KIITPOLYTECHNIC

LECTURE NOTES

ON

CIRCUIT AND NETWORK THEORY

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CONTENTS

Magnetic Circuits

Introduction: Magnetic flux lines always form closed loops. The closed path followed by the flux lines is called a magnetic circuit. Thus, a magnetic circuit provides a path for magnetic flux, just as an electric circuit provides a path for the flow of electric current. In general, the term magnetic circuit applies to any closed path in space, but in the analysis of electro-mechanical and electronic system this term is specifically used for circuits containing a major portion of ferromagnetic materials. The study of magnetic circuit concepts is essential in the design, analysis and application of electromagnetic devices like transformers, rotating machines, electromagnetic relays etc.

Magneto motive Force (M.M.F) :

Flux is produced round any current – carrying coil. In order to produce the required flux density, the coil should have the correct number of turns. The product of the current and the number of turns is defined as the coil magneto motive force (m.m.f).

If I = Current through the coil (A)

N = Number of turns in the coil.

Magneto motive force = Current x turns

So M.M.F = I X N

The unit of M.M.F. is ampere–turn (AT) but it is taken as Ampere(A) since Nhas no dimensions.

Magnetic Field Intensity

Magnetic Field Intensity is defined as the magneto-motive force per unit length of themagnetic flux path. Its symbol is H.

$$\text{Magnetic field Intensity (H)} = \frac{\text{Magneto motive force}}{\text{Mean length of the magnetic path}}$$

- $H = \frac{F}{l} = \frac{I.N.}{l}$ A/m

Where l is the mean length of the magnetic circuit in meters. Magnetic field intensity is also called magnetic field strength or magnetizing force.

Permeability:-

Every substance possesses a certain power of conducting magnetic lines of force. For example, iron is better conductor for magnetic lines of force than air (vacuum). Permeability of a material (μ) is its conducting power for magnetic lines of force. It is the ratio of the flux density. (B) Produced in a Material to the magnetic field strength (H) i.e. $\mu = B/H$

<u>Reluctance:</u>

Reluctance (s) is akin to resistance (which limits the electric Current). Flux in a magnetic circuit is limited by reluctance. Thus reluctance(s) is a measure of the opposition offered by a magnetic circuit to the setting up of the flux.

Reluctance is the ratio of magneto motive force to the flux. Thus

$$S = \text{Mmf}/\phi$$

Its unit is ampere turns per weber (or AT/wb)

<u>Permeance</u>:-

The reciprocal of reluctance is called the permeance (symbol A).

Permeance (A) = 1/S   wb/AT

Turn T has no unit.

Hence permeance is expressed in wb/A or Henerys(H).

Electric Field versus Magnetic Field.

Similarities

	Electric Field		Magnetic Field
1)	Flow of Current (I)	1)	Flow of flux (∅)
2)	Emf is the cause of flow of current	2)	MMf is the cause of flow of flux
3)	Resistance offered to the flow of Current, is called resistance (R)	3)	Resistance offered to the flow of flux, is called reluctance (S)
4)	Conductance $(\sigma) = \frac{1}{R}$	4)	Permitivity(μ) = $1/S$
5)	Current density is amperes per square meter.	5)	Flux density is number of lines per square meter.
6)	Current (I) - EMF/R	6)	Flux (∅) = $\frac{MMF}{S}$

Dissimilarities

1)	Current actually flows in an electric Circuit.	1)	Flux does not actually flow in a magnetic circuit.
2)	Energy is needed as long as current flows	2)	Energy is initially needed to create the magnetic flux, but not

			to maintain it.
3)	Conductance is constant and independent of current strength at a particular temperature.	3)	Permeability (or magnetic conductance) depends on the total flux for a particular temperature.

B.H. Curve:

Place a piece of an unmagnetised iron bar AB within the field of a solenoid to magnetise it. The field H produced by the solenoid, is called magnetising field, whose value can be altered (increased or decreased) by changing (increasing or decreasing) the current through the solenoid. If we increase slowly the value of magnetic field (H) from zero to maximum value, the value of flux density (B) varies along 1 to 2 as shown in the figure and the magnetic materials (i.e iron bar) finally attains the maximum value of flux density (Bm) at point 2 and thus becomes magnetically saturated.

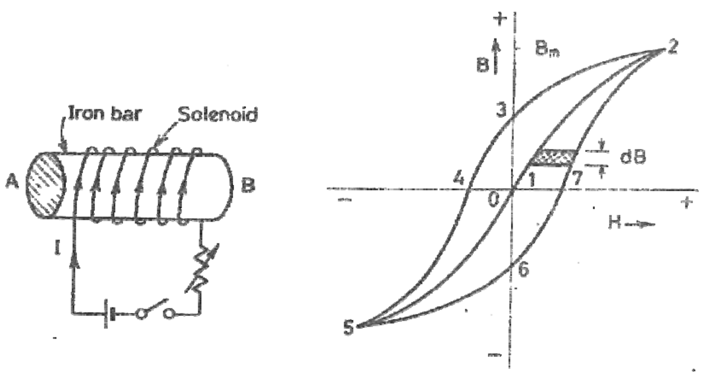


Fig. 2.1

Now if value of H is decreased slowly (by decreasing the current in the solenoid) the corresponding value of flux density (B) does not decreases along 2-1 but decreases some what less rapidly along 2 to 3. Consequently during the reversal of magnetization, the value of B is not zero, but is '13' at H= 0. In other wards, during the period of removal of magnetization force (H), the iron bar is not completely demagnetized.

In order to demagnetise the iron bar completely, we have to supply the demagnetisastion force (H) in the opposite direction (i.e. by reserving the direction of current in the solenoid). The value of B is reduced to zero at point 4, when H='14'. This value of H required to clear off the residual magnetisation, is known as coercive force i.e. the tenacity with which the material holds to its magnetism.

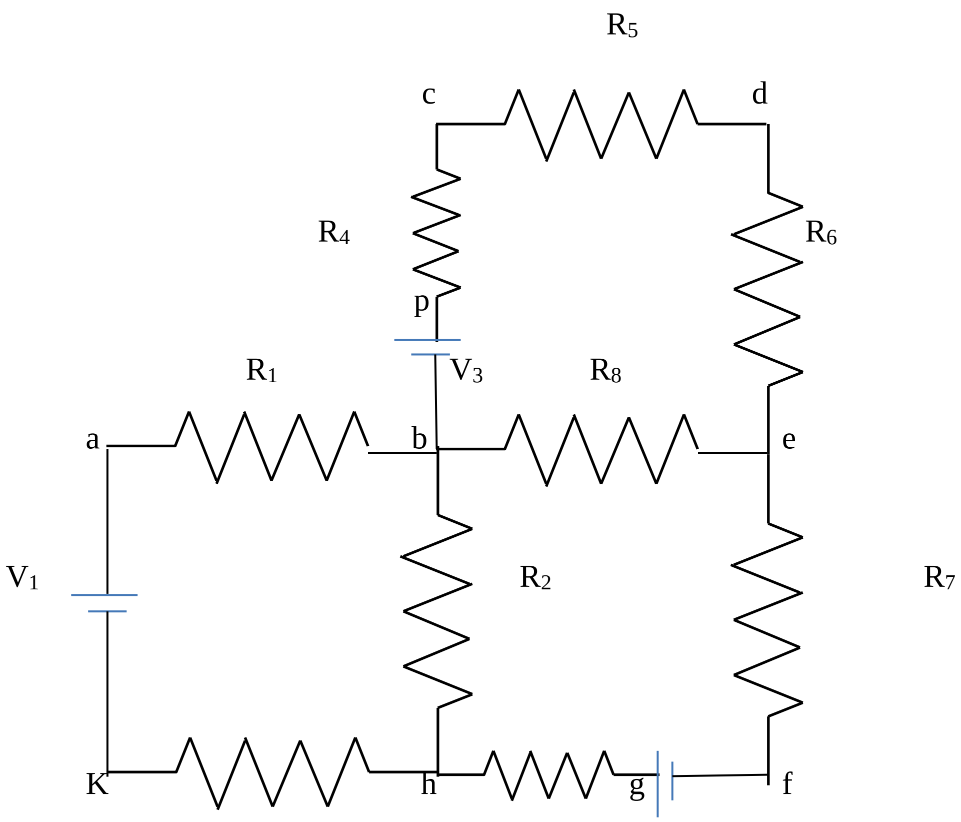
If after obtaining zero value of magnetism, the value of H is made more negative, the iron bar again reaches, finally a state of magnetic saturation at the point 5, which represents negative saturation. Now if the value of H is increased from negative saturation (= '45') to positive saturation (= '12') a curve '5,6,7,2' is obtained. The closed loop "2,3,4,5,6,7,2" thus represents one complete cycle of magnetisation and is known as hysteresis loop.

CIRCUIT ELEMENTS AND ANALYSIS

Different terms are defined below:

1. **Circuit:** A circuit is a closed conducting path through which an electric current either flow or is intended flow
2. **Network:** A combination of various electric elements, connected in any manner.Whatsoever, is called an electric network
3. **Node:** it is an equipotential point at which two or more circuit elements are joined.
4. **Junction:** it is that point of a network where three or more circuit elements are joined.
5. **Branch:** it is a part of a network which lies between junction points.
6. **Loop:** It is a closed path in a circuit in which no element or node is accounted more thanonce.
7. **Mesh:** It is a loop that contains no other loop within it.

Example 3.1 In this circuit configuration of figure 3.1, obtain the no. of i) circuit elements ii)nodes iii) junction points iv) branches and v) meshes.



