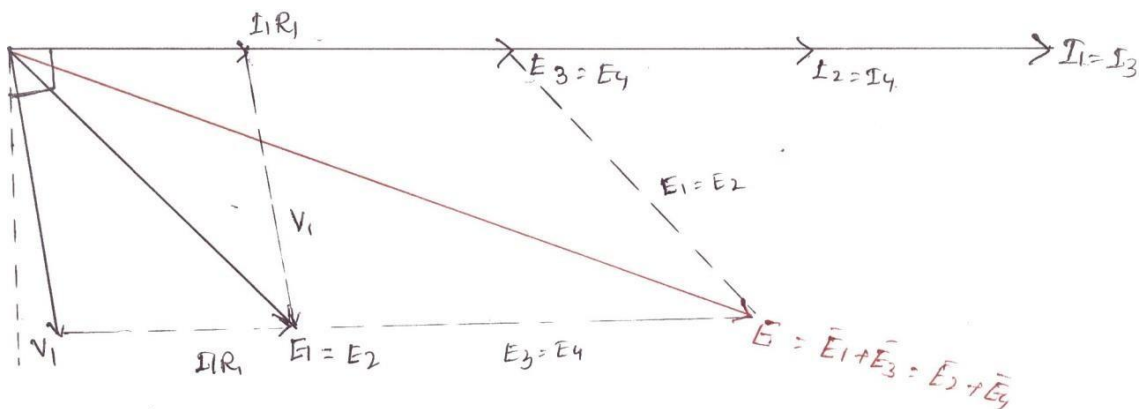


Fig 2.20 Phasor diagram of Modified Desauty's bridge

$$R_1^1 = (R + r_1)$$

$$R_2^1 = (R + r_2)$$

At balance condition, $(R_1 + \frac{1}{j\omega C_1})R_4 = R_3 (R_2 + \frac{1}{j\omega C_2})$

$$R_1 R_4 + \frac{R_4}{j\omega C_1} = R_3 R_2 + \frac{R_3}{j\omega C_2}$$

Comparing the real term, $R_1 R_4 = R_3 R_2$

$$R_1 = \frac{R_3 R_2}{R_4}$$

$$R + r_1 = \frac{(R_2 + r_2)R_3}{R_4}$$

Comparing imaginary term,

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2}$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

Dissipation factor $D = \omega C_1 r_1$

Advantages

- ✓ r_1 and c_1 are independent of frequency.
- ✓ They are independent of each other.
- ✓ Source need not be pure sine wave.

Schering bridge

$$E_1 = I_1 r_1 - jI_1 X_4$$

$C_2 = C_4 =$ Standard capacitor (Internal resistance=0)

$C_4 =$ Variable capacitance.

$C_1 =$ Unknown capacitance.

$r_1 =$ Unknown series equivalent resistance of the capacitor.

$R_3=R_4=$ Known resistor.

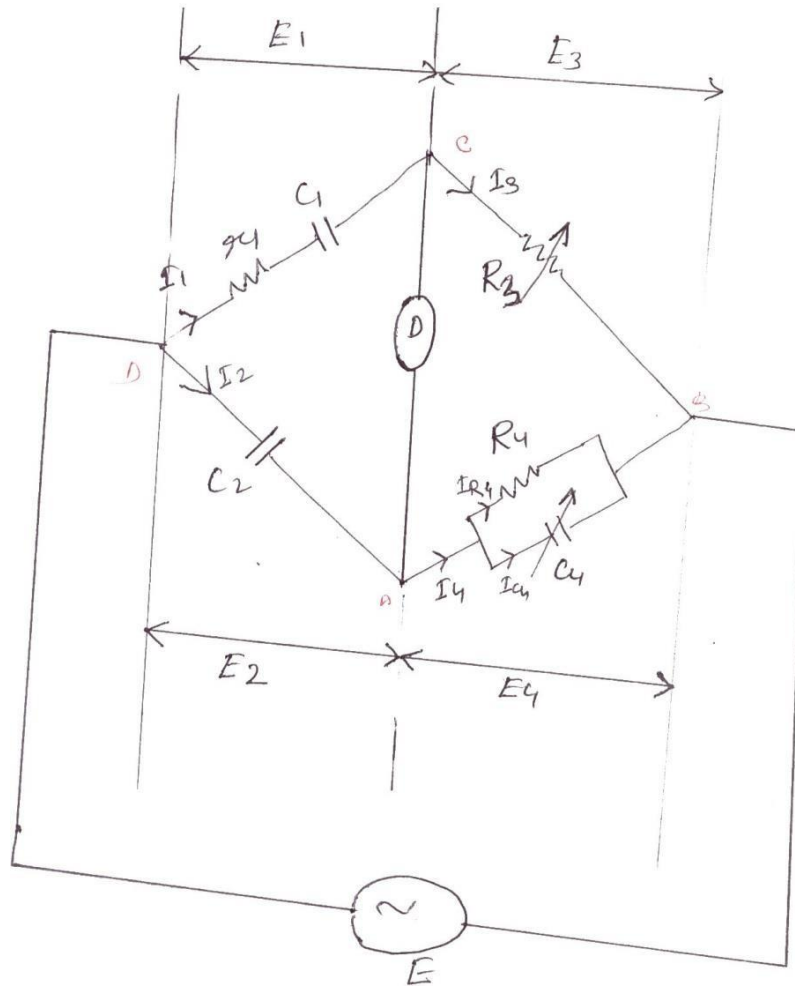


Fig 2.21 Schering bridge

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_4}{1 + j\omega C_4 R_4}$$

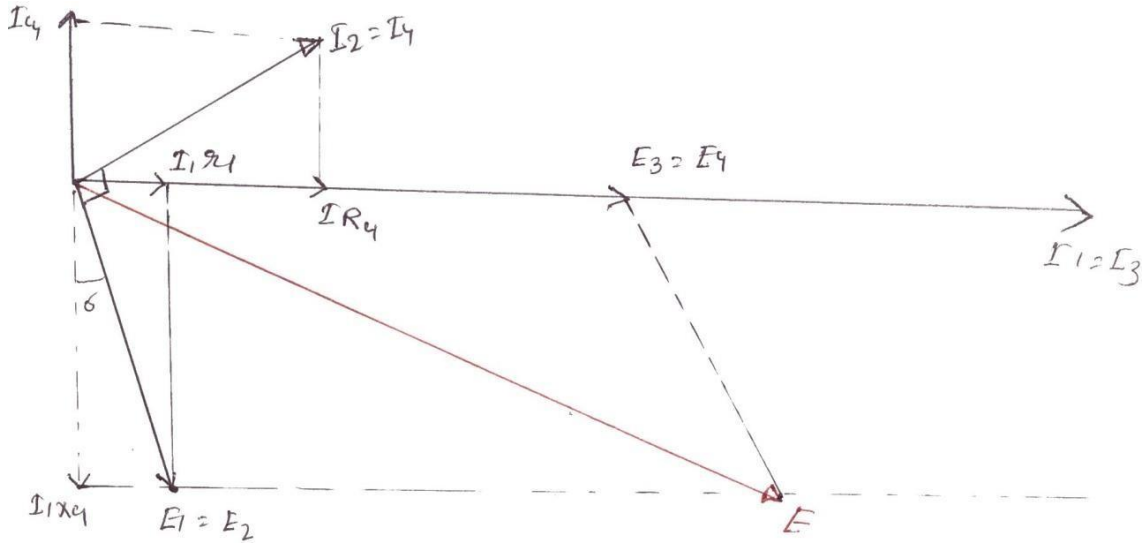


Fig 2.22 Phasor diagram of Schering bridge

At balance condition, $Z_1 Z_4 = Z_2 Z_3$

$$\frac{1 + j\omega C_1 r_1}{j\omega C_1} \times \frac{R_4}{1 + j\omega C_4 R_4} = \frac{R_3}{j\omega C_2}$$

$$(1 + j\omega C_1 r_1) R_4 C_2 = R_3 C_1 (1 + j\omega C_4 R_4)$$

$$R_2 C_2 + j\omega C_1 r_1 R_4 C_2 = R_3 C_1 + j\omega C_4 R_4 R_3 C_1$$

Comparing the real part,

$$\therefore C = \frac{R_4 C_2}{R_3}$$

Comparing the imaginary part,

$$\omega C_1 r_1 R_4 C_2 = \omega C_4 R_3 R_4 C_1$$

$$r_1 = \frac{C_4 R_3}{C_2}$$

Dissipation factor of capacitor,

$$D = \omega C_1 r_1 = \omega \times \frac{R_4 C_2}{R_3} \times \frac{C_4 R_3}{C_2}$$

$$\therefore D = \omega C_4 R_4$$

Advantages

- ✓ In this type of bridge, the value of capacitance can be measured accurately.
- ✓ It can measure capacitance value over a wide range.
- ✓ It can measure dissipation factor accurately.

Disadvantages

- ✓ It requires two capacitors.
- ✓ Variable standard capacitor is costly.

Measurements of frequency

Wein's bridge

Wein's bridge is popularly used for measurements of frequency of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to measure is connected as shown in the figure.

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

At balance condition, $Z_1 Z_4 = Z_2 Z_3$

$$\frac{j\omega C_1 r_1 + 1}{j\omega C_1} \times R_4 = \frac{R_2}{1 + j\omega C_2 R_2} \times R_3$$

$$(1 + j\omega C_1 r_1)(1 + j\omega C_2 R_2)R_4 = R_2 R_3 \times j\omega C_1$$

$$\left[1 + j\omega C_1 R_2 + j\omega C_2 r_1 - \omega^2 C_1 C_2 r_1 R_2 \right] = j\omega C_1 \frac{R_2 R_3}{R_4}$$

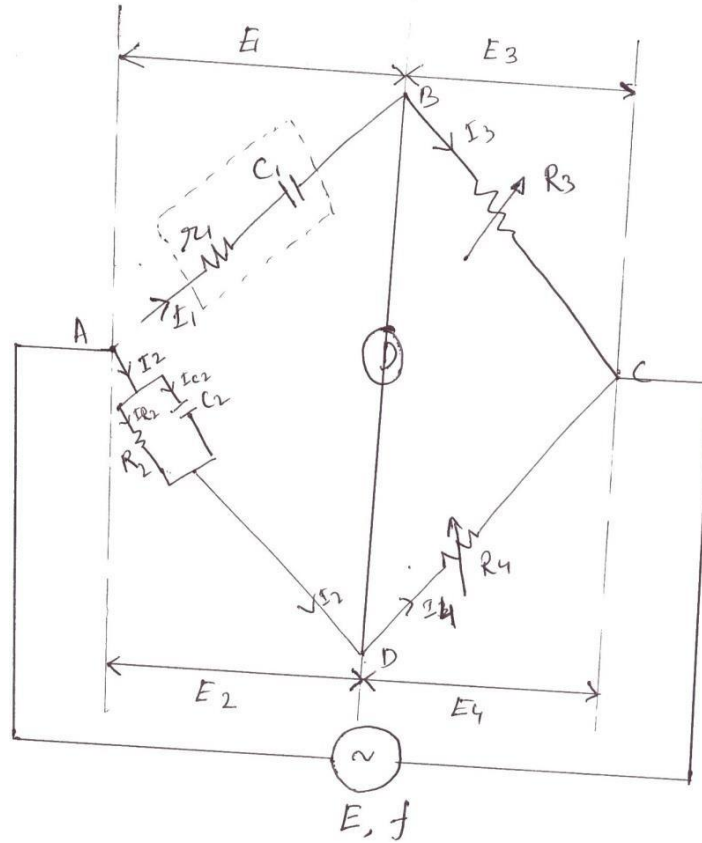


Fig 2.23 Wein's bridge

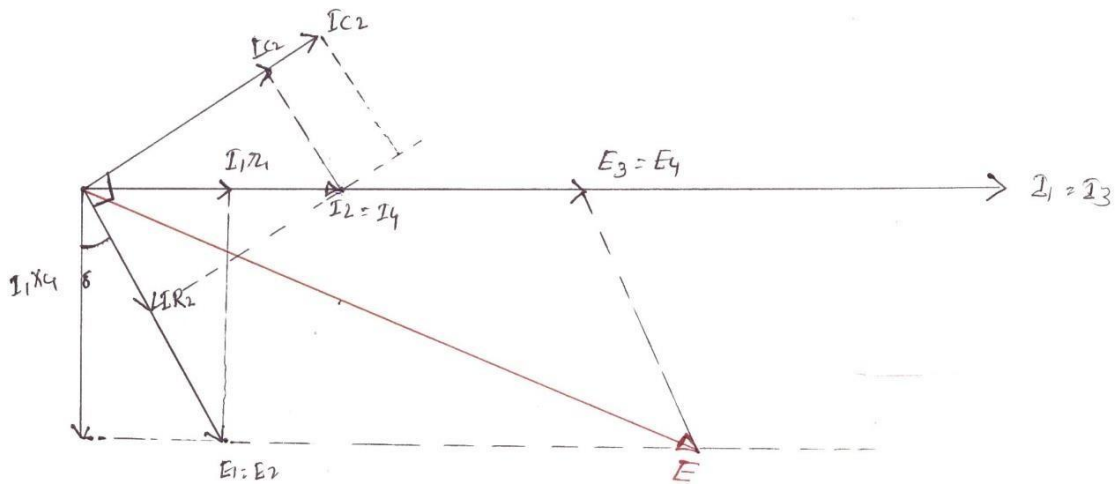


Fig 2.24 Phasor diagram of Wein's bridge

Comparing real term,

$$1 - \omega^2 C_1 C_2 r_1 R_2 = 0$$

$$\omega^2 C_1 C_2 r_1 R_2 = 1$$

$$\omega^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$\omega = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}}, \quad f = \frac{1}{2\pi \sqrt{C_1 C_2 r_1 R_2}}$$

NOTE

The above bridge can be used for measurements of capacitance. In such case, r_1 and C_1 are unknown and frequency is known. By equating real terms, we will get R_1 and C_1 . Similarly by equating imaginary term, we will get another equation in terms of r_1 and C_1 . It is only used for measurements of Audio frequency.