R_3 R_9 V_2

Solution: i) no. of circuit elements = 12 (9 resistors + 3 voltage sources)

ii) no. of nodes =10 (a, b, c, d, e, f, g, h, k, p)

iii) no. of junction points =3 (b, e, h)

iv) no. of branches = 5 (bcde, be, bh, befgh, bakh)

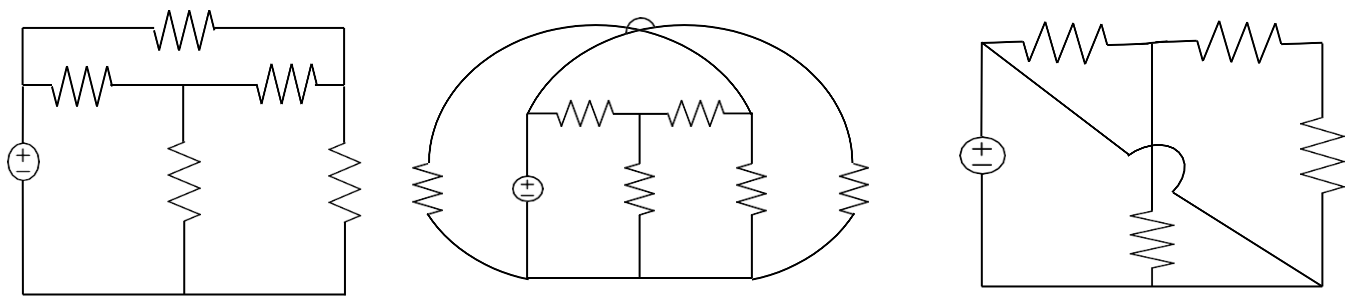
v) no. of meshes = 3 (abhk, bcde, befh)

MESH ANALYSIS

Mesh and nodal analysis are two basic important techniques used in finding solutions for a network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources .If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources,if, on the other hand, the network hasmore current sources,nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planar circuits mesh analysis is not applicable .A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without acrossover.

Figure 3.2 (a) is a planar circuit. Figure 3.2 (b) is a non-planar circuit and fig. 3.2 (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.



(a) (b) (c)

Figure 3.2

Observation of the Fig.3.2 indicates that there are two loops abefa,and bcdeb in the network .Let us assume loop currents I_1 and $I_2$with directions as indicated in the figure.

Considering the loop abefa alone, we observe that current I_1 is passing through $R_{1,}$ and $(I_1-I_2)$is passing through $R_{2.}$ By applying Kirchhoff's voltage law, we can write

$$V_{s.} = I_1R_1+R_2(I_1-I_2) \quad (3.1)$$

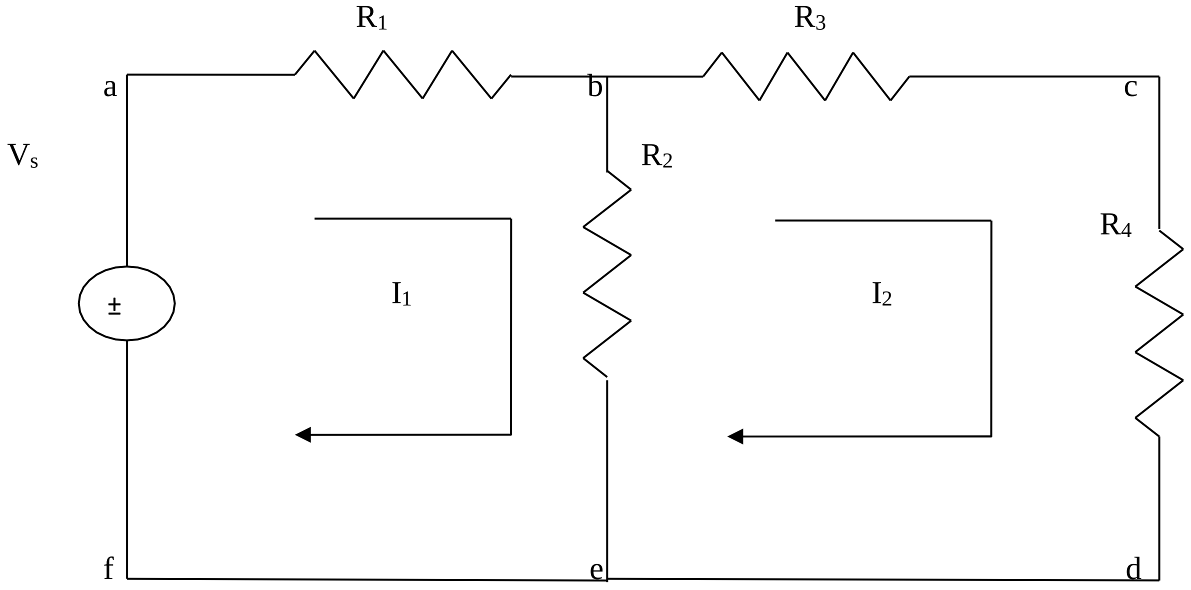


Figure 3.3

Similarly, if we consider the second mesh bcdeb, the current I_2 is passing through R_3 and $R_{4,}$ and $(I_2 – I_1)$ is passing through $R_{2.}$ By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2 (I_2-I_1) + R_3I_2 +R_4I_2 = 0 \quad (3.2)$$

By rearranging the above equations,the corresponding mesh current

equations are$I_1 (R_1+R_2) - I_2R_2 =V_{s.}$

$$-I_1R_2 +(R_2+R_3+R_4) I_2=0 \quad (3.3)$$

By solving the above equations, we can find the currents I_1 and $I_{2,.}$If we observe Fig.3.3, the circuit consists of five branches and four nodes, including the reference node.The number of mesh currents is equal to the number of mesh equations.

And the number of equations=branches-(nodes-1).in Fig.3.3, the required number of mesh current would be 5-(4-1)=2.

In general we have B number of branches and N number of nodes including thereference node than number of linearly independent mesh equations M=B-(N-1).

Example 3.2 Write the mesh current equations in the circuit shown in fig 3.4 and determine the currents.

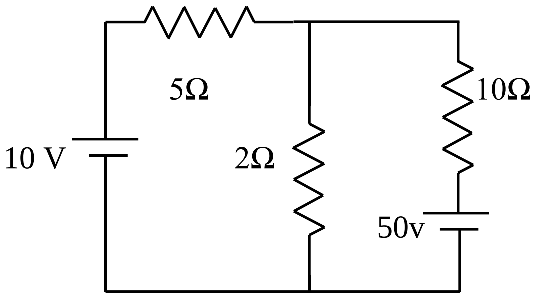


Figure 3.4

Solution: Assume two mesh currents in the direction as indicated in fig. 3.5. The mesh current equations are

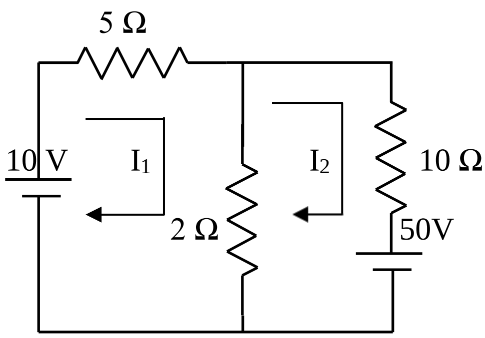


Figure 3.5

$5I_1 + 2(I_1-I_2) = 10$

$101_2 + 2(1_2-1_1) + 50 = 0$ (3.4)

We can rearrange the above equations as

$7I_1 - 2I_2 = 10$

$-2I_1 + 12I_2 = -50$ (3.5)

By solving the above equations, we have $I_1 = 0.25$ A, and $I_2 = -4.125$

Here the current in the second mesh I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of fig .3.5.

Example 3.3 Determine the mesh current I_1 in the circuit shown in fig.3.6.

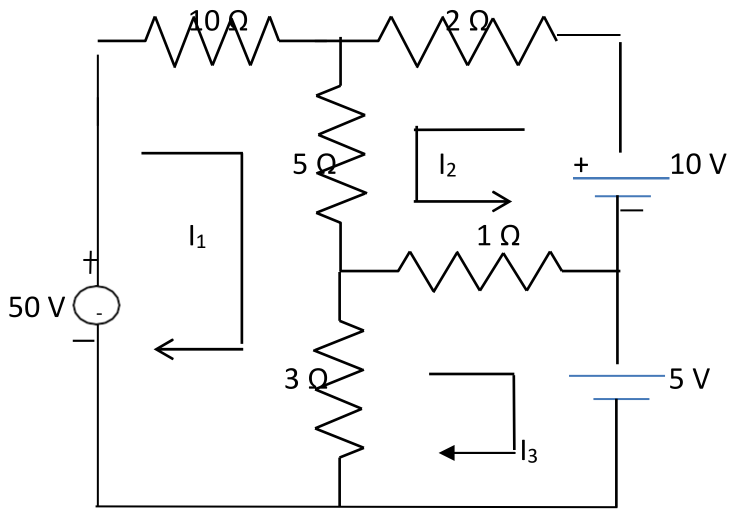


Figure 3.6

Solution: From the circuit, we can from the following three mesh equations

$$10I_1+5(I_1+I_2)+3(I_1-I_3) = 50 \quad (3.6)$$

$$2I_2+5(I_2+I_1)+1(I_2+I_3) = 10 \quad (3.7)$$

$$3(I_3-I_1)+1(I_3+I_2) = -5 \quad (3.8)$$

Rearranging the above equations we get

$$18I_1+5I_2-3I_3=50 \quad (3.9)$$

$$5I_1+8I_2+I_3=10 \quad (3.10)$$

$$-3I_1+I_2+4I_3=-5 \quad (3.11)$$

According to the Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{1175}{356}$$

Or I_1= 3.3 A Similarly,

$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{-355}{356}$$

Or I_2=-0.997A (3.12)

$$I_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{525}{356}$$

Or I_3=1.47A (3.13)

$\therefore I_1$=3.3A, I_2=-0.997A, I_3=1.47A

MESH EQUATIONS BY INSPECTION METHOD The mesh equations for a general planar network can be writtenby inspection without going through the detailed steps. Consider a three mesh networks as shown in figure 3.7

The loop equation are $I_1R_1+ R_2(I_1-I_2) \quad =V_1$

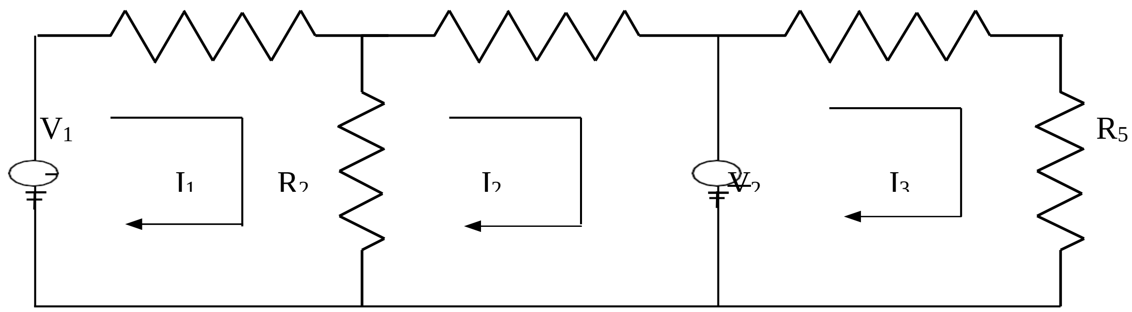


Figure 3.7

$R_2(I_2-I_1)+I_2R_3= -V_2$ 3.14

$R_4I_3+R_5I_3=V_2$ 3.15

Reordering the above equations, we have

$(R_1+R_2)I_1-R_2I_2=V_1$ 3.16

$-R_2I_1+(R_2+R_3)I_2=-V_2$ 3.17

$(R_4+R_5)I_3=V_2$ 3.18

The general mesh equations for three mesh resistive network can be written as

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3= V_a$ 3.19

$\pm R_{21}I_1+R_{22}I_2 \pm R_{23}I_3= V_b$ 3.20

$\pm R_{31}I_1 \pm R_{32}I_2+R_{33}I_3= V_c$ 3.21

By comparing the equations 3.16, 3.17 and 3.18 with equations 3.19, 3.20 and 3.21 respectively, the following observations can be taken into account.

1. The self-resistance in each mesh
2. The mutual resistances between all pairs of meshes and
3. The algebraic sum of the voltages in each mesh.

The self-resistance of loop 1, $R_{11}=R_1+R_2$, is the sum of the resistances through which I_1 passes.

The mutual resistance of loop 1, $R_{12}= -R_2$, is the sum of the resistances common to loop currents I_1 and I_2. If the directions of the currents passing through the common resistances are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a=V_1$ is the voltage which drives the loop 1. Here the positive sign is used if the direction of the currents is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly $R_{22}=R_2+R_3$ and $R_{33}=R_4+R_5$ are the self-resistances of loops 2 and 3 respectively. The mutual resistances $R_{13}=0$, $R_{21}= -R_2$, $R_{23}=0$, $R_{31}=0$, $R_{32}=0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b= -V_2$, $V_c= V_2$ are the sum of the voltages driving their respective loops.

Example 3.4 writes the mesh equation for the circuit shown in fig. 3.8

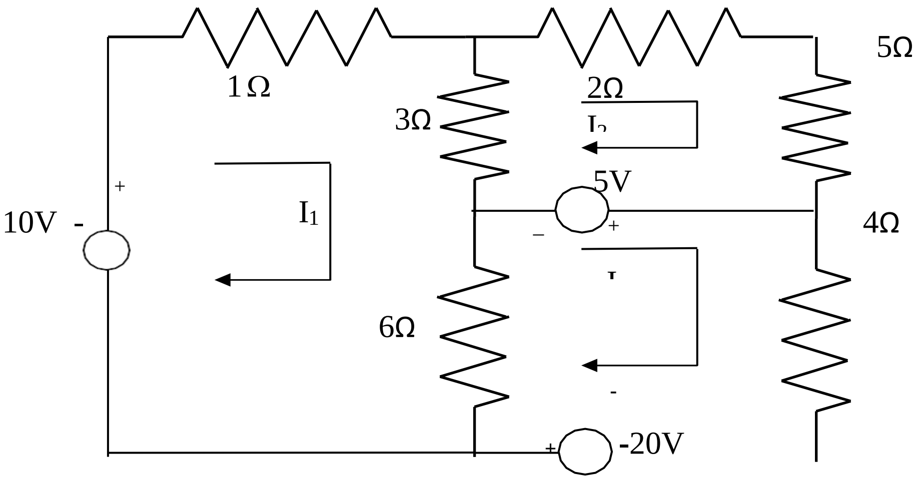


Figure 3.8

Solution : the general equation for three mesh equation are

$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a$ (3.22)

$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b$ (3.23)

$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c$ (3.24)

Consider equation 3.22

R_{11}=self resistance of loop 1=(1Ω+ 3 Ω +6 Ω) =10 Ω

R_{12}= the mutual resistance common to loop 1 and loop 2 = -3 Ω

Here the negative sign indicates that the currents are in opposite direction .

R_{13}= the mutual resistance common to loop 1 & 3= -6 Ω

V_a= +10 V, the voltage the driving the loop 1.

Here he positive sign indicates the loop current I_1 is in the same direction as the source element.

Therefore equation 3.22 can be written as

$10\ I_1 - 3I_2 - 6I_3 = 10\ V$ (3.25)

Consider Eq. 3.23

R_{21}= the mutual resistance common to loop 1 and loop 2 = -3 Ω

R_{22}= self resistance of loop 2=(3Ω+ 2 Ω +5 Ω) =10 Ω
R_{23}=0, there is no common resistance between loop 2 and 3.
V_b = -5 V, the voltage driving the loop 2.
Therefore Eq. 3.23 can be written as
$-3I_1 + 10I_2 = -5V$ (3.26)
Consider Eq. 3.24
R_{31}= the mutual resistance common to loop 1 and loop 3 = -6 Ω
R_{32}= the mutual resistance common to loop 3 and loop 2 = 0
R_{33}= self resistance of loop 3=(6Ω+ 4 Ω) =10 Ω
V_c= the algebraic sum of the voltage driving loop 3
=(5 V+20V)=25 V (3.27)
Therefore, Eq3.24can be written as $-6I_1 + 10I_3 = 25V$
$-6I_1 - 3I_2 - 6I_3 = 10V$
$-3I_1 + 10I_2 = -5V$
$-6I_1 + 10I_3 = 25V$

SUPERMESH ANALYSIS

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equation as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. On way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in the figure 3.9.

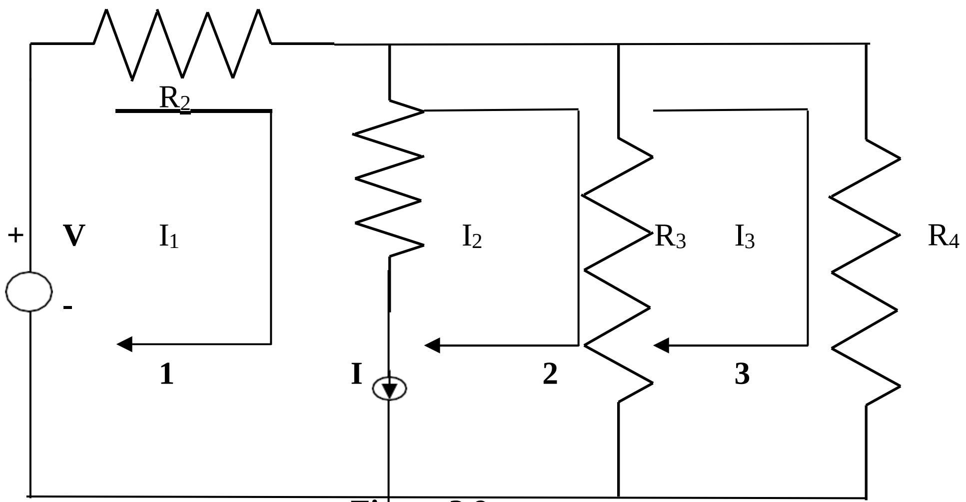


Figure 3.9

Here the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$R_1I_1 + R_3(I_2-I_3)=V$

Or $R_1I_1 + R_3I_2 - R_4I_3= V$

Considering mesh 3, we have

$R_3(I_3-I_2)+ R_4I_3=0$

Finally the current I from current source is equal to the difference between two mesh currents i.e.

$I_1-I_2=I$

we have thus formed three mesh equations which we can solve for the three unknown currents in the network.

Example 3.5. Determine the current in the 5Ω resistor in the network given in Fig. 3.10

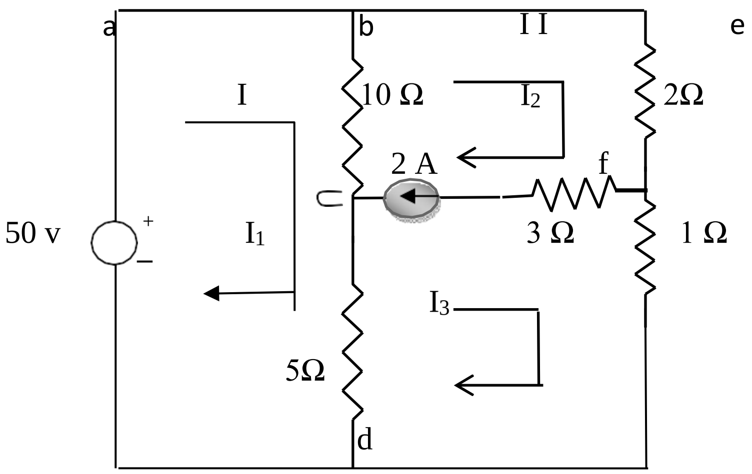


Figure 3.10

Solution: - From the first mesh, i.e. abcda, we have

$$50 = 10(I_1-I_2) + 5(I_1-I_3)$$

$$\text{Or } 15I_1-10I_2 -5I_3 =50 \quad (3.28)$$

From the second and third meshes. we can form a super mesh

$$10(I_2-I_1)+2I_2 +I_3+5(I_3-I_1)=0$$

$$\text{Or } -15I_1+12I_2 +6I_3 =0 \quad (3.29)$$

The current source is equal to the difference between II and III mesh currents

i.e. $I_2 - I_3 = 2A$ (3.30)

Solving 3.28.,3.29 and 3.30. we have

$I_1 = 19.99A, I_2 = 17.33$ A, and $I_3 = 15.33$ A

The current in the 5Ω resistor $= I_1 - I_3$

$= 19.99 - 15.33 = 4.66A$

The current in the 5Ω resistor is 4.66A.

Example 3.6. Write the mesh equations for the circuit shown in fig. 3.11 and determine the currents, I_1, I_2 and I_3.