A.F=20 HZ to 20 KHZ

R.F=>>> 20 KHZ

Comparing imaginary term,

$$\omega C_2 R_2 + \omega C_1 r_1 = \omega C_1 \frac{R_2 R_3}{R_4}$$

$$C_2 R_2 + C_1 r_1 = \frac{C_1 R_2 R_3}{R_4} \dots\dots\dots (2.19)$$

$$C_1 = \frac{1}{\omega^2 C_2 r_1 R_2}$$

Substituting in eqn. (2.19), we have

$$C_2 R_2 + \frac{r_1}{\omega^2 C_2 r_1 R_2} = \frac{R_2 R_3}{R_4} C_1$$

Multiplying $\frac{R_4}{R_2 R_3}$ in both sides, we have

$$C_2 R_2 \times \frac{R_4}{R_2 R_3} + \frac{1}{\omega^2 C_2 R_2} \times \frac{R_4}{R_2 R_3} = C_1$$

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{R_4}{w C_2 R_2 R_3}$$

$$w^2 C_1 r_1 C_2 R_2 = 1$$

$$r_1 = \frac{1}{w C_2 R_2 C_1} = \frac{1}{w C_2 R_2 \left[\frac{C_2 R_4}{R_3} + \frac{R_4}{w C_2 R_2 R_3} \right]}$$

$$= \frac{1}{w^2 C_2^2 R_2 \left[\frac{R_4}{R_3} + \frac{R_4}{R_2 R_3} \right]}$$

$$\therefore r_1 = \frac{1}{R_3 \left[w^2 C_2^2 R_2 + \frac{1}{R_4} \right]}$$

$$\therefore r_1 = \frac{R_3}{R_4 \left(w^2 C_2^2 R_2 + \frac{1}{R_4} \right)}$$

High Voltage Schering Bridge

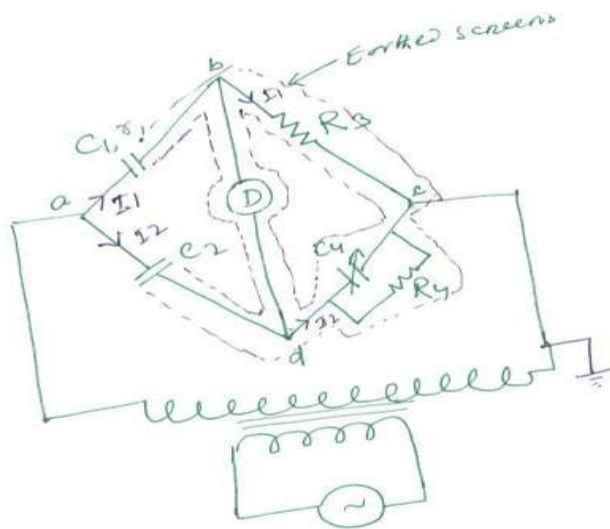


Fig 2.25 High Voltage Schering bridge

(1) The high voltage supply is obtained from a transformer usually at 50 HZ.

Wagner earthing device:

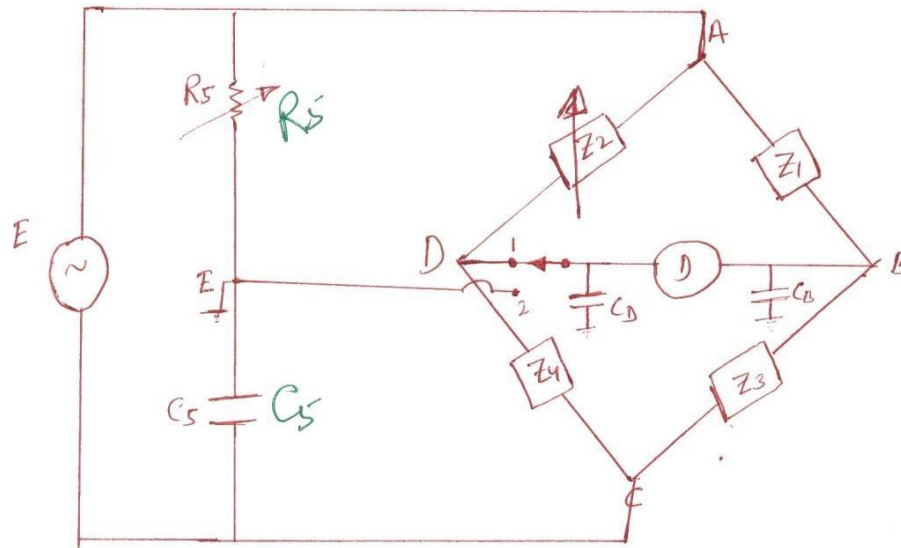


Fig 2.26 Wagner Earthing device

Wagner earthing consists of 'R' and 'C' in series. The stray capacitance at node 'B' and 'D' are C_B , C_D respectively. These Stray capacitances produced error in the measurements of 'L' and 'C'. These error will predominant at high frequency. The error due to this capacitance can be eliminated using wagner earthing arm.

Close the change over switch to the position (1) and obtained balanced. Now change the switch to position (2) and obtained balance. This process has to repeat until balance is achieved in both the position. In this condition the potential difference across each capacitor is zero. Current drawn by this is zero. Therefore they do not have any effect on the measurements.

What are the sources of error in the bridge measurements?

- ✓ Error due to stray capacitance and inductance.
- ✓ Due to external field.
- ✓ Leakage error: poor insulation between various parts of bridge can produced this error.
- ✓ Eddy current error.
- ✓ Frequency error.

- ✓ Waveform error (due to harmonics)
- ✓ Residual error: small inductance and small capacitance of the resistor produce this error.

Precaution

- ✓ The load inductance is eliminated by twisting the connecting the connecting lead.
- ✓ In the case of capacitive bridge, the connecting lead are kept apart. ($QC = \frac{A \in 0 \in r}{d}$)
- ✓ In the case of inductive bridge, the various arm are magnetically screen.
- ✓ In the case of capacitive bridge, the various arm are electro statically screen to reduced the stray capacitance between various arm.
- ✓ To avoid the problem of spike, an inter bridge transformer is used in between the source and bridge.
- ✓ The stray capacitance between the ends of detector to the ground, cause difficulty in balancing as well as error in measurements. To avoid this problem, we use wagner earthing device.

Ballistic galvanometer

This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension is used for this meter. Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphorous bronze wire is used for suspension.

When the D.C. voltage is applied to the terminals of moving coil, current flows through it. When a current carrying coil kept in the magnetic field, produced by permanent magnet, it experiences a force. The coil deflects and mirror deflects. The light spot on the glass scale also move. This deflection is proportional to the current through the coil.

$$i = \frac{Q}{t}, Q = it = \int idt$$

$$\theta \propto Q, \text{ deflection} \propto \text{Charge}$$

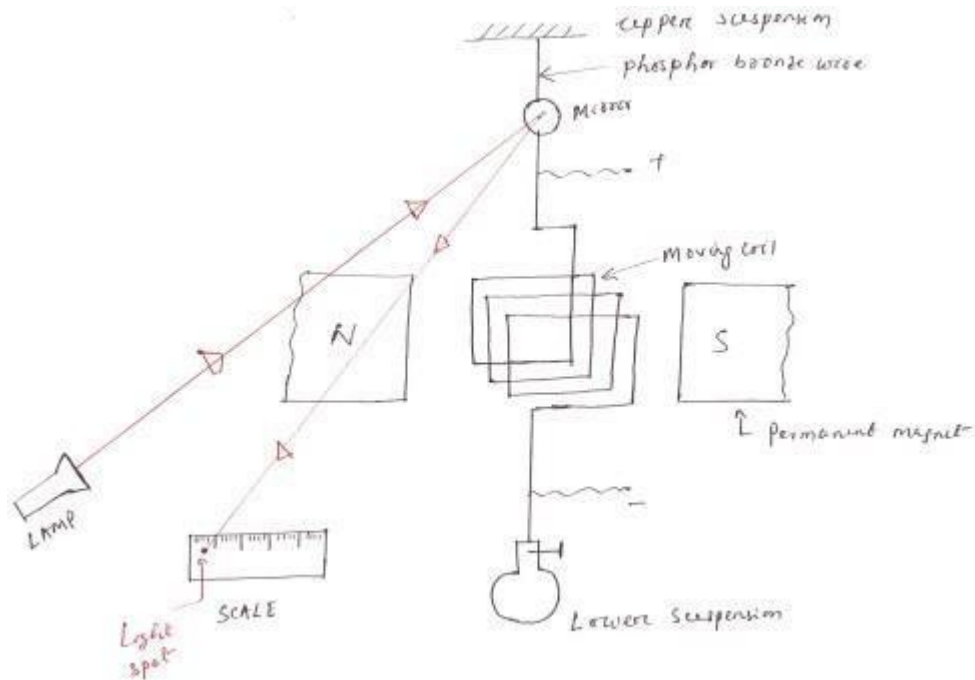


Fig 2.27 Ballistic galvanometer

Measurements of flux and flux density (Method of reversal)

D.C. voltage is applied to the electromagnet through a variable resistance R_1 and a reversing switch. The voltage applied to the toroid can be reversed by changing the switch from position 2 to position '1'. Let the switch be in position '2' initially. A constant current flows through the toroid and a constant flux is established in the core of the magnet.

A search coil of few turns is provided on the toroid. The B.G. is connected to the search coil through a current limiting resistance. When it is required to measure the flux, the switch is changed from position '2' to position '1'. Hence the flux reduced to zero and it starts increasing in the reverse direction. The flux goes from $+\phi$ to $-\phi$, in time 't' second. An emf is induced in the search coil, since the flux changes with time. This emf circulates a current through R_2 and B.G. The meter deflects. The switch is normally closed. It is opened when it is required to take the reading.

Plotting the BH curve

The curve drawn with the current on the X-axis and the flux on the Y-axis, is called magnetization characteristics. The shape of B-H curve is similar to shape of magnetization characteristics. The residual magnetism present in the specimen can be removed as follows.

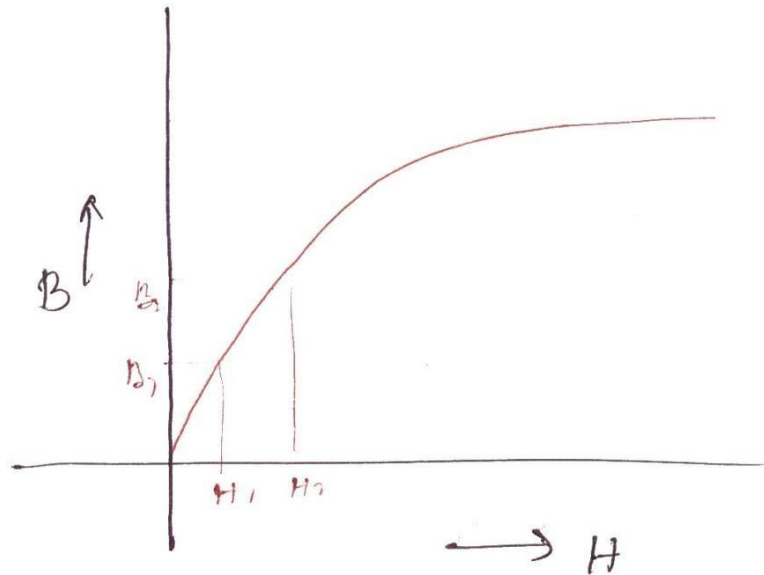


Fig 2.28 BH curve

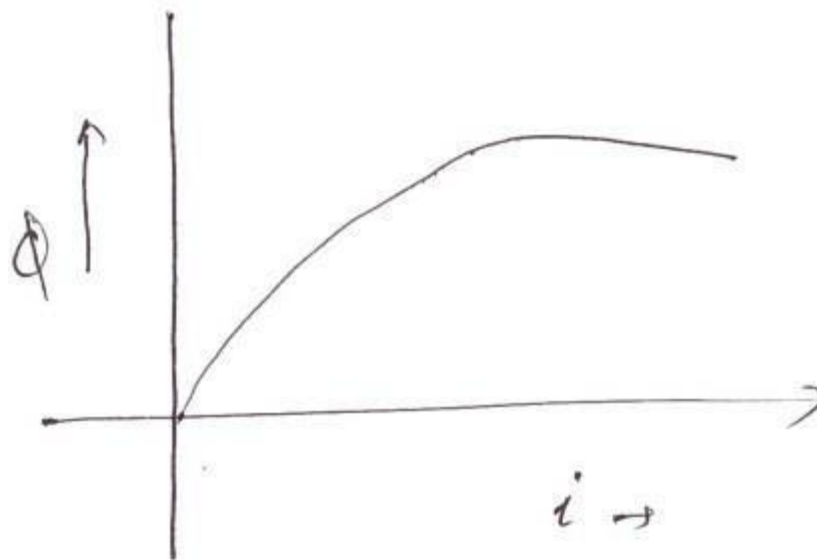


Fig 2.29 Magnetization characteristics

Close the switch 'S₂' to protect the galvanometer, from high current. Change the switch S₁ from position '1' to '2' and vice versa for several times.

To start with the resistance 'R₁' is kept at maximum resistance position. For a particular value of current, the deflection of B.G. is noted. This process is repeated for various value of current. For each deflection flux can be calculated. ($B = \frac{\phi}{A}$)

Magnetic field intensity value for various current can be calculated.().The B-H curve can be plotted by using the value of 'B' and 'H'.