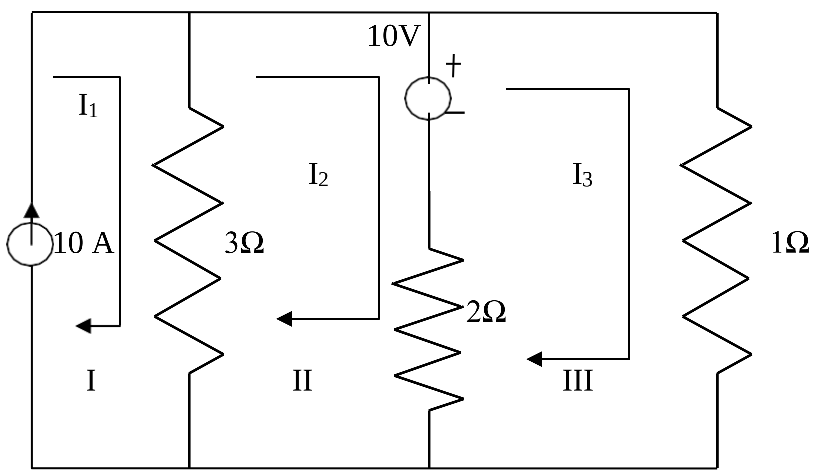


Figure 3.11

Solution ; In fig 3.11, the current source lies on the perimeter of the circuit, and the first mesh is ignored. Kirchhoff's voltage law is applied only for second and third meshes .

From the second mesh, we have

$3(I_2 - I_1) + 2(I_2 - I_3) + 10 = 0$

Or $-3I_1 + 5I_2 - 2I_3 = -10$ (3.31)

From the third mesh, we have

$I_3 + 2(I_3 - I_2) = 10$

Or $-2I_2 + 3I_3 = 10$ (3.32)

From the first mesh, I_1 =10A (3.33)

From the abovethree equations, we get

I_1=10A, I_2 =7.27, I_3 =8.18A

NODALANALYSIS

In the chapter I we discussed simple circuits containing only two nodes, including the reference node. In general, in a N node circuit, one of the nodes is chosen as the reference or datum node, then it is possible to write N -1nodal equations by assuming N-1 node voltages. For example,a10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential. In the circuit shown in fig. 3.12, node 3 is assumed as the Reference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1, the current entering is the current leaving (See Fig.3.13)

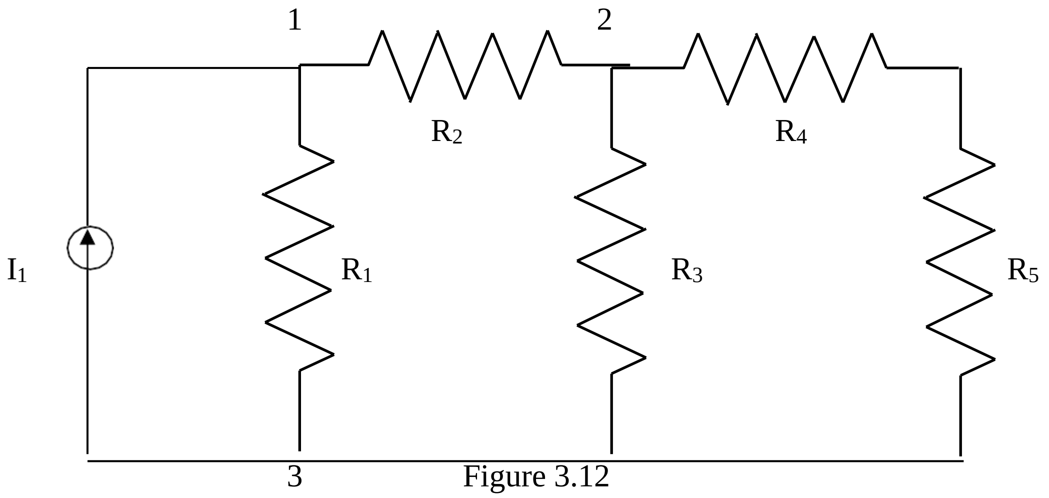


Figure 3.12

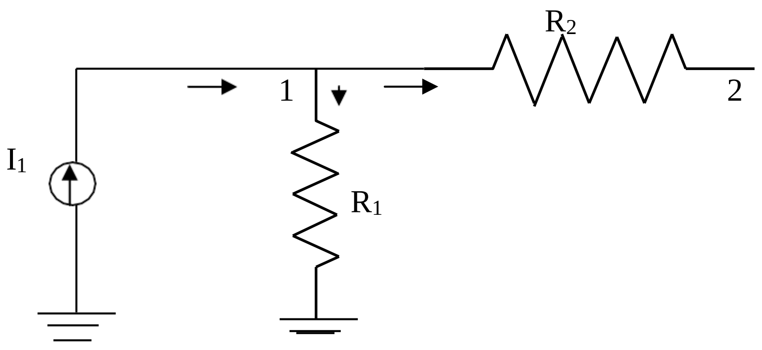


Figure 3.13

$I_1 = V_1/R_1 + (V_1-V_2)/R_2$

Where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2.the current entering is equal to the current leaving as shown in fig. 3.14

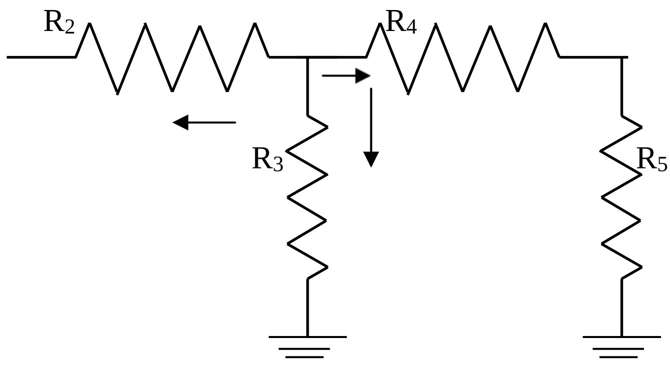


Figure 3.14

$(V_2-V_1)/R_2 + V_2/R_3 + V_2/(R_4+R_5) = 0$

Rearranging the above equations, we have

$V_1[1/R_1+1/R_2]-V_2(1/R_2)= I_1$

$-V_1(1/R_2) + V_2[1/R_2+1/R_3+1/(R_4+R_5)]=0$

From the above equations we can find the voltages at each node.

Example 3.7 Determine the voltages at each node for the circuit shown in fig 3.15

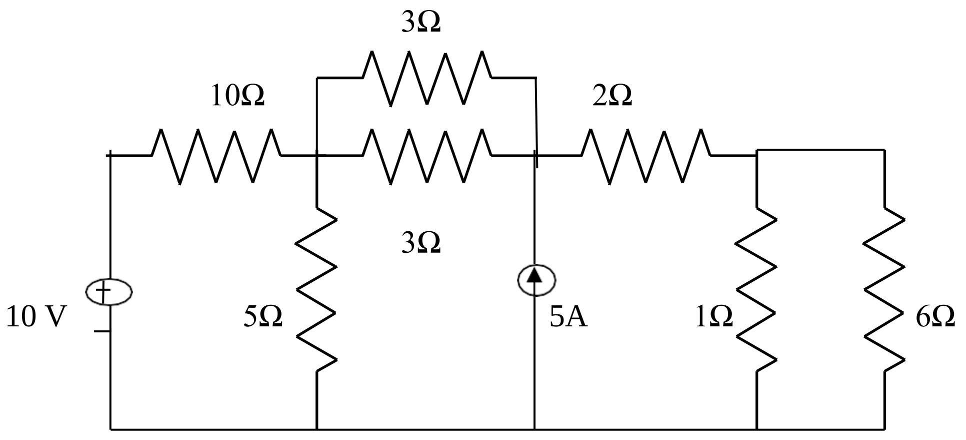


Figure 3.15

Solution : At node 1, assuming that all currents are leaving, we have

$(V_1-10)/10 + (V_1-V_2)/3 + V_1/5 + (V_1-V_2)/3 = 0$

Or $V_1[1/10 + 1/3 + 1/5 + 1/3] - V_2[1/3 + 1/3] = 1$

$0.96V_1 - 0.66V_2 = 1$ (3.36)

At node 2, assuming that all currents are leaving except the current from current source, we have

$(V_2-V_1)/3 + (V_2-V_1)/3 + (V_2-V_3)/2 = 5$

$-V_1[2/3] + V_2[1/3 + 1/3 + 1/2] - V_3(1/2) = 5$

$-0.66V_1 + 1.16V_2 - 0.5V_3 = 5$ (3.37)

At node 3 assuming all currents are leaving, we have

$(V_3-V_2)/2 + V_3/1 + V_3/6 = 0$

$-0.5V_2 + 1.66V_3 = 0$ (3.38)

Applying Cramer's rule we get

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{7.154}{0.887} = 8.06$$

Similarly,

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{9.06}{0.887} = 10.2$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{2.73}{0.887} = 3.07$$

NODAL EQUATIONS BY INSPECTION METHOD The nodal equations for a general planar network can also be written by inspection without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in fig 3.16

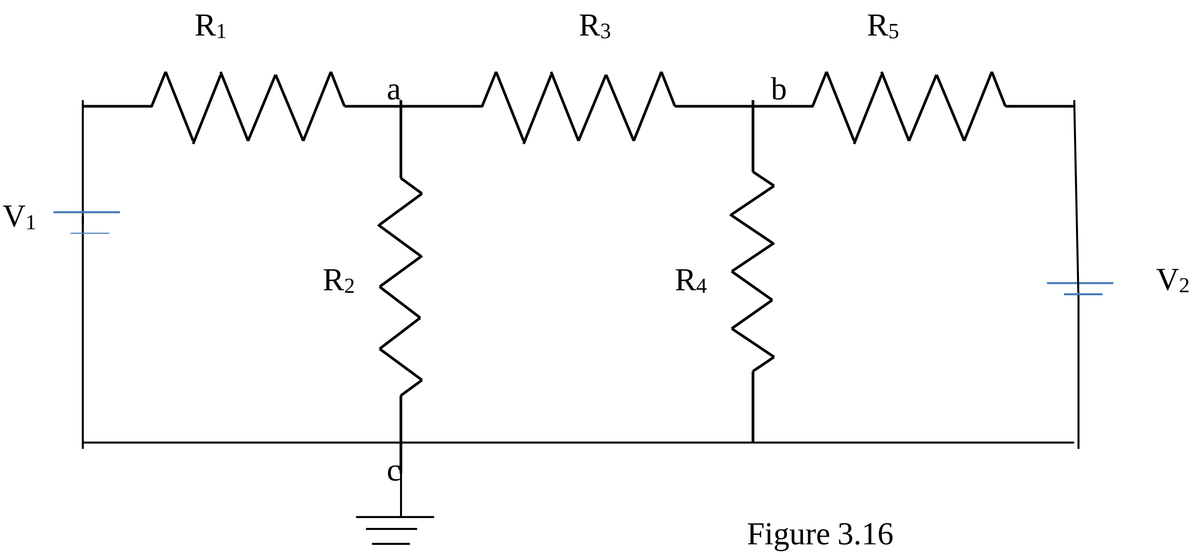


Figure 3.16

In fig. 3.16 the points a and b are the actual nodes and c is the reference node.

Now consider the nodes a and b separately as shown in fig 3.17(a) and (b)

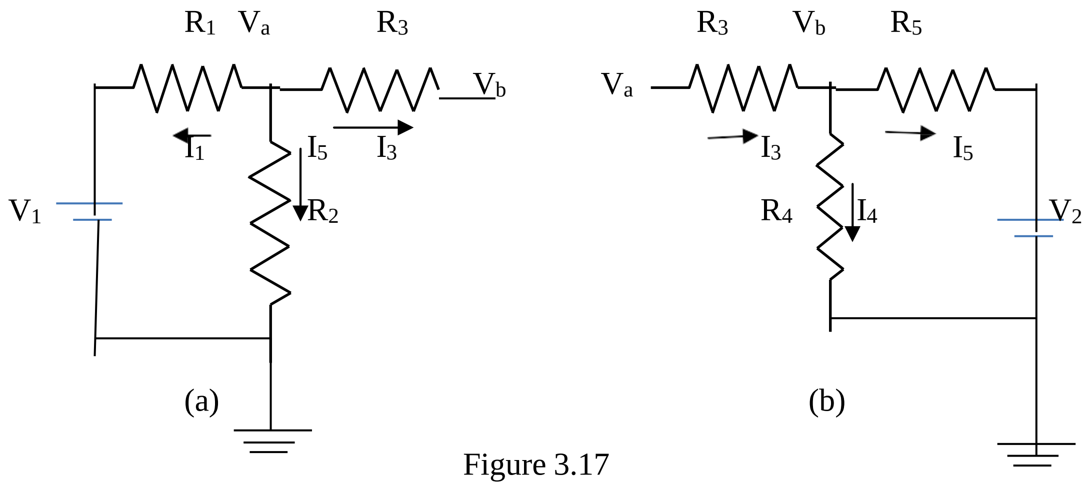


Figure 3.17

In fig 3.17 (a), according to Kirchhoff's current law we have

$I_1+I_2+I_3=0$

$(V_a-V_1)/R_1 + V_a/R_2 + (V_a-V_b)/R_3 = 0$ (3.39)

In fig 3.17 (b) , if we apply Kirchhoff's current law

$I_4 + I_5 = I_3$

$\therefore (V_b-V_a)/R_3 + V_b/R_4 + (V_b-V_2)/R_5 = 0$ (3.40)

Rearranging the above equations we get

$(1/R_1+1/R_2+1/R_3)V_a-(1/R_3)V_b=(1/R_1)V_1$ (3.41)

$(-1/R_3)V_a + (1/R_3+1/R_4+1/R_5)V_b = V_2/R_5$ (3.42)

In general, the above equation can be written as

$G_{aa}V_a + G_{ab}V_b = I_1$ (3.43)

$G_{ba}V_a + G_{bb}V_b = I_2$ (3.44)

By comparing Eqs 3.41,3.42 and Eqs 3.43, 3.44 we have the self conductance at node a, $G_{aa}=(1/R_1 + 1/R_2 + 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb}= (1/R_3 + 1/R_4 +1/R_5)$ is the sum of the conductances connected to node b. $G_{ab}=(-1/R_3)$ is the sum of the mutual conductances connected to node *a* and node *b*. Here all the mutual conductances have negative signs. Similarly, $G_{ba}= (-1/R_3)$ is also a mutual conductance connected between nodes *b* and *a*. I_1 and I_2 are the sum of the source currents at node *a* and node *b*, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

Example 3.8 for the circuit shown in the figure 3.18 write the node equations by the inspection method.

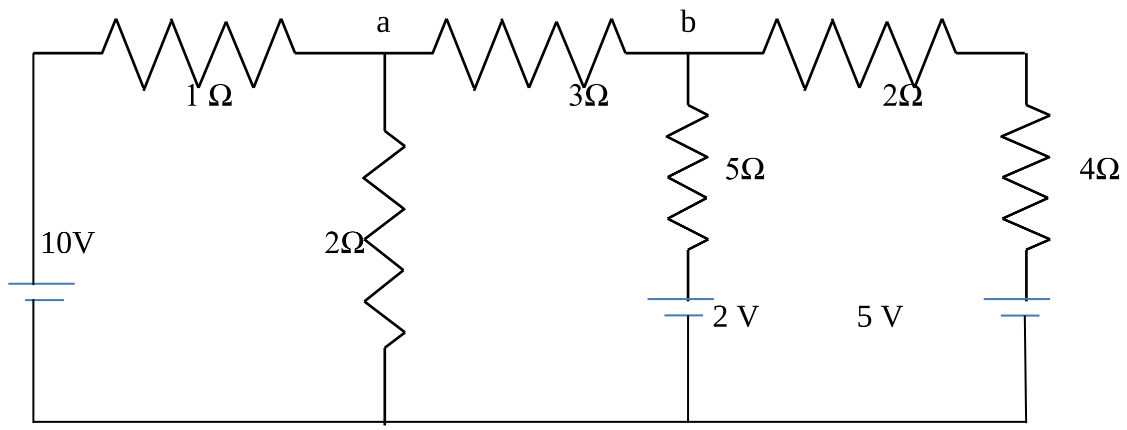


Fig 3.18

Solution:-

The general equations are

$G_{aa}V_a+G_{ab}V_b=I_1$ (3.45)

$G_{ba}V_a + G_{bb}V_b=I_2$ (3.46)

Consider equation 3.45

G_{aa}=(1+ 1/2 +1/3) mho. The self conductance at node *a* is the sum of the conductances connected to node *a*.

G_{bb} = (1/6 + 1/5 + 1/3) mho the self conductance at node *b* is the sum of conductances connected to node *b*.

G_{ab} =-(1/3) mho, the mutual conductances between nodes *a* and *b* is the sum of the conductances connected between node *a* and *b*.

Similarly G_{ba} = -(1/3), the sum of the mutual conductances between nodes *b* and *a*.

I_1=10/1 =10 A, the source current at node *a*,

I_2=(2/5 + 5/6) = 1.23A, the source current at node *b*.

Therefore, the nodal equations are

$1.83V_a - 0.33V_b = 10$ (3.47)

$-0.33V_a + 0.7V_b = 1.23$ (3.48)

SUPERNODE ANALYSIS

Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of fig. 3.19

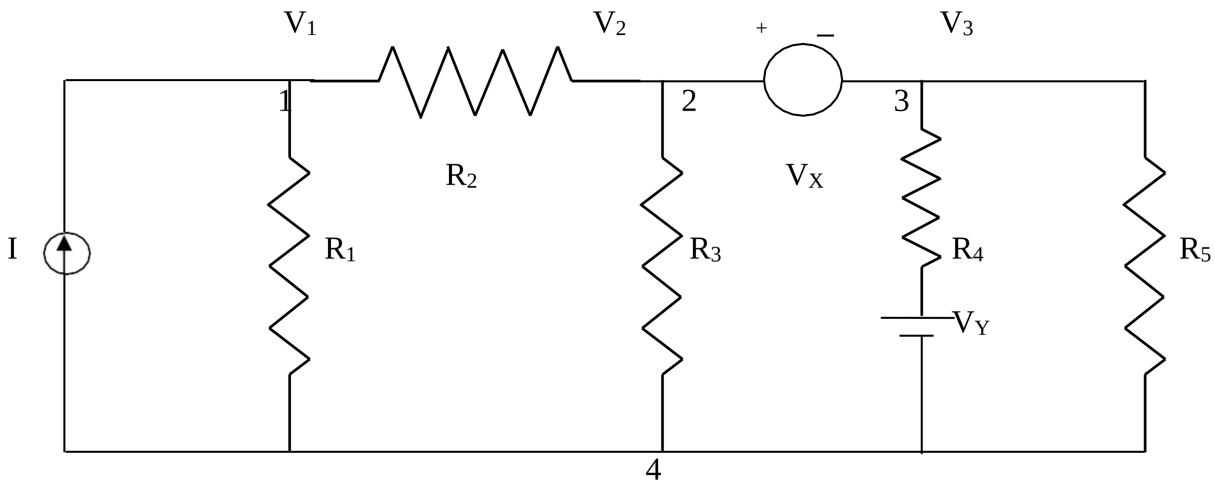


FIG 3.19

It is clear from the fig.3.19, that node 4 is the reference node. Applying Kirchhoff's current law at node 1, we get

$I = (V_1/R_1) + (V_1 - V_2)/R_2$

Due to the presence of voltage source V_χ in between nodes 2 and 3 , it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly, we can write the combined equation for nodes 2 and 3 as under.

$(V_2-V_1)/R_2 + V_2/R_3 + (V_3-V_y)/R_4 + V_3/R_5 = 0$

The other equation is

$V_2-V_3 = V_x$

From the above three equations, we can find the three unknown voltages.