Measurements of iron loss:

Let R_P= pressure coil resistance

R_S = resistance of coil S₁

E= voltage reading= Voltage induced in S₂

I= current in the pressure coil

V_P= Voltage applied to wattmeter pressure coil.

W= reading of wattmeter corresponding voltage V

W₁= reading of wattmeter corresponding voltage E

$$W \rightarrow V \quad \frac{W_1}{W} = \frac{E}{V} \Rightarrow W = \frac{E \times W_1}{V}$$

W₁ → E_P W V 1 V

W₁=Total loss=Iron loss+ Cupper loss.

The above circuit is similar to no load test of transformer.

In the case of no load test the reading of wattmeter is approximately equal to iron loss. Iron loss depends on the emf induced in the winding. Science emf is directly proportional to flux. The voltage applied to the pressure coil is V. The corresponding of wattmeter is 'W'. The iron loss corresponding E is $E = \frac{WE}{V}$. The reading of the wattmeter includes the losses in the pressure

coil and copper loss of the winding S₁. These loses have to be subtracted to get the actual iron loss.

Galvanometers

D-Arsonval Galvanometer

Vibration Galvanometer

Ballistic C

D-aronval galvanometer (d.c. galvanometer)

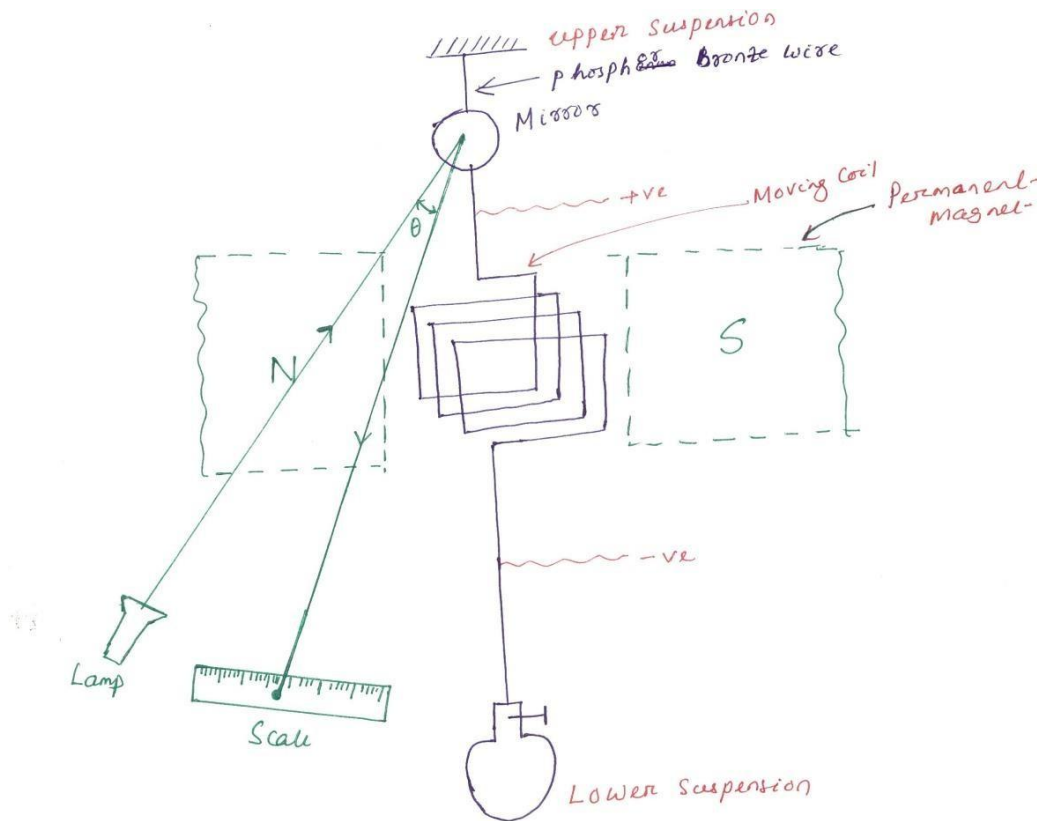


Fig 2.30 D-Arsonval Galvanometer

Galvanometer is a special type of ammeter used for measuring A or mA. This is a sophisticated instruments. This works on the principle of PMMC meter. The only difference is the type of suspension used for this meter. It uses a sophisticated suspension called taut suspension, so that moving system has negligible weight.

Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphors bronze is used for suspension.

When D.C. voltage is applied to the terminal of moving coil, current flows through it. When current carrying coil is kept in the magnetic field produced by P.M. , it experiences a force. The light spot on the glass scale also move. This deflection is proportional to the current through the coil. This instrument can be used only with D.C. like PMMC meter.

The deflecting Torque,

$$T_D = BINA$$

$$T_D = GI, \quad \text{Where } G = BAN$$

$$T_C = K_S \theta = S\theta$$

$$\text{At balance, } T_C = T_D \Rightarrow S\theta = GI$$

$$\therefore \theta = \frac{GI}{S}$$

Where G= Displacements constant of Galvanometer

S=Spring constant

Vibration Galvanometer (A.C. Galvanometer)

The construction of this galvanometer is similar to the PMMC instrument except for the moving system. The moving coil is suspended using two ivory bridge pieces. The tension of the system can be varied by rotating the screw provided at the top suspension. The natural frequency can be varied by varying the tension wire of the screw or varying the distance between ivory bridge piece.

When A.C. current is passed through coil an alternating torque or vibration is produced. This vibration is maximum if the natural frequency of moving system coincide with supply frequency. Vibration is maximum, science resonance takes place. When the coil is vibrating , the mirror oscillates and the dot moves back and front. This appears as a line on the glass scale. Vibration galvanometer is used for null deflection of a dot appears on the scale. If the bridge is unbalanced, a line appears on the scale

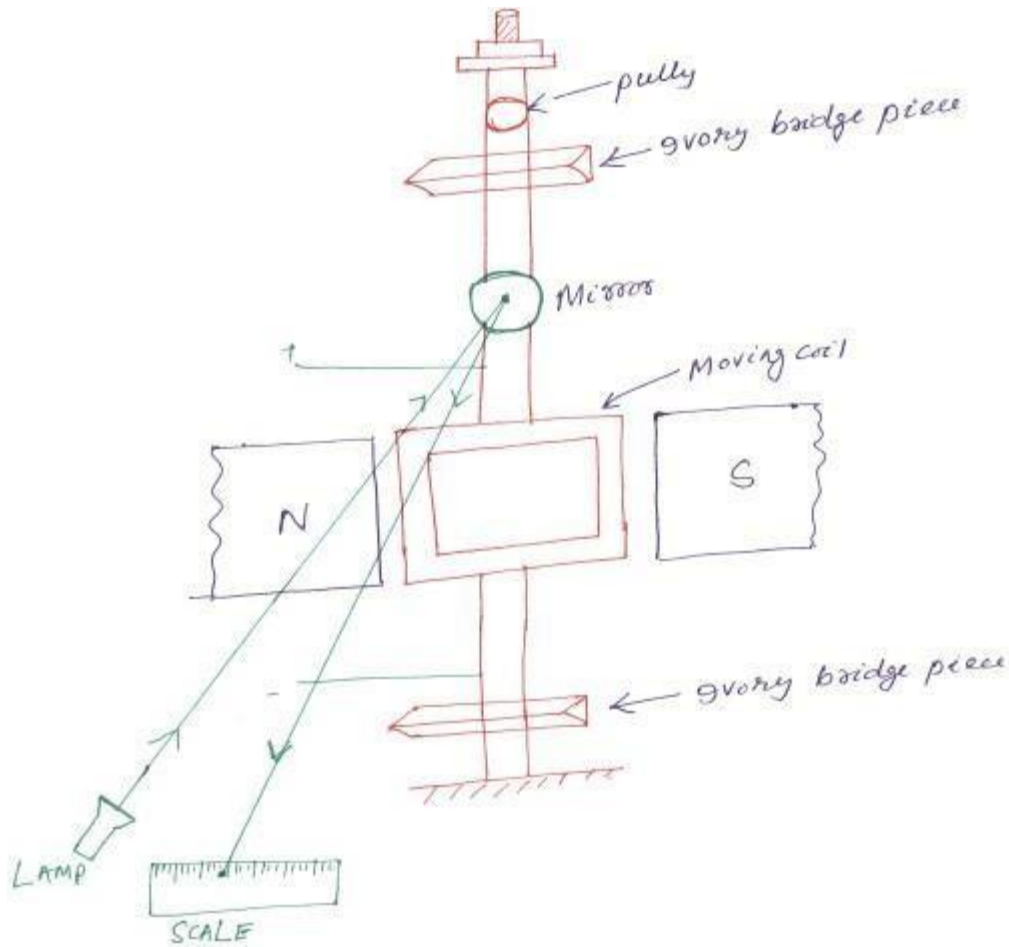


Fig 2.31 Vibration Galvanometer

Example 2.2-In a low- Voltage Schering bridge designed for the measurement of permittivity, the branch 'ab' consists of two electrodes between which the specimen under test may be inserted, arm 'bc' is a non-reactive resistor R_3 in parallel with a standard capacitor C_3 , arm CD is a non-reactive resistor R_4 in parallel with a standard capacitor C_4 , arm 'da' is a standard air capacitor of capacitance C_2 . Without the specimen between the electrode, balance is obtained with following values , $C_3=C_4=120$ pF, $C_2=150$ pF, $R_3=R_4=5000\Omega$. With the specimen inserted, these values become $C_3=200$ pF, $C_4=1000$ pF, $C_2=900$ pF and $R_3=R_4=5000\Omega$. In such test $\omega=5000$ rad/sec. Find the relative permittivity of the specimen?

Sol: Relative permittivity(ϵ_r) = $\frac{\text{capacitance measured with given medium}}{\text{capacitance measured with air medium}}$

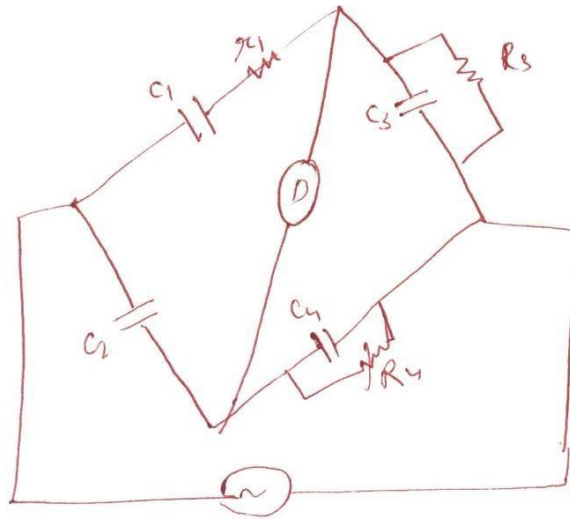


Fig 2.32 Schering bridge

$$C_1 = C_2 \left(\frac{R_4}{R_3} \right)$$

Let capacitance value C_0 , when without specimen dielectric.

Let the capacitance value C_S when with the specimen dielectric.

$$C_0 = C_2 \left(\frac{R_4}{R_3} \right) = 150 \times \frac{5000}{5000} = 150 \text{ pF}$$

$$C_S = C_2 \left(\frac{R_4}{R_3} \right) = 900 \times \frac{5000}{5000} = 900 \text{ pF}$$

$$\epsilon_r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

Example 2.3- A specimen of iron stamping weighting 10 kg and having a area of 16.8 cm² is tested by an Epstein square. Each of the two winding S_1 and S_2 have 515 turns. A.C. voltage of 50 HZ frequency is given to the primary. The current in the primary is 0.35 A. A voltmeter connected to S_2 indicates 250 V. Resistance of S_1 and S_2 each equal to 40 Ω . Resistance of pressure coil is 80 k Ω . Calculate maximum flux density in the specimen and iron loss/kg if the wattmeter indicates 80 watt?

$$\text{Sol}^n - E = 4.44 f \phi_m N$$

$$B = \frac{E}{4.44 f AN} = 1.3 \text{wb} / \text{m}^2$$

$$\text{Iron loss} = W \left(1 + \frac{R_S}{R_P}\right) - \frac{E^2}{(R_S + R_P)}$$

$$= 80 \left(1 + \frac{40}{80 \times 10^3}\right) - \frac{250^2}{(40 + 80 \times 10^3)} = 79.26 \text{watt}$$

$$\text{Iron loss/ kg} = 79.26/10 = 7.926 \text{ w/kg.}$$

CHAPTER-1

TRANSDUCERS

Transducer is a device which transforms energy from one form to another. It also converts any non-electrical quantity to electrical quantity.

- i.e.
- (1) Mechanical energy to Electrical signal
 - (2) Heat to Electrical signal.
 - (3) Liquid level to Electrical Signal.
 - (4) Flow rate to Electrical Signal.

There are two types of transducers:

- (1) Electrical Transducers
- (2) Mechanical Transducers

Electrical Transducers

Non-electrical quantities such as temperature, pressure, displacement, speed, fluid flow cannot be measured directly. The electrical Quantities such as voltage, current, resistance, inductance capacitance can be measured directly. For measurement of non-electrical quantities these quantities are to be converted into the electrical quantities and then measured. This function of converting non-electrical quantities to electrical quantities is accomplished by a device called as the Electrical Transducers.

Advantages of Electrical Transducers

- The electrical output from the transducer can be amplified into any desired level of voltage and current.
- Very small power is required for the controlling of the electrical and electronic systems.
- Frictional effect is reduced to minimum.
- The output can be measured at a distance from the sensing medium through wired or wireless medium.