Example 3.9 Determine the current in the 5 Ω resistor for the circuit shown in fig. 3.20

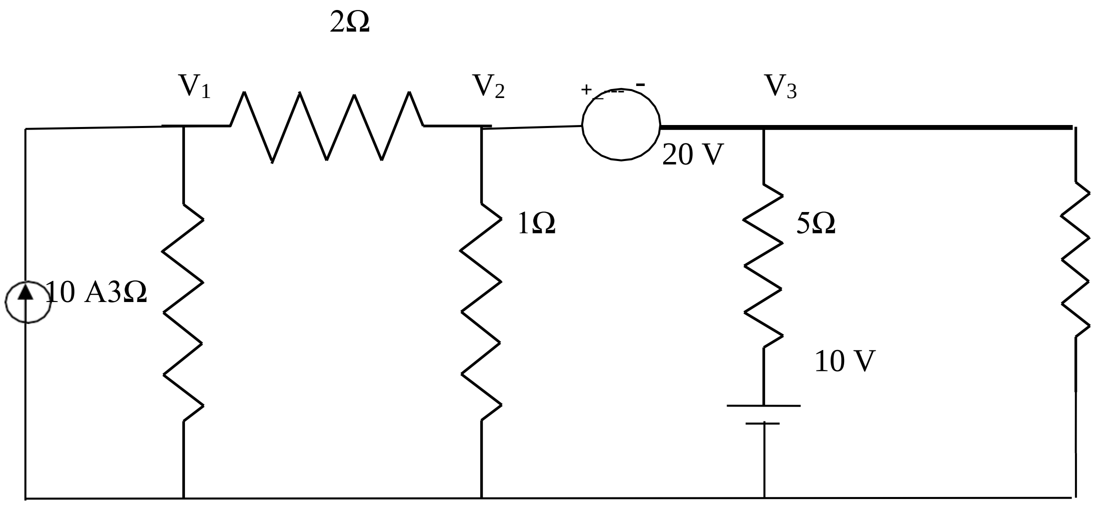


fig. 3.20

Solution. At node 1

$10 = V_1/3 + (V_1-V_2)/2$

Or $V_1[1/3 + 1/2]-(V_2/2)-10=0$

$0.83V_1-0.5V_2-10 = 0$ (3.49)

At node 2 and 3, the supernode equation is

$(V_2-V_1)/2 + V_2/1 + (V_3-10)/5 + V_3/2 = 0$

Or $-V_1/2 + V_2[(1/2)+1] + V_3[1/5 + 1/2]=2$

Or $-0.5V_1 + 1.5V_2 + 0.7V_3 - 2 = 0$ (2.50)

The voltage between nodes 2 and 3 is given by

$V_2-V_3=20$ (3.51)

The current in 5Ω resistor $I_5 = (V_3 - 10)/5$

Solving equation 3.49, 3.50 and 3.51, we obtain

$V_3 = -8.42$ V

∴ Currents $I_5 = (-8.42-10)/5 = -3.68$ A (current towards node 3) i.e the current flows towards node 3.

SOURCE TRANSFORMATION TECHNIQUE

In solving networks to find solutions one may have to deal with energy sources. It has already been discussed in chapter 1 that basically, energy sources are either voltage sources or current sources. Sometimes it is necessary to convert a voltage source to a current source or vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in figure3.21. R_v and R_i represent the internal resistances of the voltage source V_s , and current source I_s ,respectively.

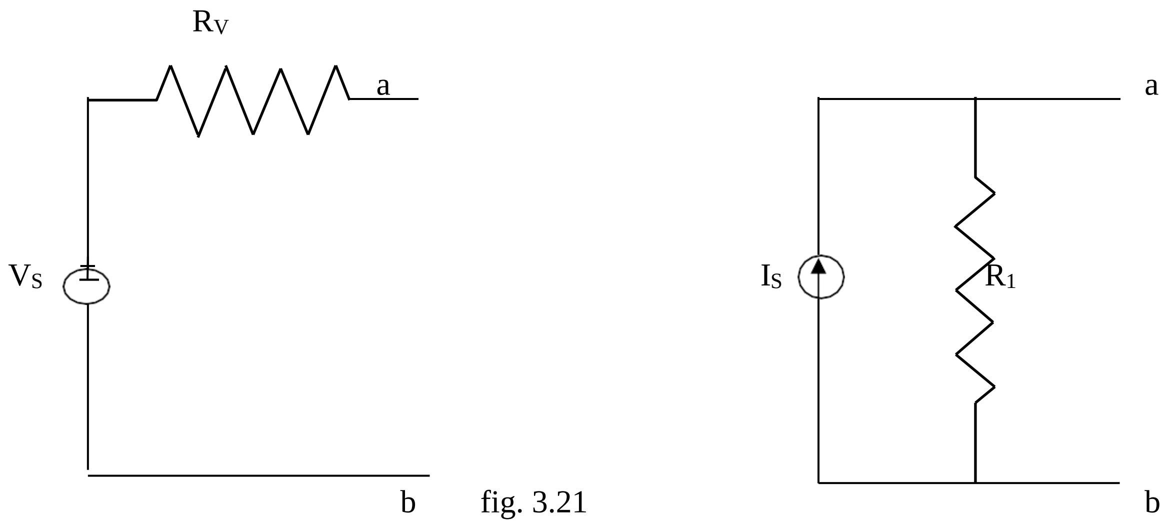


fig. 3.21

Any source, be it a current source or a voltage source, drives current through its load resistance, and the magnitude of the current depends on the value of the load resistance. Fig 3.22 represents a practical voltage source and a practical current source connected to the same load resistance R_L.

R_V

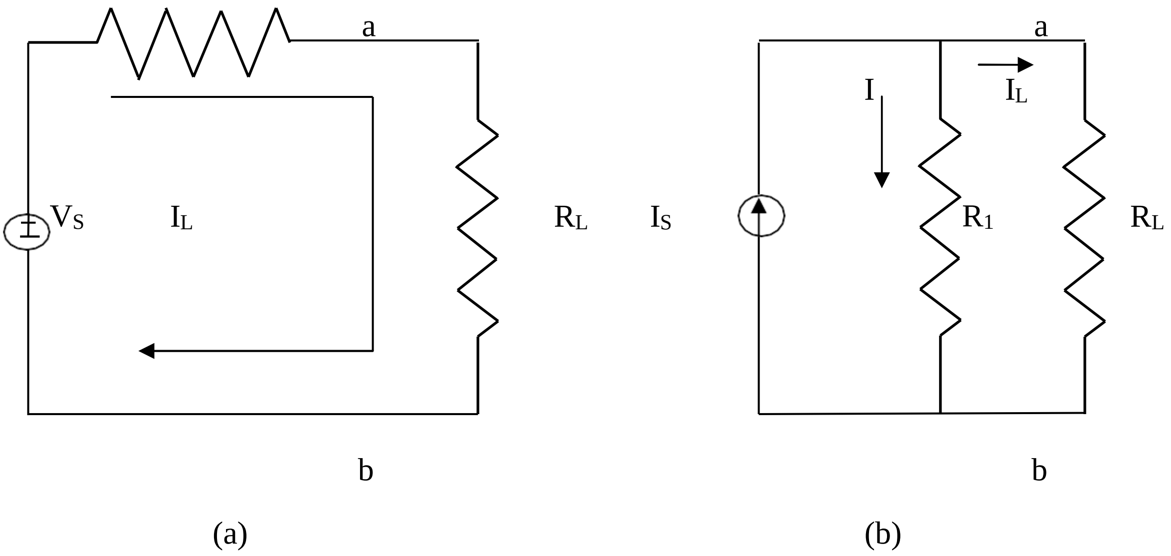


Figure 3.22

From fig 3.22 (a) the load voltage can be calculated by using Kirchhoff's voltage law as

$V_{ab}=V_s-I_LR_v$

The open circuit voltage $V_{oc}=V_s$

The short circuit current $I_{sc}=\frac{V_s}{R_v}$

from fig 3.22 (b)

$I_L=I_s-I=I_s-(V_{ab}/R_1)$

The open circuit voltage $V_{oc}= I_sR_1$

The short circuit current $I_{sc}=I_s$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open circuit votages and short circuit currents of the above two sources we obtain

$V_{oc}=I_sR_1=V_s$

$I_{sc}=I_s=V_s/R_v$

It follows that

$R_1=R_v=R_s$; $V_s=I_sR_s$

where R_s is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage V_s and internal series resistance R_s can be replaced by a current source $I_s=V_s/R_s$ in parallel with an internal resistance R_s. The reverse tansformation is also possible. Thus, a practical current source in parallel with an internal resistance R_s can be replaced by a voltage source $V_s=I_sR_s$ in series with an internal resistance R_s.

Example 3.10 Determine the equivalent voltage source for the current source shown in fig 3.23

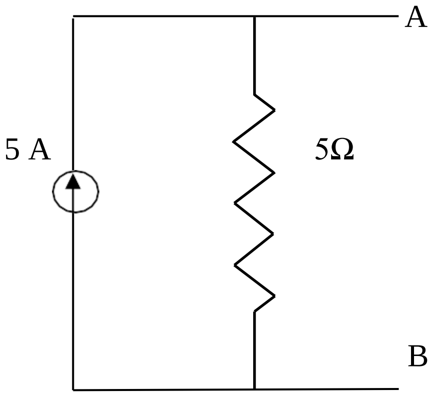


Figure 3.23

Solution: The voltage across terminals A and B is equal to 25 V. since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. The equivalent voltage source is shown in fig. 3.24.

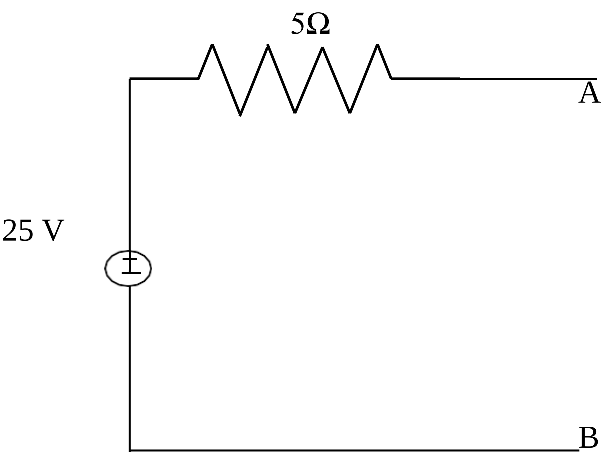
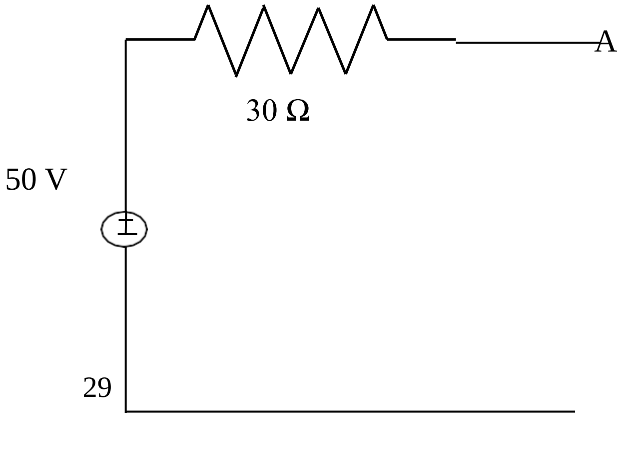


Fig 3.24

Example 3.11 Determine the equivalent current source for the voltage source shown in fig. 3.25



Solution : the short circuit current at terminals A and B is equal to

I= 50/30 = 1.66 A

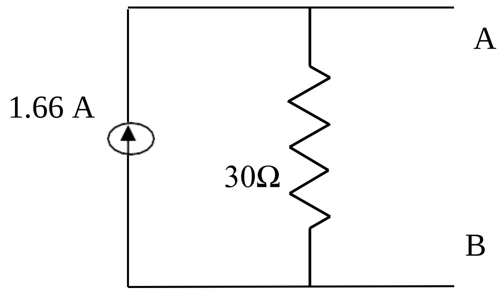


Fig 3.26

Since the internal resistance for the voltage source is 30**Ω**, the internal resistance of the current source is also 30 **Ω**. The equivalent current source is shown in fig. 3.26

NETWORK THEOREMS

Before start the theorem we should know the basic terms of the network.