Disadvantages of Electrical Transducers

- The electrical transducers are comparatively costly than the other types of transducers.
- They have low reliability in comparison to that of the mechanical transducers.

CLASSIFICATION OF TRANSDUCERS

Classification of Transducers on the basis of method of application

On the basis of method of application, the transducers can be classified as:

- (1) Primary Transducers
- (2) Secondary Transducers

Primary Transducers

When the input is sensed by transducer and the physical phenomenon is converted into electrical signal then such transducers is called as primary transducers.

e.g.: Thermistors: It senses the temperature directly and causes change in resistance with the change in temperature.

Secondary Transducers

When the input is first sensed by a sensor, output being of some form other than input signal is given to the transducer for conversion into electrical form. Such transducer is called as secondary transducers. It consists of two parts: sensing element and transduction element.

E.g.: LVDT (Linear Variable Differential Transducers): Displacement is converted to the electrical signal by LVDT.

Sensing element: It corresponds to the change in physical phenomenon.

Transduction element: It changes the output of the sensing element into electrical signal.

Classification of Transducers on the basis of energy conversion:

On the basis of energy conversion, the transducers can be classified as:

- (1) Active Transducers
- (2) Passive Transducers

Active Transducers

An active transducer does not require any external power source for its operation. They develop their own voltage or current. An active transducer can generate an electrical signal directly in response to the physical quantity to be measured.

E.g.: Thermocouple, Tacho-generator, Photovoltaic cell.

Passive Transducers

These transducers require an external power source for its conversion. In these transducers electrical parameters such as resistance, Inductance, Capacitance changes and it causes a change in voltage and current.

E.g.: Resistive transducer, Inductive transducer and capacitive transducer.

Classification of Transducers on the basis of nature of output signal

On the basis of energy conversion, the transducers can be classified as:

- (1) Analog Transducers
- (2) Digital Transducers

Analog Transducers

It converts input signal to output signal which is continuous function of time.

E.g.: Strain gauge, thermocouple, strain gauge.

Digital Transducers

It converts input signal into output signal in the form of pulses which gives discrete output. These are more popular in use in the form of digital measuring instruments.

RESISTIVE TRANSDUCERS

In resistive transducers, resistance changes due to change in some of the physical phenomenon.

Resistance of any metal conductor, $R = \rho \frac{L}{A}$

Where ρ = Specific resistance of the material (resistivity)

L = Length of the wire

A = Cross-sectional Area of the wire

If there is a change in ρ, L, A then there is a change in resistance. Thus it can be said that input signal to the transducer causes the variation in the resistances by changing the value of any one quantity of ρ, L, A .

E.g.: Potentiometer: Length varies so there is a change in resistance.

Strain Gauges: When the gauge is strained then there is a change in cross sectional area and hence there is a change in resistance.

For temperature measurement there is a change in resistivity ' ρ '.

POTENTIOMETER

Potentiometer is a very simple device where there is a change in output voltage due to change in displacement. It is a passive transducer which requires an external power source for its operation. It is a very cheap form of transducer and is very widely used.

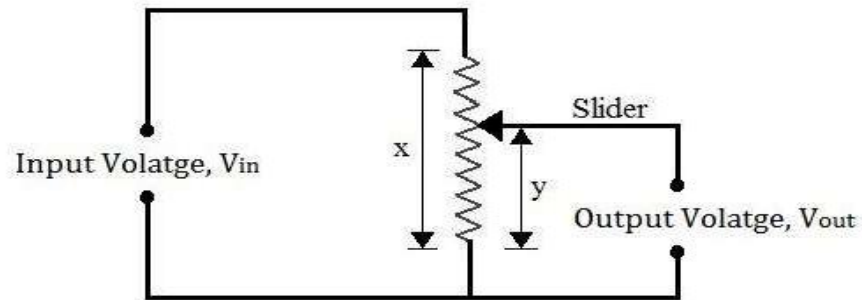
It converts linear displacement to the voltage by change in resistance and also rotational displacement to voltage by change in resistance.

It is of the following types:

- Linear potentiometer.
- Rotational Potentiometer
- Helipot

Linear Potentiometer

Linear potentiometers are used for the measurement of displacement. It is simple and cheap. Its output voltage is a function of linear displacement.



x = Total length of resistive material.
 y = Relative position of the slider on the resistive material.

Then,

$$V_{out} = \frac{y}{x} V_{in}$$

Fig. 1.1: Potetiometric arrangemet

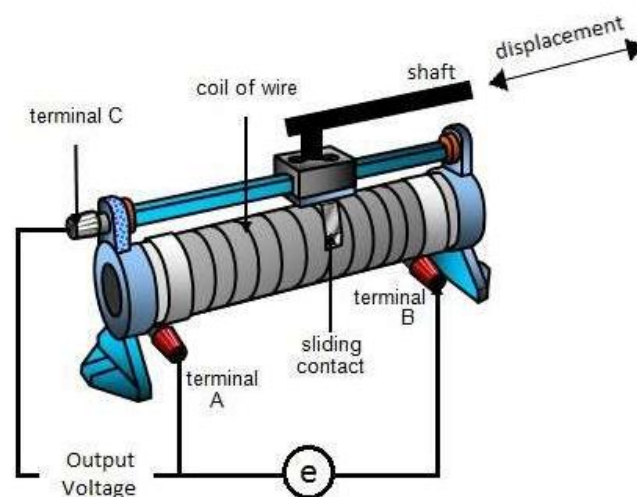


Fig. 1.2: Potentiometer connection using a rheostat

Construction:

It consists of a resistance element provided with a sliding contact called as wiper. The resistance element is a wire made up of platinum or nickel alloy. This resistance element is wound uniformly throughout the length of the sliding contact. Displacement of the slider/wiper determines the change in the voltage drop related to the position of slider. It is mostly used in the laboratory demonstrations and is not suitable for industrial use.

Rotary Potentiometer:

This kind of potentiometer operates in the same way as that of the linear potentiometer. In rotary potentiometer the resistance element is connected a circular manner in case of rotary potentiometer. This kind of potentiometer has a rotary movement and is useful in the measurement of angular displacement. The angular measurement can be measured between 1° and 357° .

When the motion of the wiper is in both translational and rotational way then the potentiometer is called as helipot.

Advantage of potentiometer:

- It can measure large displacement.
- It has high electrical efficiency.
- It is simple in construction.
- It is cheap and easy to operate.

Disadvantages of potentiometer:

- Sliding contact may wear out and get contaminated or generate noise.
- They require large force to move their sliding contact.

STRAIN GAUGES

If a metal conductor is stretched or compressed, its resistance changes due to the change in both length and diameter of the conductor.

The change in the value of the resistance by straining of the gauge may be partially explained. As the gauge is under a positive strain, the length of it increases and its area of cross section decreases. Since the resistance of the conductor is proportional to the length and inversely proportional to its area of cross section, the resistance of the gauge increases with positive strain.

Let us consider a strain gauge made up of circular wire

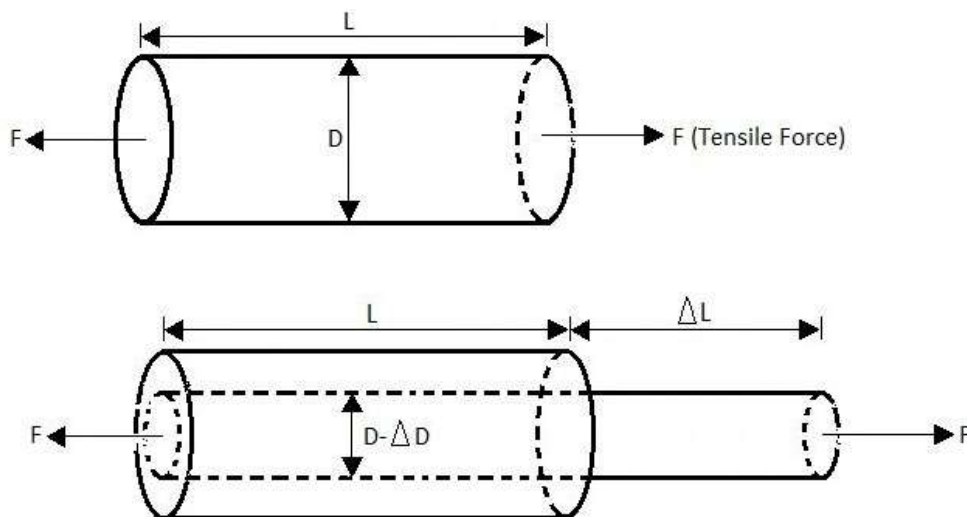


Fig. 1.3: Change in dimensions of a strain gauge due to force 'F'

L = Original length of wire

D = Original Diameter of wire

F = Tensile Force

ΔL = Change in length

ΔD = Change in Diameter

ΔR = Change in Resistance

ΔA = Change in cross sectional area

We know,

$$R = \rho \frac{L}{A}$$

Differentiating with respect to stress s

$$\frac{dR}{ds} = \frac{\rho}{A} \frac{\partial L}{\partial s} - \frac{\rho L}{A^2} \frac{\partial A}{\partial s} + \frac{L}{A} \frac{\partial \rho}{\partial s}$$

Dividing by $R = \rho \frac{L}{A}$

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{1}{A} \frac{\partial A}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

Per unit change in length = $\frac{\Delta L}{L}$

Per unit change in area = $\frac{\Delta A}{A}$

Per unit change in Resistivity = $\frac{\Delta \rho}{\rho}$

$$A = \frac{\pi}{4} D^2$$

$$\frac{\partial A}{\partial s} = 2 \frac{\pi}{4} D \frac{\partial D}{\partial s}$$

$$\frac{1}{A} \frac{\partial A}{\partial s} = \frac{\left(\frac{2\pi}{4}\right) D \frac{\partial D}{\partial s}}{\left(\frac{\pi}{4}\right) D^2} = \frac{2}{D} \frac{\partial D}{\partial s}$$

We know,

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{1}{A} \frac{\partial A}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

$$\text{Poisson's ratio, } \nu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = -\frac{\frac{\partial D}{D}}{\frac{\partial L}{L}}$$

$$\frac{\partial D}{D} = -\nu \times \frac{\partial L}{L}$$

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \nu \frac{2}{L} \frac{\partial L}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

The above relation can be written as:

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$$

Gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

$$G_f = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}}$$

Therefore, $G_f = 1 + 2\nu + \frac{\Delta \rho / \rho}{\Delta L / L}$

$$\frac{\Delta L}{L} = \epsilon = \text{Strain}$$

Thus $G_f = 1 + 2\nu + \frac{\Delta \rho / \rho}{\epsilon}$

Unbonded Metal Strain Gauges

This gauge consists of a wire stretched between two points in a insulating medium such as air. The wires may be made up of copper-nickel, chrome-nickel or nickel-iron alloys. The wires are tensioned to avoid buckling when they experience a compressive force.

The unbonded metal wire gauges used almost exclusively in transducer applications, employ preload resistance wires connected in a wheat-stone bridge. At initial period, the strains and resistances of the four arms are normally equal, with this result, output voltage of the bridge, $e_0 = 0$. When displacement occurs, it increases tension in two wires and decreases in the other two, thereby increase the resistance of the two wires which are in tension and decreasing the resistance of the two wires. This causes an unbalance of the bridge producing an output voltage which is proportional to the input displacement and hence to the applied pressure.

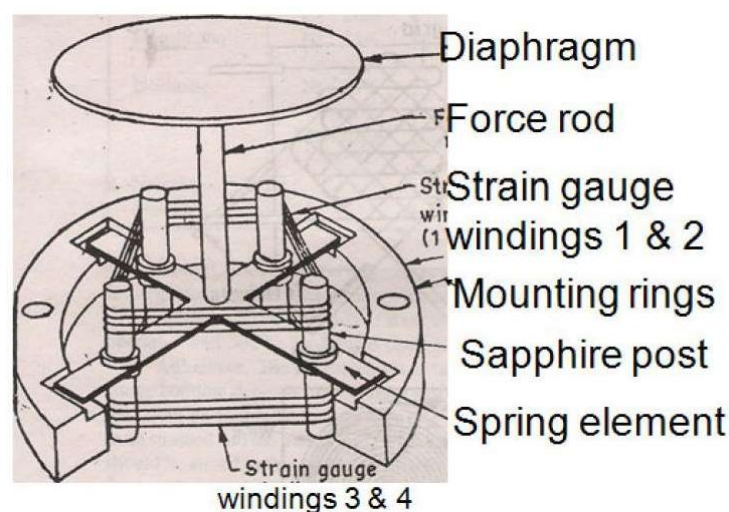


Fig. 1.4: Constructional diagram of Unbonded metal strain gauge

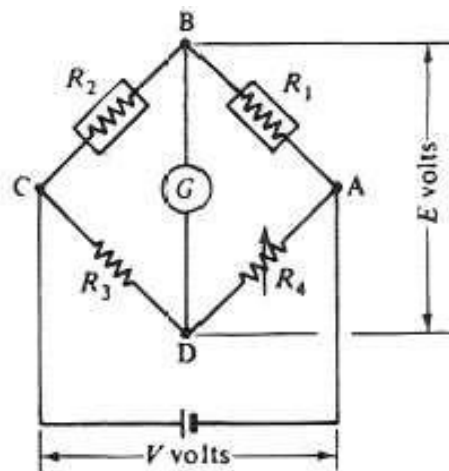


Fig. 1.5: Schematic diagram of unbonded metal strain gauge

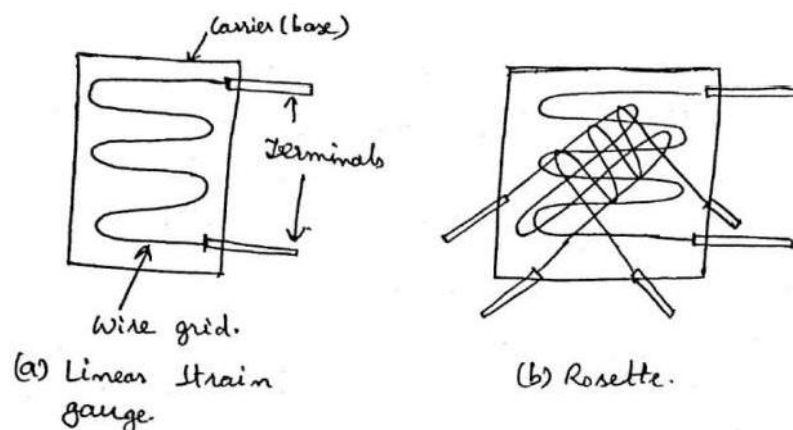
Bonded Wire Strain Gauges

It is used for both stress analysis and for constructing transducers.

A resistance wire strain gauge consists of grid of fine resistances wire of about 0.025mm in diameter or less. The grid is cemented to carrier (base) which may be thin sheet of paper, a thin sheet of Bakelite or sheet of Teflon. The wire is covered on top with a thin sheet of material so as to prevent it from any mechanical damage. The spreading of the wire permits an uniform distribution of stress over the grid. The carrier is bonded with an adhesive material to the specimen under study. The wires cannot buckle as they are embedded in a matrix of cement and hence faithfully follow both the tensile and the compressive strains of the specimen. The materials and the wire sizes used for the banded wire strain gauges are the same as used for the unbonded wire strain gauges.

It is desirable that resistance wire strain gauges should have the following characteristics:

- (1) Strain gauge should have a high value of gauge factor G_f . A high gauge factor indicates high sensitivity.
- (2) The resistances of the strain gauges should be as high as possible since this minimizes the effects of undesirable variations of resistance in the measurement circuit.



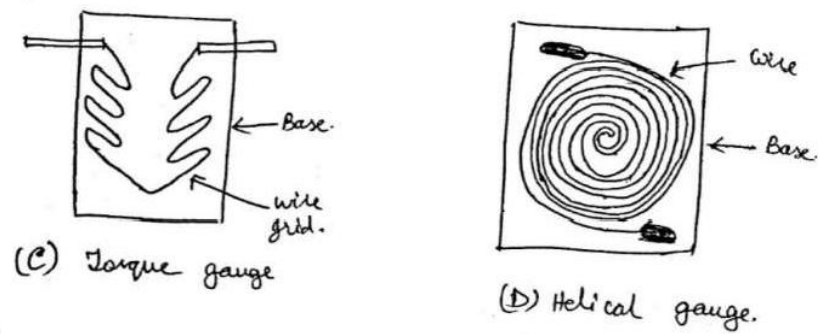


Fig. 1.6: Different types of bonded strain gauge

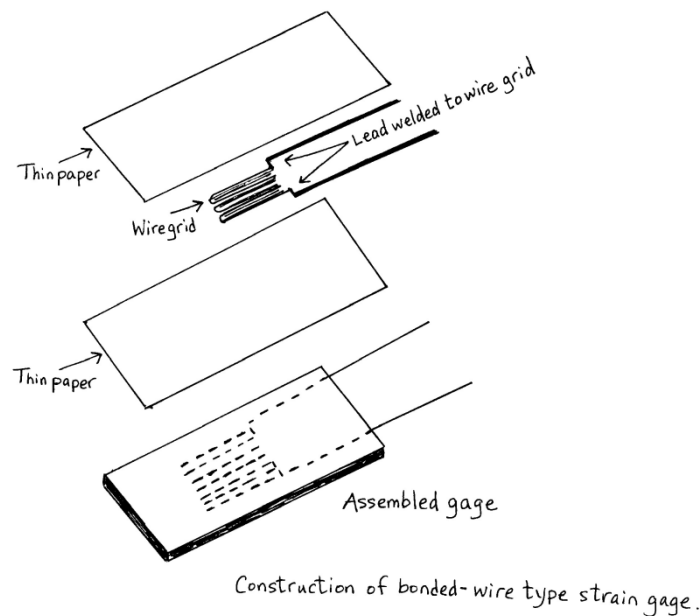


Fig. 1.7: Construction of Bonded strain gauge

PLATINUM RESISTANCE THERMOMETER

The resistance type thermometer bulbs are sensing elements in the form of the wires or foils. The films deposited on insulating surfaces are also used for temperature sensing. In the wire type, the arrangement are commonly a helical coil wound as a double wire to avoid inductive effects. The laboratory type resistance thermometers have the temperature sensing element wound on a cross mica former and enclosed in Pyrex tube shown in fig. 1.

The industrial type of thermometer shown in fig. 2. Here the former is of grooved ceramic and the wire is being protected by a glass coating or by a stainless steel tube. The element is normally sealed in glass when used for temperatures upto 150°C and ceramic for use in temperatures upto 850°C .

Resistive elements are also made up of thin etched grids of metal foils similar in shape of the foil type strain gauges. They are constructed of the platinum and may be bonded to plastic booking for attachment to a surface.

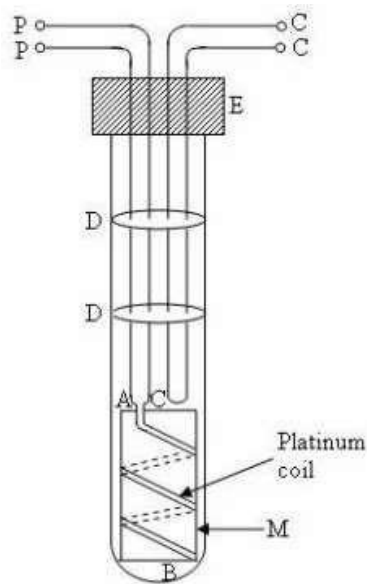


Fig. 1.8: Schematic diagram of Platinum Resistance Thermometer

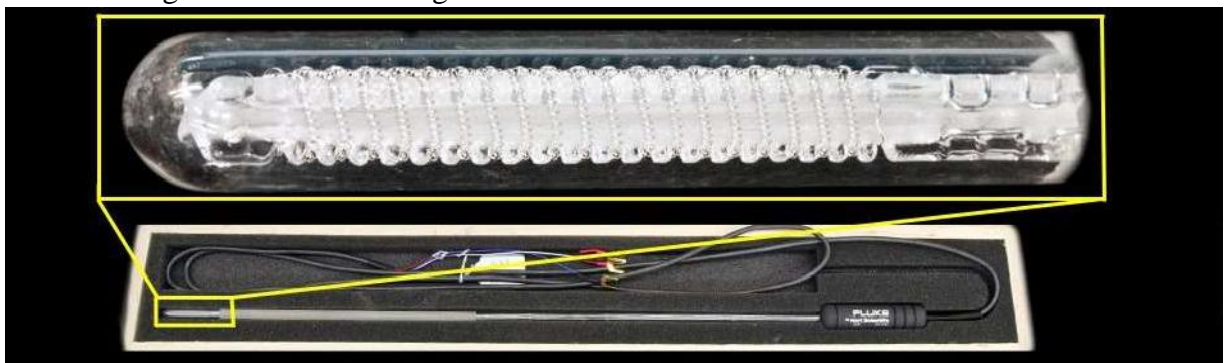


Fig. 1.9: Platinum Resistance thermometer

THERMISTOR

Thermistors are generally composed of semi-conductors. Most thermistors have a negative temperature coefficient i.e. their resistance decreases with increase of temperature. This allows thermistor circuits to detect very small changes in temperature which could be observed using a RTD or a thermocouple.

Thermistors are widely used to measure temperature ranging from -600°C to 150°C . Resistance of thermistors ranges from 0.5Ω to $0.75\text{M}\Omega$.

Construction of thermistors:

These are composed of sintered mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in variety of shapes and sizes. They may be in the form of beads, rods and disks. Thermistors in the form of beads are the smallest in size.

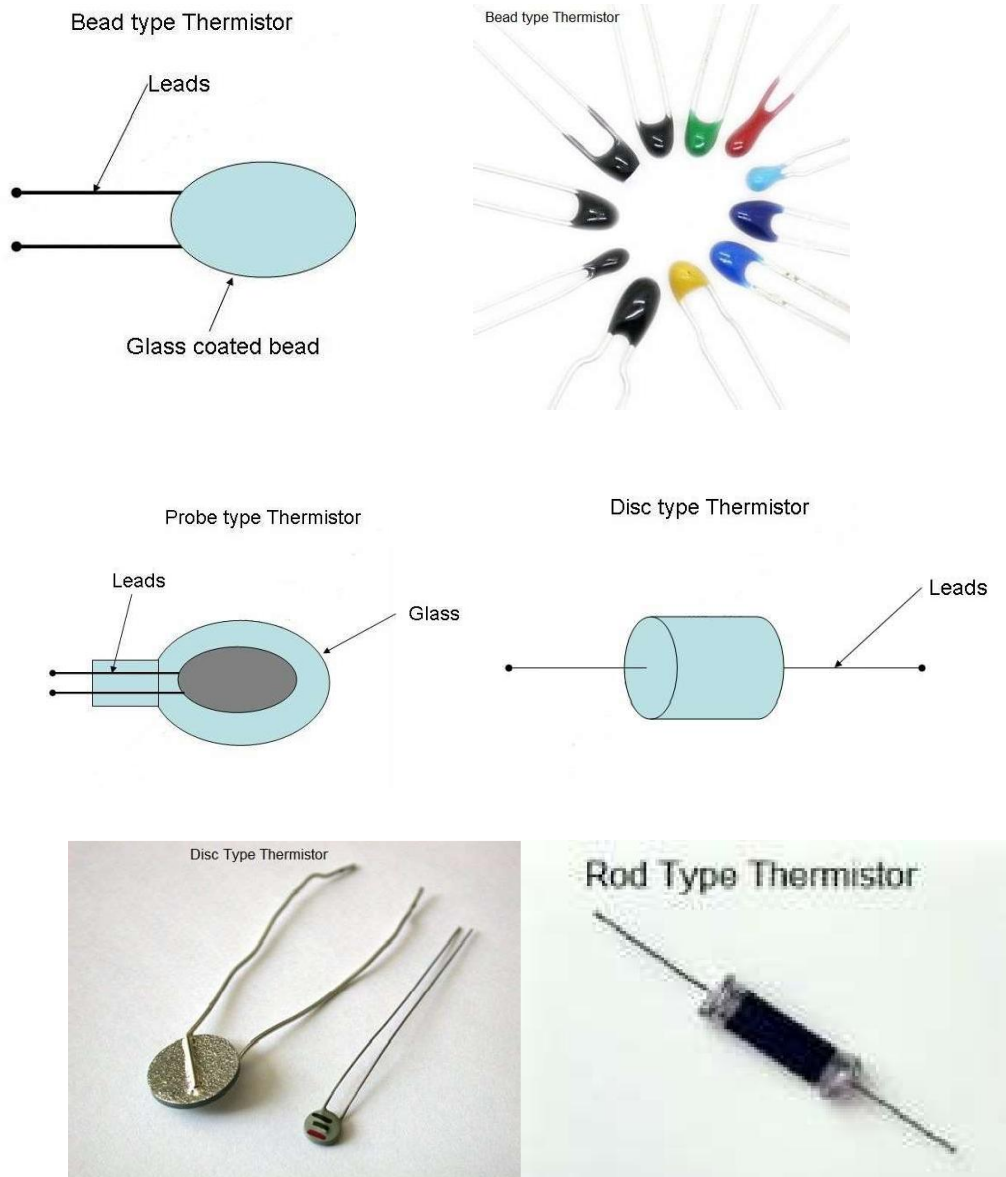


Fig. 1.10: Different types and construction of Thermistors

Resistance temperature characteristics of thermistor

It is given by,

$$R_{T_1} = R_{T_2} \exp\left[\beta \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

R_{T_1} = Resistance of the thermistor at temperature T_1 (OK)

R_{T_2} = Resistance of the thermistor at temperature T_2 (OK)

β = Constant, depending upon the material of thermistor.

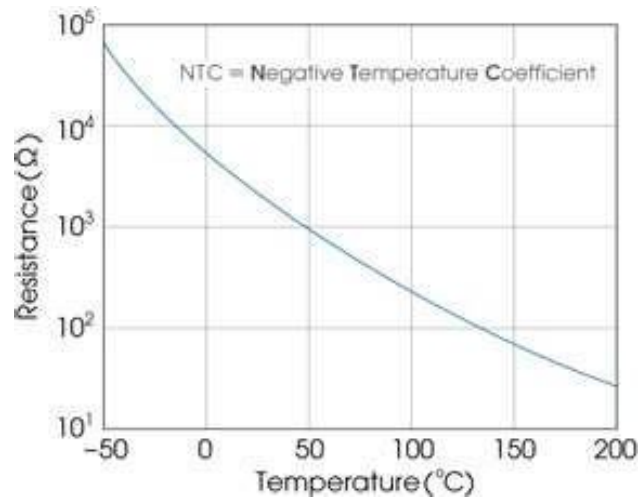


Fig. 1.11: Resistance temperature characteristics of thermistor

Thus a thermistor has a very high negative temperature coefficient of resistance making it an ideal temperature transducer. Between -100°C to 400°C , the thermistor changes between its resistivity from 10^4 to $10^{-4}\Omega\text{m}$ which explains high sensitivity of thermistors for measurement of temperature.

INDUCTIVE TRANSDUCER

The most commonly used inductive transducer is Linear Variable Differential Transducer (LVDT).

LVDT is widely used for translating linear motion and displacement into electrical signals. It is a passive transducer.