Circuit: It is the combination of electrical elements through which current passes is called circuit.

Network: It is the combination of circuits and elements is called network.

Unilateral: It is the circuit whose parameter and characteristics change with change in the direction of the supply application.

Bilateral: It is the circuit whose parameter and characteristics do not change with the supply in either side of the network.

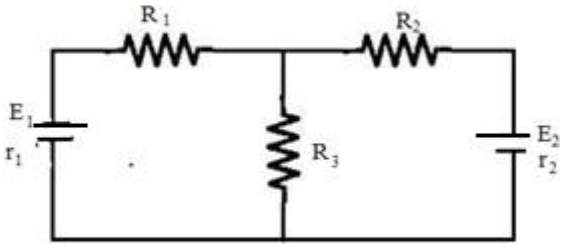
Node: It is the inter connection point of two or more than two elements is called node.

Branch: It is the interconnection point of three or more than three elements iscalled branch.

Loop: It is a complete closed path in a circuit and no element or node is takenmore than once.

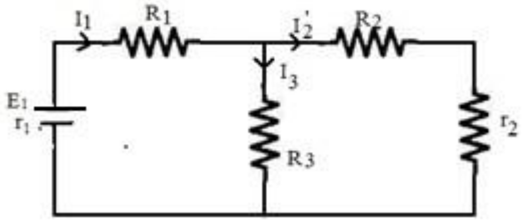
Super-Position Theorem:

Statement :" It states that in a network of linear resistances containing more than one source the current which flows at any point is the sum of all the currents which would flow at that point if each source were considered separately and all other sources replaced for time being leaving its internal resistances if any".



Explanation:

Considering E_1 source



Step 1.

R_2&r are in series and parallel with R_3 and again series with R_1

$(R_2+r_2) \parallel R_3$

$$= \frac{(R_2+r_2)R_3}{R_2+r_2+R_3} = m \quad \text{(say)}$$

$$Rt_1 = m + R_1 + r_1$$

$$I_1 = \frac{E_1}{Rt_1}$$

$$I_2 = \frac{I_1 \times R_3}{R_2 + r_2 + R_3}$$

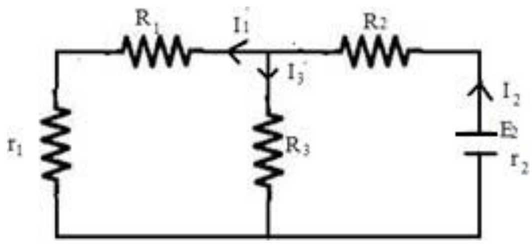
$$I_3 = \frac{I_1(R_2 + r_2)}{R_2 + r_2 + R_3}$$

Step – 2

Considering E2 source, R_1&r_2 are series and R_3 parallel and R_2 in series

$(R_1+r_1) \parallel R_3$

$$= \frac{(R_1+r_1)R_3}{R_1+r_1+R_3} = n \quad \text{(say)}$$



$$Rt_2 = n + R_2 + r_2$$

$$I_2 = \frac{E_2}{Rt_2}$$

$$I_3^{/} = \frac{I_{21}(R_1 + r_1)}{R_1 + r_1 + R_3}$$

$$I_1^{/} = \frac{I_2 \times R_3}{R_1 + r_1 + R_3}$$

Step – 3

Current in R_1 branch = $I_1 - I_1^{/}$

Current in R_2 branch = $I_2 - I_2^{/}$

Current in R_3 branch = $I_3 - I_3^{/}$

The direction of the branch current will be in the direction of the greater value current.

Thevenin's Theorem:

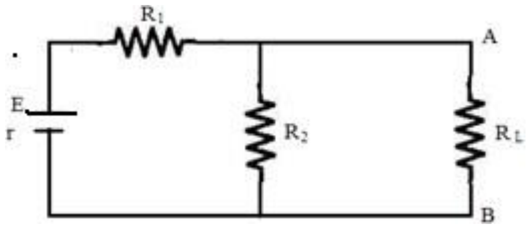
The current flowing through the load resistance R_1 connected across any two terminals A and B of a linear active bilateral network is given by

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{oc}}{R_i + R_L}$$

Where $V_{th} = V_{oc}$ is the open. circuit voltage across A and B terminal when R_L is removed.

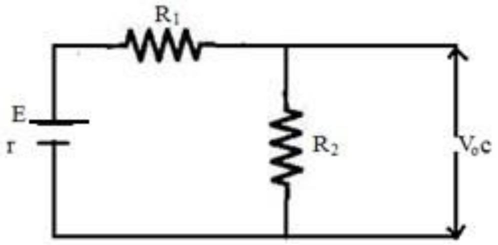
$R_i = R_{th}$ is the internal resistances of the network as viewed back into the open circuit network from terminals A & B with all sources replaced by their internal resistances if any.

Explanation :



<u>Step – 1 </u>for finding V_{oc}

Remove R_L temporarily to find V_{oc}.

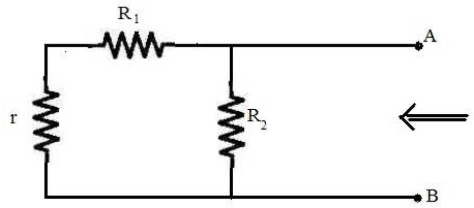


$$I = \frac{E}{R_1 + R_2 + r}$$

$$V_{oc} = IR_2$$

<u>Step – 2 </u>finding R_{th}

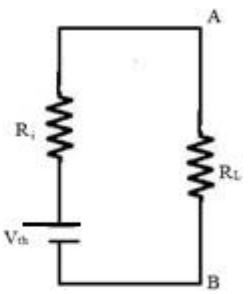
Remove all the sources leaving their internal resistances if any and viewed from open circuit side to find out R_i or R_{th}.



$$R_i = (R_1 + r) \parallel R_2$$

$$R_i = \frac{(R_1 + r)R_2}{R_1 + r + R_2}$$

<u>Step – 3</u>



Connect internal resistances and Thevenin’s voltage in series with load resistance R_L.

Where R_{th}=thevenin resistance

V_{th}=thevenin voltage

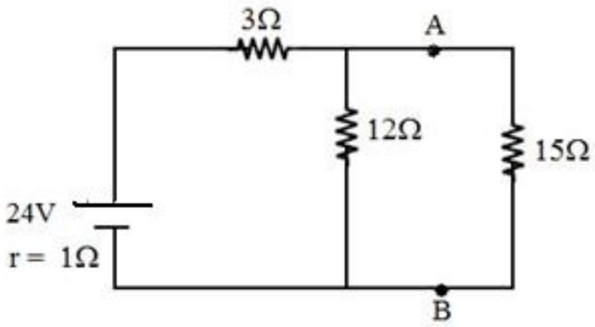
I_{th}=thevenin current

$R_i = (R_1 + r) \parallel R_2$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{oc}}{R_i + R_L}$$

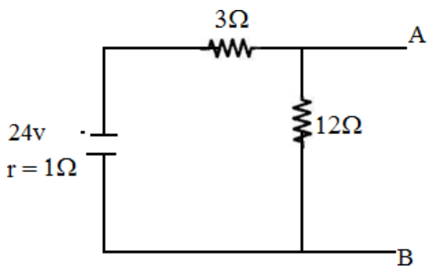
Example 01- Applying thevenin theorem find the following from given figure

(i) the Current in the load resistance R_L of 15 Ω



Solution : (i) Finding Voc

→ Remove 15Ω resistance and find the Voltage across A and B



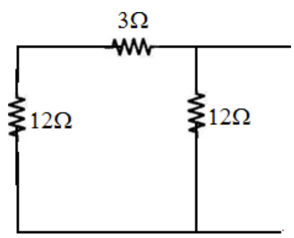
V_{oc} is the voltage across 12Ω resister

$$V_{oc} = \frac{24 \times 12}{12 + 3 + 1} = 18V$$

(ii) Finding R_{th}

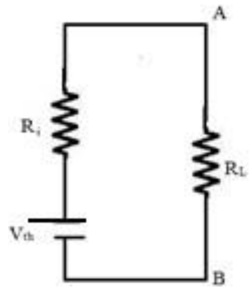
R_{th} is calculated from the terminal A & B into the network.

The 1Ω resister and 3Ω in are series and then parallel



R_{th} = 3+1 // 12

$$= \frac{4 \times 12}{16} = 3\Omega$$



(iii) $I_{th} = = \frac{Voc}{R_L + R} = \frac{18}{15+3} = 1$ A.

<u>Example 02:</u> Determine the current in 1Ω resistor across AB of the network shown in fig(a) using thevenin theorem.

Solution:The circuirt can be redrawn as in fig (b).