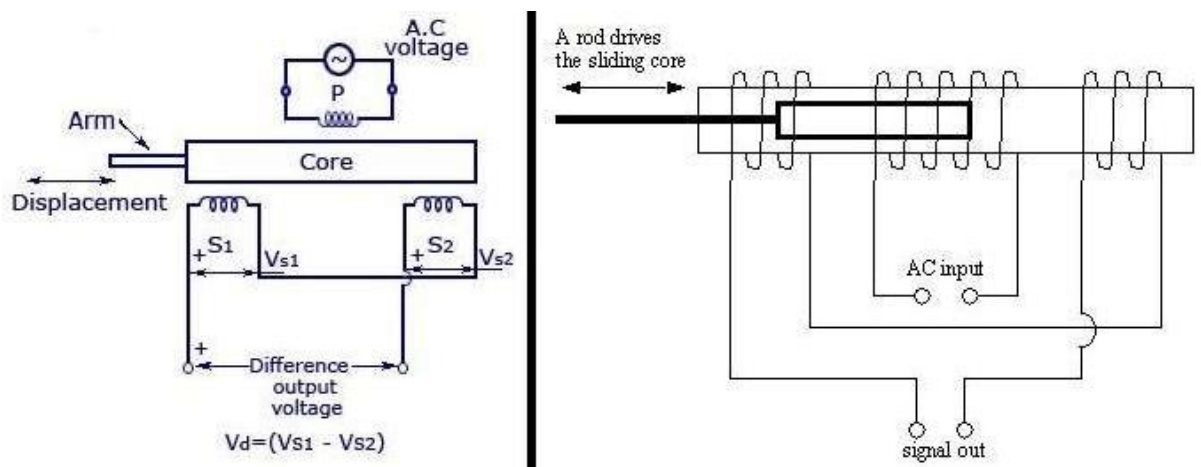


Fig. 1.12: Schematic diagram of LVDT

Construction:

LVDT consists of two windings: (a) One Primary Winding, P (b) Two secondary winding S_1 & S_2 . These two windings are wound on hollow cylindrical former known as 'bobbin' which is made up of either non-magnetic material or insulating material.

The secondary windings S1 & S2 have equal number of turns and are placed on either side of the primary winding. Primary winding is connected to the AC source.

A moveable cylindrical shape soft iron core is attached to the sensing element of the transducer. The core slides freely within the hollow portion of the bobbin. Core is made up of nickel iron alloy to reduce the eddy current loss. The displacement to be measured is attached to the soft iron core. In order to get a differential output voltage, the two single voltage signals V_{s1} and V_{s2} from the two secondary windings S_1 and S_2 are connected in series in a particular fashion. Such a circuit connection has been adopted to ensure that we get a difference of the two output voltages V_{s1} and V_{s2} and not a summation.

Working:

Any physical displacement of the core causes the voltage of one secondary winding to increase while simultaneously reducing the voltage in the other secondary winding. The difference of the two voltages appear across the output terminal of the transducer and gives a measure of the physical position of the core and hence the displacement.

When the core is at the normal position the flux linking with both secondary windings is equal and hence equal EMFs are induced. So the net output is $V_0 = V_{s1} - V_{s2} = 0$

If the core is moved to the left of the null (normal) position, then more flux gets linked with the winding ' S_1 ' and less with the winding ' S_2 '. So $V_{s1} > V_{s2}$. Hence we get a positive voltage at the output.

Similarly, if the core is moved to the right of the null (normal) position, then more flux gets linked with the winding ' S_2 ' and less with the winding ' S_1 '. So $V_{s1} < V_{s2}$. Hence we get a negative voltage at the output.

So when

$$\begin{aligned} V_{s1} > V_{s2} \text{ then } V_0 &= V_{s1} - V_{s2} = +ve \\ V_{s1} < V_{s2} \text{ then } V_0 &= V_{s1} - V_{s2} = -ve \end{aligned}$$

The difference of the output voltage of the secondary windings gives the amount of displacement.

Applications:

It is widely used for the measurement of displacement ranging from few mm to few cm.

Advantages:

- Its output is very high. So no need of amplifying the output.
- The device consumes less power.
- It is very simple, light in weight and easy to maintain.
- It can tolerate high degree of shock and vibration.

Disadvantages:

- These devices are very sensitive to stray magnetic field.

CAPACITIVE TRANSDUCER

It is a device in which the capacitance is varied by the non-electrical quantity being measured. If there is a change in a physical quantity such as force displacement and pressure then there is a change in capacitance.

Operating principle of capacitive transducer is based on the principle of parallel plate capacitor.

$$\text{Capacitance, } C = \frac{\epsilon_0 \epsilon_r A}{D}$$

Where C = Capacitance (in F)

A = Overlapping area of the plate (in m^2)

D = Distance between the plates

ϵ_r = Relative permittivity

ϵ_0 = Permittivity of free space

Capacitive transducer work on the principle of the change of capacitance which may be caused by:

- (1) Change in overlapping area (A).
- (2) Change in distance (D) between the two plates.
- (3) Change in dielectric constant (ϵ).

All these changes are caused due to changes in physical quantities like pressure, force which causes change in capacitance and hence the non-electric quantities get changed to electrical quantities.

Transducer using change in area of plate

For cylindrical Tube:

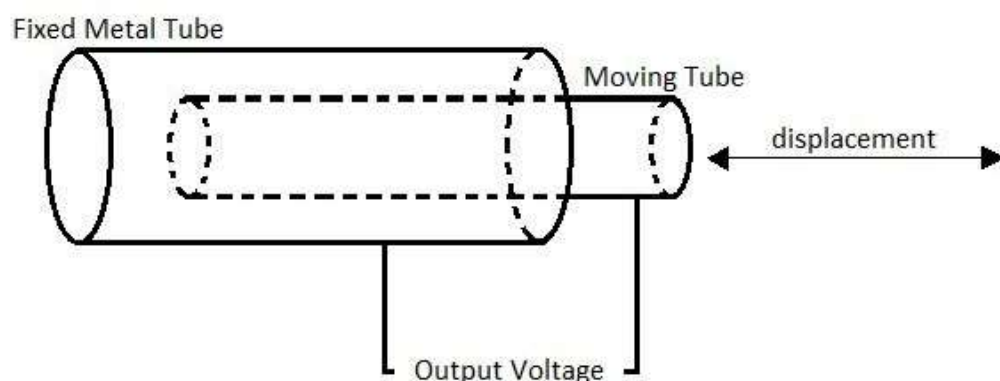


Fig. 1.13: Change in Capacitance due to change in area of the cylindrical plates

In this diagram the moving tube is inside a fixed tube. The movement in the moving tube causes a change in the overlapping area (A) due to which the capacitance changes and hence there is a change in the output voltage.

$$C = \frac{\epsilon_0 \epsilon_r A}{D}$$

$$C \propto A \text{ and } C \propto \frac{1}{D}$$

For parallel plate:

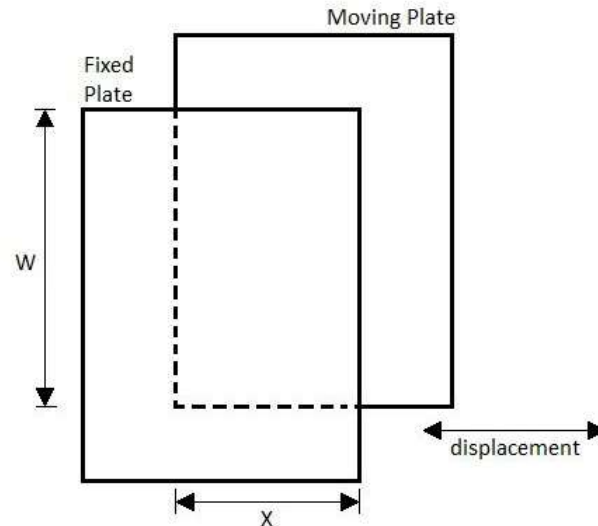


Fig. 1.14: Change in Capacitance due to change in area of the parallel plates

In this diagram the capacitance changes due to the changes in the overlapping area (A). The overlapping area changes due to the displacement in the moving plate. Currently the overlapping area (A) is $W \times x$. Thus the capacitance is given by:

$$C = \frac{\epsilon_0 \epsilon_r (W \times x)}{D}$$

Hence when there is a change in W or x then there is a change in the capacitance due to the change in the overlapping area (A).

Transducer using change in distance between the two plates

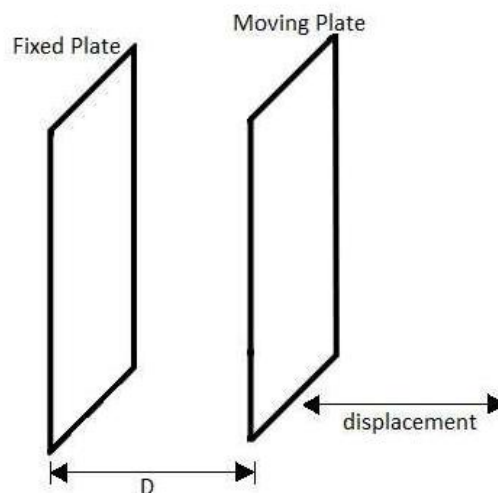


Fig. 1.15: Change in Capacitance due to change in distance between the plates

In this mechanism if there is a change in the distance between the two plates then there is a change in capacitance.

Since we know,

$$C = \frac{\epsilon_0 \epsilon_r A}{D}$$

Hence, from the formula it can be observed that,

$$C \propto \frac{1}{D}$$

So it is quite clear that the capacitance is inversely proportional to the distance between the plates. So when the distance increases then there is a decrease in capacitance and vice versa.

Application:

- Capacitive transducers are used for measurement of small displacement.
- These are used for linear displacement.

Advantages of capacitive transducer:

- Very small force is required to operate the system.
- It is highly sensitive.

Disadvantages of capacitive transducer:

- Capacitive transducers are very sensitive to vibrations and temperature.
- Metallic parts of capacitor must be insulated from each other.
- Performance of capacitive transducers is affected by humidity, dirt and other contaminations.

PIEZOELECTRIC TRANSDUCER

A piezoelectric material is one in which an electric potential appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible i.e. conversely, if varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. The effect is known as piezo-electric effect.

Common piezo-electric materials are: Rochelle salt, ammonium dihydrogen phosphate, Lithium sulphate, dipotassiumtartarate, quartz and ceramics A and B. except for quartz and ceramics A and B, the rest are man-made crystals grown from aqueous solution under carefully controlled conditions.

The materials that exhibit a significant and useful piezo-electric effect are divided into two categories:

- (1) Natural group – quartz and Rochelle salt.
- (2) Synthetic group – Lithium sulphate, Ethylene diaminetartarate

The piezo-electric effect can be made to respond to the mechanical deformation s of the material. It is used for converting mechanical motion to electrical signals, which can be thought like charge generator and a capacitor.

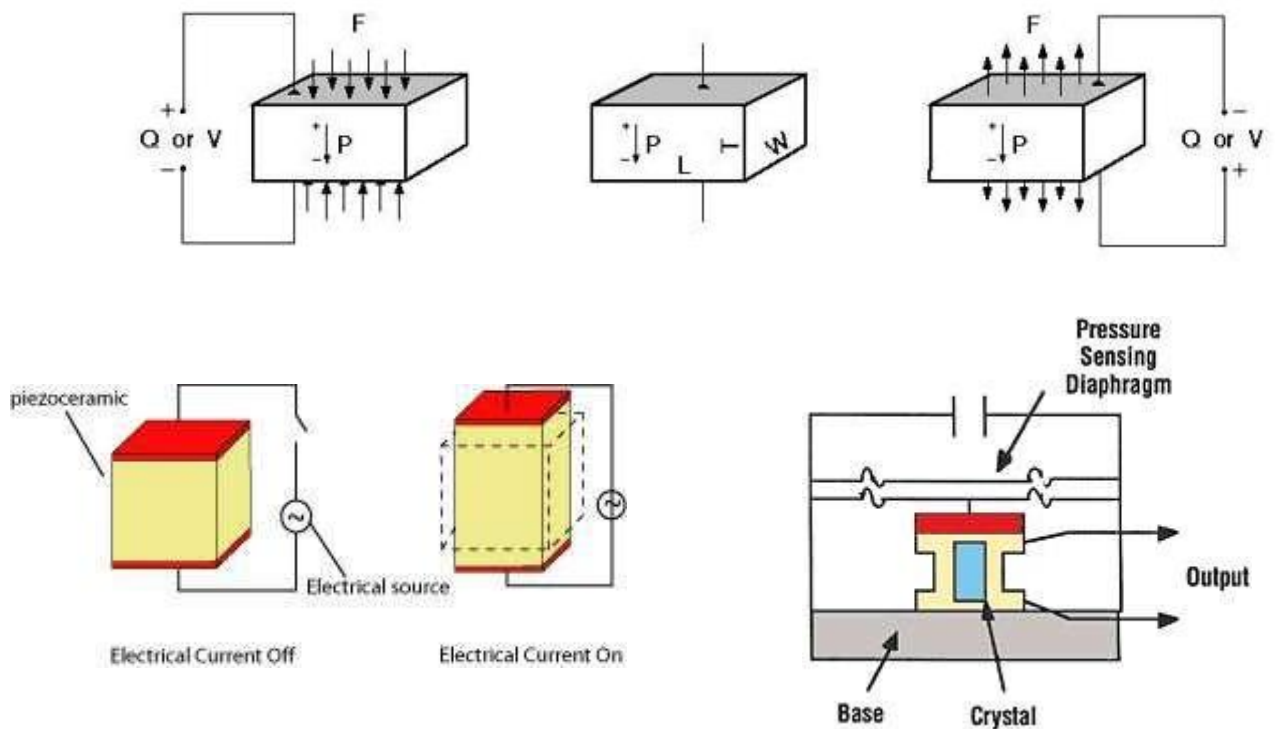


Fig 1.16: Schematic of Piezo-electric Transducers



Fig. 1.17: Piezoelectric Transducer

Mechanical deformation generates a charge and this charge appears as voltage across the electrodes voltage, $E = \frac{Q}{C}$

A tensile force produces a voltage of one polarity while a compressive force produces a voltage of opposite polarity.

$$\text{Charge, } Q = d \times F \quad \dots (1)$$

d = Charge sensitivity of the crystal (Constant for a given crystal) [Unit: C/N]

F = Applied force(N)

$$F = \frac{AE}{t} \Delta t \quad (N) \quad \dots (2)$$

A = Area of crystal (m^2)

t = Thickness of crystal (m)

E = Young's modulus (N/m^2)

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \frac{1}{\Delta t/t} = \frac{Ft}{A\Delta t} \quad (N/m^2)$$

Also, $A = w.l$ Where w = Width of the crystal
 l = Length of the crystal

From equations (1) and (2),

$$Q = dAE(\Delta t/t)$$

Output voltage, $E_0 = Q/C_p$

C_p = Capacitance between electrodes

Also, $C_p = \epsilon_r \epsilon_0 A/t$

$$E_0 = \frac{Q}{C_p} = \frac{dF}{\epsilon_r \epsilon_0 A/t} = \frac{dt}{\epsilon_r \epsilon_0 A} F$$

$F/A = P =$ Pressure or stress in N/m^2 .

$$E_0 = \frac{dt}{\epsilon_r \epsilon_0} \cdot P$$

$$E_0 = g \cdot t \cdot P$$

Where $E_0 = \frac{d}{\epsilon_r \epsilon_0} \quad \dots (3)$

g = Voltage sensitivity of the crystal (This is a constant for a given crystal. Its unit is Vm/N)

$$g = \frac{E_0}{t \cdot P} = \frac{E_0/t}{P} = \frac{\epsilon}{P}$$

Where $\epsilon = \frac{E_0}{t}$ = electric field

$$g = \frac{\text{electric field}}{\text{stress}} = \frac{\epsilon}{P}$$

Crystal voltage sensitivity, g , can be defined as the ratio of electric field intensity to pressure (or stress). The units of g are Vm/N .

From (3),

$$d = g \epsilon_r \epsilon_0 = \text{Charge sensitivity}$$

OPTO-ELECTRONIC TRANSDUCER

Optoelectronics is the study and application of electronic devices that source, detect and control light, usually considered a sub-field of photonics. In this context, light often includes invisible forms of radiation such as gamma rays, X-rays, ultraviolet and infrared, in addition to visible light. Optoelectronic devices are electrical-to-optical or optical-to-electrical transducers, or instruments that use such devices in their operation.

Photoelectric transducers are an integral part of the optoelectronic transducers. They are the most used transducers for detection of light and controlling various systems.

Photoelectric Transducers

It is a device which allows flow of current only when the light falls on the device. The magnitude of current varies with the intensity of the light.

Photoelectric transducers operate on the principle that when light strikes on the photosensitive surface the following may result:

- Electrons may flow-in photoemissive cell
- A voltage may be generated-photovoltaic cell
- A change of resistance may take place-Photoconductive cell

This kind of photoelectric transducers have application in many of the electronic devices.

- The photoemissive Transducers are based on the emission of electrons from the metal cathode when it is exposed to light or other radiation. These electrons are collected by the positively charged anode. So this kind of transducers are used in sound reproduction of picture film, counting device and turn on and off a circuit etc.
- Photovoltaic cell-This generates an output voltage proportional to intensity of light or radiation intensity. Extremely used as a source of power for space aircraft. Other than that it is also used in TV, calculators and automatic control system etc.
- Photoconductive cell-In this device the resistance of the semiconductor decreases when intensity of light increases. Such devices have applications in industrial and laboratory control applications. These are used for relay operation. When light falls on the semiconductor then the resistance reduces and the relay operates otherwise it has high resistance. Also used to switch transistors on or off.
- Optocoupler: It is device in which the transistor is switched on using the light from the LED. It transfers electrical signals between two isolated circuits by using light. It consists of opto-transistor which turns on when light falls on its base terminal.

CHAPTER-2

Cathode Ray Tube (CRT)

A Cathode Ray Tube is the main part of Cathode Ray Oscilloscope (CRO) with additional circuitry to operate the circuit.

Its main parts are:

- (1) Electron gun assembly.
- (2) Deflection plate assembly.
- (3) Fluorescent Screen.
- (4) Glass envelope.
- (5) Base for making the connection.

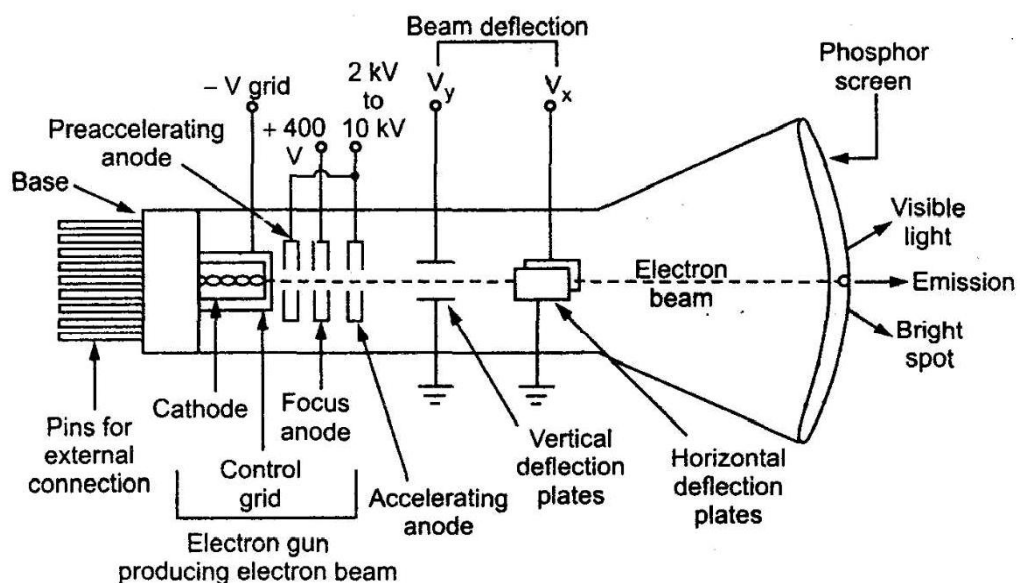


Fig. 2.1: Block Diagram of Cathode Ray Tube

Cathode ray tube is a vacuum tube and convert an electrical signal into visual one. Cathode ray tube makes available plenty of electrons. The electrons are accelerated to high velocity and are focussed on a fluorescent screen. Electron beam produces a spot of light wherever it strikes. The electron beam is deflected on its journey in response to the electrical signal under study. Hence the electrical signal is displayed.

Electron gun assembly

The arrangement of electron which produces a focussed beam of electrons is called the electron gun. It consists of:

- (1) An indirectly heated cathode.
- (2) A control grid surrounding the cathode.
- (3) A focusing anode.
- (4) An accelerating anode.

Control grid is held at negative potential with respect to cathode. Two anodes focusing anode and accelerating anode are maintained at high positive potential with respect to cathode.

It has the following parts:

- Cathode with the heater.
- Control grid
- Focusing anode
- Accelerating anode.

Cathode: It consists of a nickel cylinder coated with oxide coating of barium and strontium emit plenty of electron. Rate of emission of electrons depend upon magnitude cathode current which can be controlled by the control grid.

Control Grid: It is a nickel cylinder with a centrally located hole. It encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. It is 15mm diameter and 15mm long with a hole of 0.25mm at the centre. Control grid is kept at negative potential with respect to the cathode and its function is to vary the electron emission. The hole in the grid is provided to allow the passage of electrons and concentrate the beam of electron along the axis of the tube.

Focusing anode: The focusing anode focuses the electron beam into sharp pin point by controlling the positive potential on it. Its value is a few hundred volts and more positive than cathode so as to accelerate the electron beam.

Accelerating anode: The positive potential on the accelerating anode is much higher than the focusing anode. It is in the range of 10KV and for this reason the anode accelerate the narrow beam of electrons to high velocity. Therefore the electron gun assembly forms a narrow accelerated beam of electron s which produces a spot of light when strikes the screen.

Deflection Plate assembly

Deflection plate is accomplished by two set of deflecting plates placed within the tube beyond accelerating anode.

- (1) One set is vertical deflection plates.
- (2) Other set is horizontal deflection plates.

Vertical deflection plates:

These plates are mounted horizontally and applied a proper potential. These plates move the electron beam in the vertical direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the vertical up and down on the fluorescent screen.

Horizontal deflection plates:

These plates are mounted vertically and applied a proper potential. These plates move the electron beam in the horizontal direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the horizontal left and right on the fluorescent screen.

Fluorescent Screen:

The end wall or inside face of tube coated with some fluorescent material called:

- Phosphor
- Zinc oxide.
- Zinc orthosilicate

When high velocity of electron beam strikes the screen a spot of light is produced at the point of impact. It absorbs the kinetic energy of the electron and convert into light. Colour of the light emitted depends on the fluorescent material used.

Glass Envelope

It is a highly evacuated glass housing in which vacuum is maintained inside. Inner wall of the CRT between neck and the screen are usually coated with conducting material called “aquadag”. This coating is electrically connected to the accelerating anode so that electrons which accidentally stroke the wall return to the anode. This prevents the wall from charging to a high negative potential. This aquadag coating prevents the formation of negative charge on the screen.

Cathode Ray Oscilloscope (CRO)

Cathode ray oscilloscope is the most useful and the most versatile laboratory instrument for studying wave shapes of alternating current, voltage, power, frequency which have amplitude and waveform.

It allows the user to see the amplitude of electrical signal as a function of time on the screen.

With CRO wave shapes of signal can be studied and can be used for measuring

- (1) Voltage
- (2) Frequency
- (3) Phase shift

Application of CRO

In laboratories it is used for

- Tracing actual waveform of current and voltage.
- Determination of amplitude of a variable quantity.
- Comparison of phase and frequency.
- Can be used to check any kind of electronic components in an electronic circuit.
 - For finding B-H curve of hysteresis loop.
 - For tracing the characteristics of any electronic components like diode, transistors etc.

For commercial purpose:

- In CRT televisions.
- In Radar.

Block diagram of General purpose CRO

It consists of the following components:

- Cathode Ray Tube
- Power supply Block
- Vertical Amplifier
- Horizontal Amplifier.
- Time Base generator.
- Trigger Circuit.

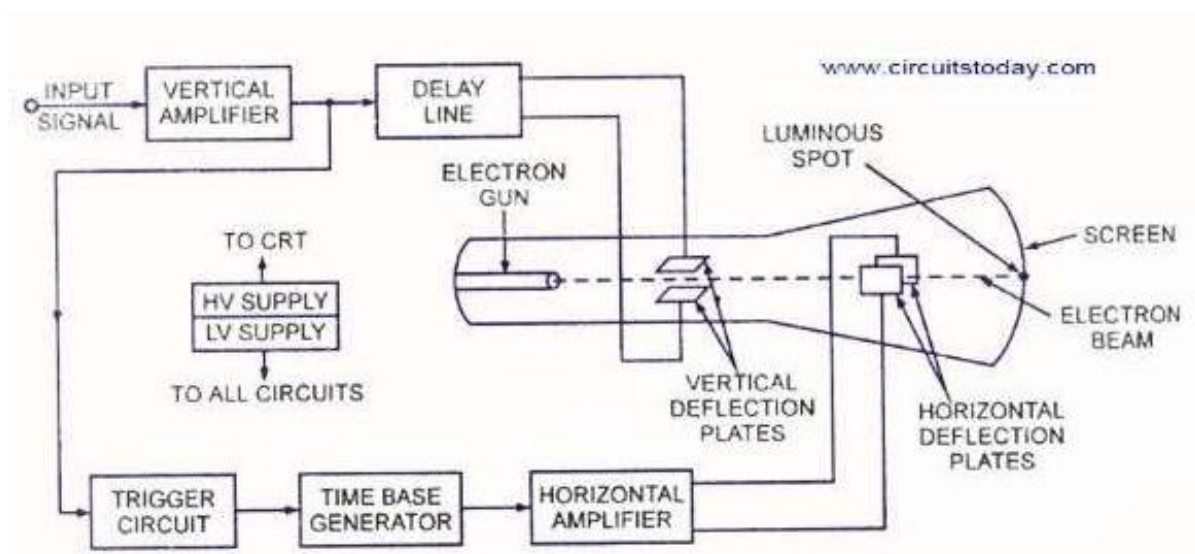


Fig 2.2: Block Diagram of Cathode Ray Oscilloscope

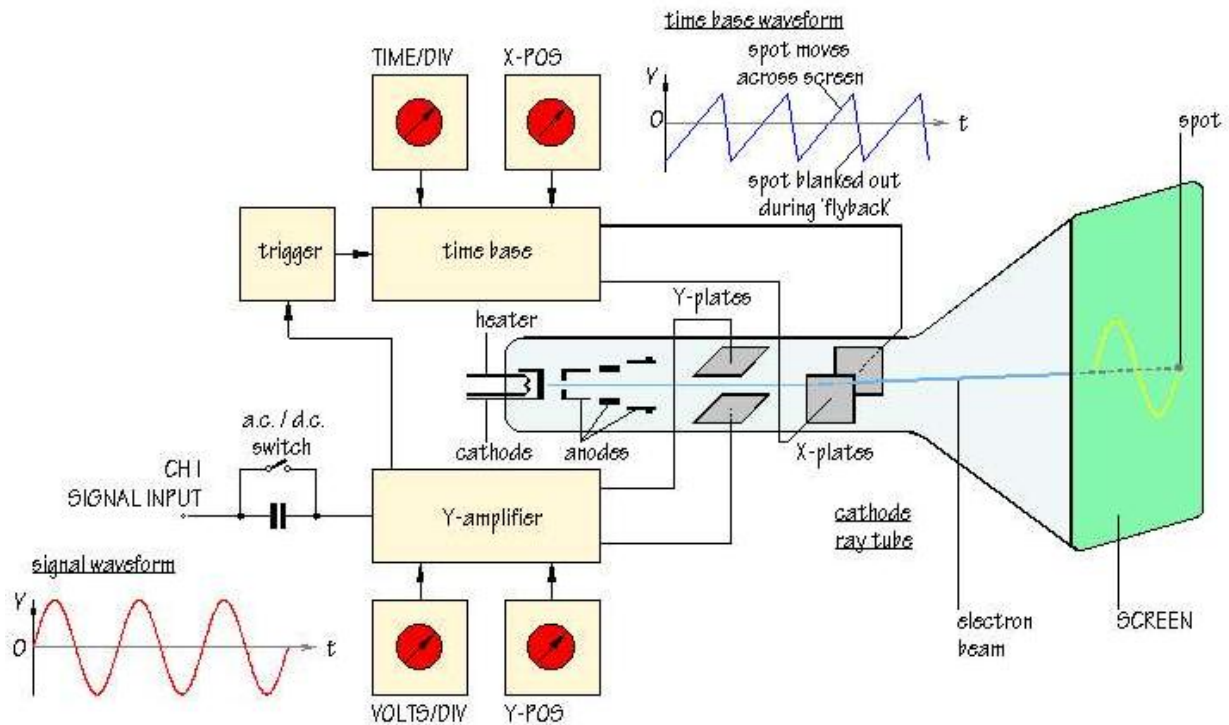


Fig. 2.3: Block Diagram of Cathode Ray Oscilloscope in a more practical way

Cathode Ray Tube:

It is the heart of the oscilloscope. It generates sharply focused electron beam and accelerate the beam to a high velocity and deflect the beam to create image. It contains the phosphor screen where the electron beam became visible. While travelling from the electron gun to screen the electron beam passes between set of

- (1) Vertical Deflection Plate.
- (2) Horizontal Deflection Plate.

Voltage applied to the vertical deflection plate to move the beam in the vertical plane and the CRT spot moves up and down.

Voltage applied to the horizontal deflection plate moves the beam in the horizontal plane.

Power supply block

It provides the voltage required by the CRT to generate and accelerate the electron beam as well as to supply the required operating voltages for the other circuit of the oscilloscope. Voltage in the range of few Kilo Volts are required by CRT for acceleration.

Low voltage for heater of electron gun of CRT which emit the electron.

Supply voltage for the other circuit is not more than few hundred volt.

Vertical Amplifier

It amplifies the signal waveform to be viewed. This is a wide band amplifier used to amplify signal in the vertical section.

Horizontal Amplifier

It is fed with saw-tooth voltage. It amplifies saw-tooth voltage before it is applied to horizontal deflection plates.

Time base generator:

It develops saw-tooth voltage waveform required to deflect the beam in horizontal section.

Trigger Circuit

The trigger circuit is used to convert the incoming signal into trigger pulses so that input frequency can be synchronised.

CRO Measurement

Various parameters which can be measured by CRO are

1. Voltage; 2. Current; 3. Time period; 4. Frequency; 5. Phase angle
6. Amplitude, 7. Peak to peak value

Voltage measurement

Voltage to be measured is applied to the V deflection plates through vertical amplifier.

The 'X' deflection plate is excited by time base generator.

⇒ After noting down the selection in vect/div from front panel the peak to peak value amplitude and r m s value of sinusoidal voltage can be obtained.

$$\text{I. Peak to peak value } V_{P-P} = \frac{\text{Volt}}{\text{Division}} \times \text{no. of division}$$

$$\text{II. Amplitude } V_m = \frac{V_{P-P}}{2}$$

$$\text{III. R. M. S value } V_{RMS} = \frac{2V_{P-P}}{\sqrt{2}} = \frac{V_P}{\sqrt{2}} + \frac{V_P}{\sqrt{2}}$$

Voltage/division = deflection Sensitivity

$$\text{Deflection factor} = \frac{1}{\text{Deflection Sensitivity}}$$

Current Measurement

A CRO has a very high input impedance and cannot be used for direct measurement. However the current can be measured in terms of voltage drop across a standard resistance

$$I = \frac{V}{R}$$

Time period measurement:

For the measurement of time period 'T' of the waveform. It is displayed on the screen such that one complete cycle is visible on the screen.

After noting the time/division selected on front panel the time period of the waveform can be obtained as Time period T = time/division X number of division occupied by one cycle.

$$\text{Frequency} = f = 1/T$$

Frequency measurement by (Lissajous Method Pattern)

The unknown frequency can be accurately determined with the help of CRO

Step – 1

Known frequency is applied to horizontal input (2000 Hz) Unknown frequency to vertical input

The number of loops cut by the horizontal line gives frequency on the vertical plates
 f_v

The number of loops cut by vertical lines gives the frequency on the horizontal plates
 f_H

$$\frac{f_v}{f_H} = \frac{\text{No of loops cut by horizontal line}}{\text{No of loops cut by vertical line}}$$

$$\frac{f_v}{2000} = \frac{1}{2}$$

$$f_v = 2000 \times \frac{1}{2}$$

Unknown frequency = 1000 Hz

It is a method to determine the unknown frequency by comparing it with known frequency.

This fog = Lissajous fog – Named after French.

Example 1:



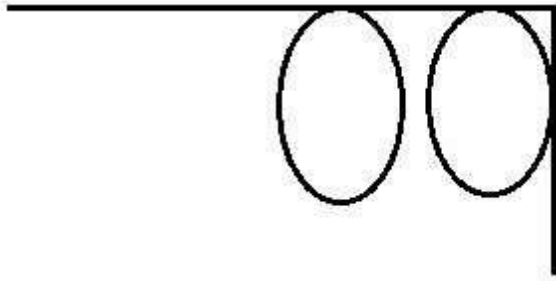
$$\frac{f_v}{f_H} = \frac{1}{1}$$

$$f_H = 1000$$

$$f_v = f_H = 1000 \text{ Hz}$$

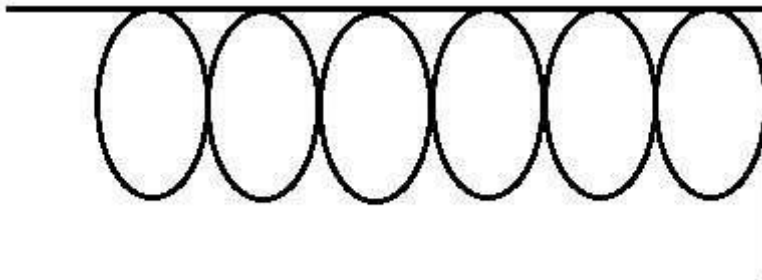
No of loops cut by horizontal and vertical line – 1

Example 2:



No. of loops cut by horizontal line = 2
 $f_v / f_H = 2/1$, $f_v / 1000 = 2/1$ $f_v = 2000 \text{ Hz}$

Example 3:



No. of cut by horizontal line = 6
 vertical = 1
 $f_v / f_H = 6/1$ $f_v / 1000 = 6/1$ $f_v = 6 \times 1000 = 6000 \text{ Hz}$.

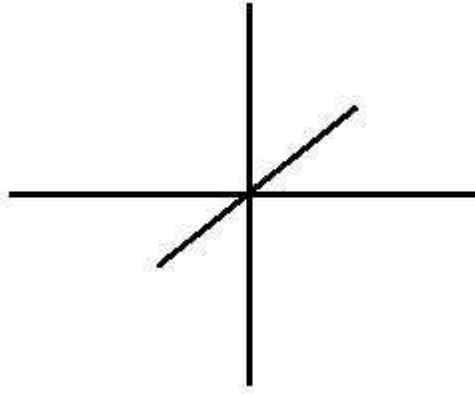
Phase and frequency measurement

An oscilloscope can be used to find the phase angle between two sinusoidal quantities of the same frequency.

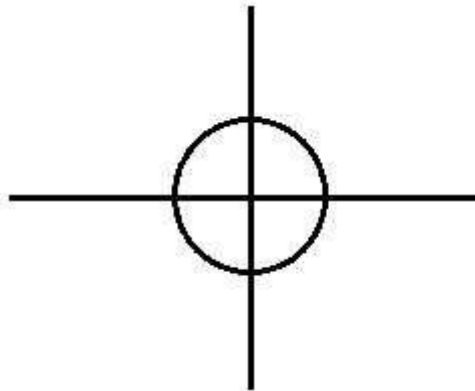
I. One of the signals is applied to Y plates time base generator is switched out and second signal is fed to X plates.

It is necessary that X & Y are equal magnitude.

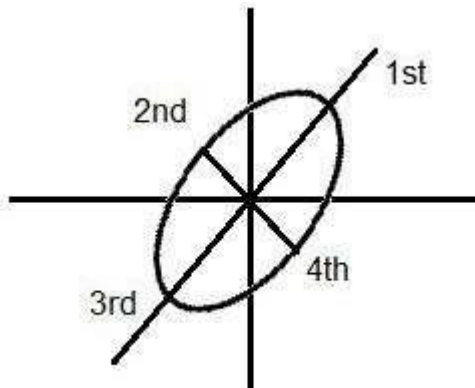
➤ If the two signals are in phase the display would be a straight line at 45° to the horizontal.



- If the phase angle is 90° the display would be a **circle**.

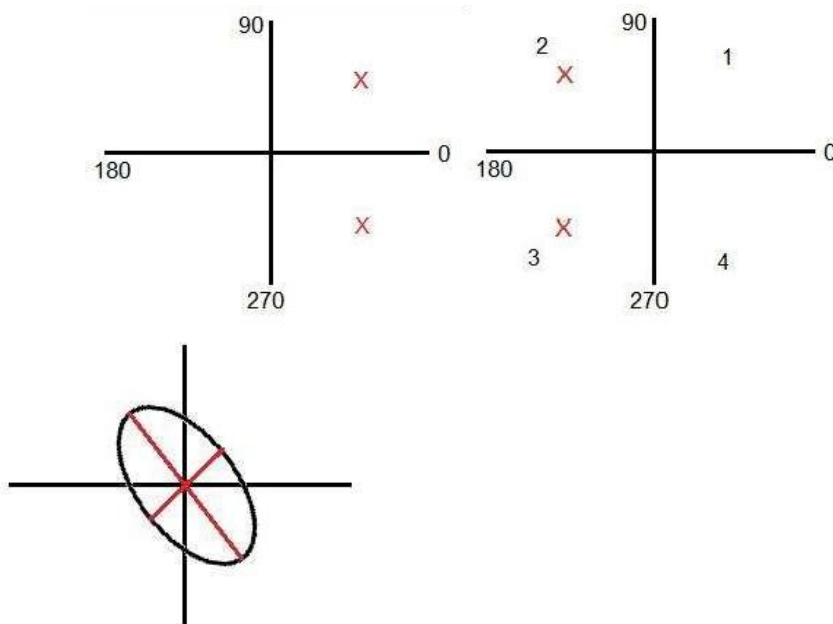


- For any other phase difference the display would be an ellipse.



If the phase angle between $(0 - 90^\circ) = (270^\circ - 360^\circ)$

The ellipse has its major axis in the **1st** quadrant and **3rd** quadrant.



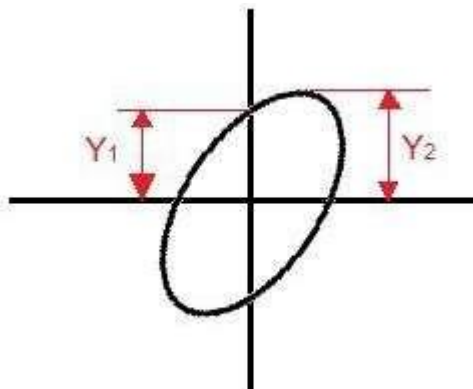
If the phase difference is between 90° and 180° the ellipse has its major axis in the 2nd and 4th quadrant.

Value of phase angle ϕ is given by

$\sin \phi = Y_1/Y_2$ are intercept.

I When the phase angle is between **0 to 90°**

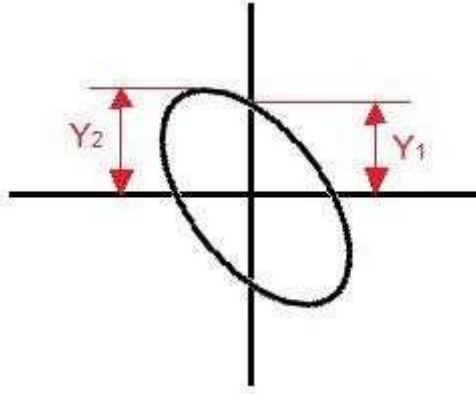
& 270 to 360° major axes lines in 1st quadrant = 3rd quadrant



$$\sin \phi = Y_1/Y_2 = 1/2 = 0.5$$

$$\phi = \sin^{-1} 0.5 = 30^\circ \text{ or } +5$$

II. When phase angle is between 90° to 180° and 180° to 270° major axis would lie in 2nd and 4th quadrant.



$$\sin \phi = Y_1/Y_2 = 0.5$$
$$\phi = 150^\circ \text{ or } 210^\circ$$

References

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