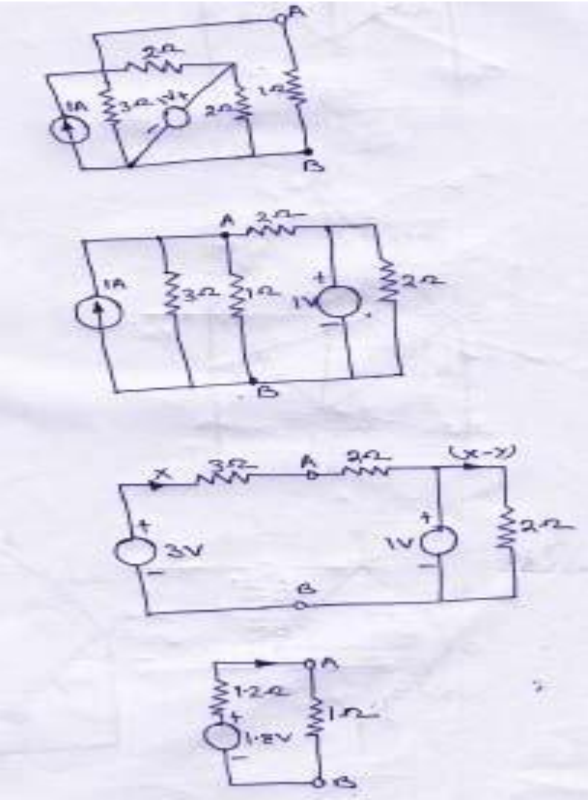


fig (a),(b),(c),(d) respectively

Step-1 remove the 1Ω resistor and keeping open circuit .The current source is converted to the equivalent voltage source as shown in fig (c)

Step-02 for finding the V_{th} we'll apply KVL law in fig (c)

then 3-(3+2)x-1=0

x=0.4A

$V_{th}=V_{AB}$ =3-3*0.4=1.8V

<u>Step03</u>-for finding the R_{th},all sources are set be zero

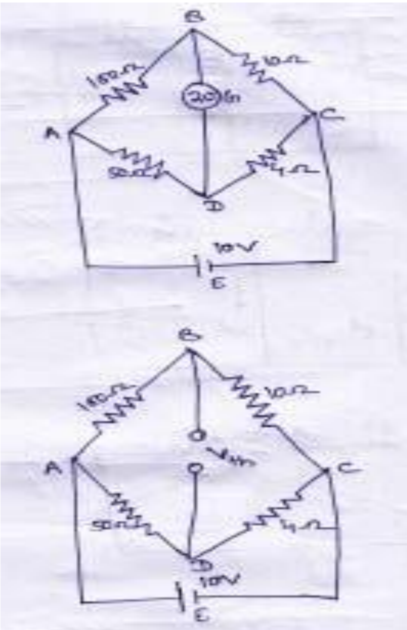
R_{th}=2//3=(2*3)/(2+3)=1.2Ω

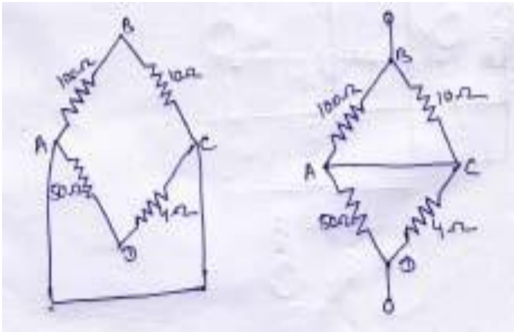
Step04- Then current I_{th}=1.8/(12.1+1)=0.82A

Example03: The four arms of a wheatstone bridge have the following resistances .

AB=100Ω,BC=10Ω,CD=4Ω,DA=50Ω.AA galvanometer of 20Ω resistance is connected across BD. Use thevenin theorem to compute the current through the galvanometer when the potential difference10V is maintained across AC.

Solution:





step 01- Galvanometer is removed.

step02-finding the V_{th} between B&D.ABC is a potential divider on which a voltage drop of 10vtakes place.

Potential of B w.r.t C=10*10/110=0.909V

Potential of D w.r.t C=10*4/54=.741V

then,

p.d between B&D is V_{th}=0.909-.741=0.168V

Step03-finding R_{th}

remove all sources to zero keeping their internal resistances.

$R_{th} = R_{BD} = 10//100 + 50//4 = 12.79\Omega$
Step04;
lastly $I_{th} = V_{th}/R_{th} + R_L = 0.168/(12.79+20) = 5mA$

Norton's Theorem

Statement : In any two terminal active network containing voltage sources and resistances when viewed from its output terminals in equivalent to a constant current source and a parallel resistance. The constant current source is equal to the current which would flow in a short circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from the open circuit side after replacing their internal resistances and removing all the sources.

OR

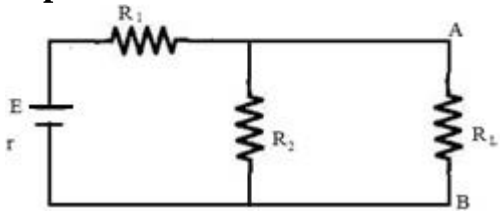
In any two terminal active network the current flowing through the load resistance R_L is given by

$$I_L = \frac{I_{sc} \times R_i}{R_i \times R_L}$$

Where R_i is the internal resistance of the network as viewed from the open ckt side A & B with all sources being replaced by leaving their internal resistances if any.

I_{sc} is the short ckt current between the two terminals of the load resistance when it is shorted

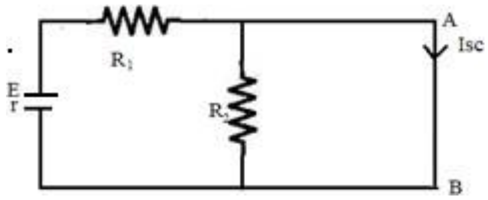
Explanation :



Step – 1

A &B are shorted by a thick copper wire to find out I_{sc}

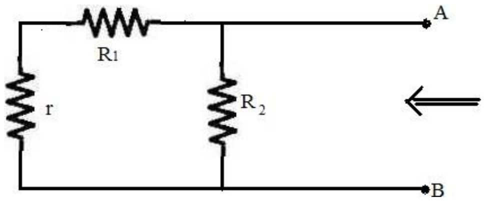
$I_{sc} = E/(R_1 + r)$



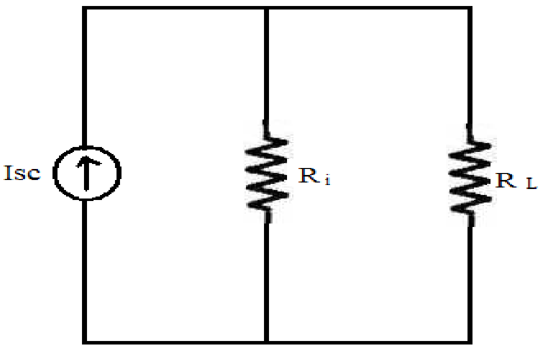
$I_{sc} = E / (R_1 + r)$

Step – 2

Remove all the source leaving its internal resistance if any and viewed from open circuit side A and B into the network to find R_i .



$$R_i = (R_1 + r) \parallel R_2$$
$$R_i = (R_1 + r)R_2 / (R_1 + r + R_2)$$



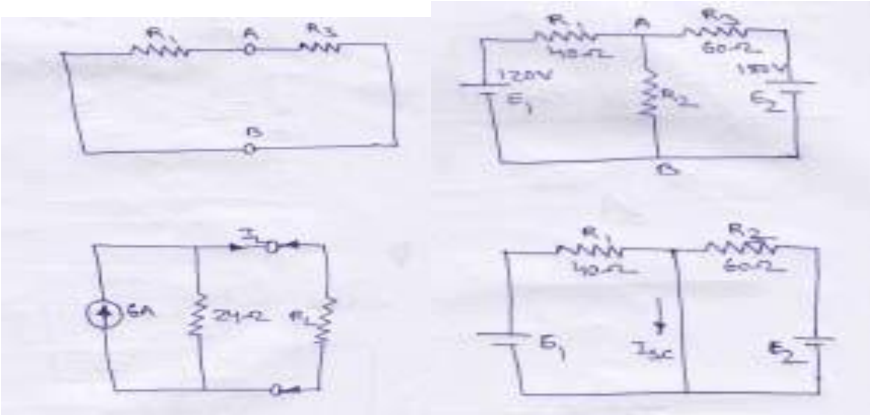
Step – 3

Connect I_{sc} & R_i in parallel with R_L

$$I_L = \frac{I_{sc} \times R_i}{R_i + R_L}$$

Example 01: Using norton's theorem find the current that would flow through the resistor R_2 whenit takes the values of 12Ω,24Ω&36Ω respectively in the fig shown below.

Solution:



Step 01-remove the load resistance by making short circuit. now terminal AB short circuited.

Step 02-Finding the short circuit current I_{sc}

First the current due to E_1 is =120/40=3A,and due to E_2 is 180/60=3A.

then I_{sc}=3+3=6A

Step 03-finding resistance R_N

It is calculated by by open circuit the load resistance and viewed from open circuit and into the network and all sources are taken zero.

R_N=40//60=(40*60)/(40+60)=24Ω

i) when R_L=12Ω, I_L=6*24/(24+36)=4A

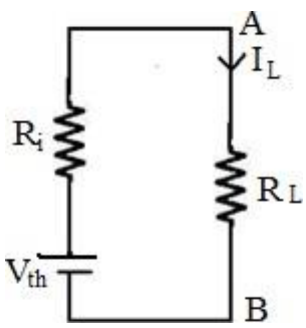
ii) when R_L=24Ω,I_L=6/2=3A

iii) when R_L=36Ω,I_L=6*24/(24+36)=2.4A

Maximum PowerTransfer Theorem

Statement : A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals(Open circuit) with all sources removed leaving their internal resistances if any

Proof:



$$I_L = \frac{V_{th}}{R_i + R_L}$$

Power delivered to the load resistance is given by

P= $$I^2 R_L = \frac{V^2 R_L}{(R_S + R_L)}$$

Power delivered to the load resistance R_L will be maximum

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\frac{V^2}{(R_S+R_L)^2}R_L$$

$$= \frac{V^2[(R_S+R_L)^2-(2R_L)(R_S+R_L)]}{(R_S+R_L)^4}$$

$$(R_S+R_L)^2-2R_L(R_S+R_L)=0$$

$$R_S{}^2+R_L{}^2+2R_SR_L-2R_SR_L-2R_L{}^2=0$$

$$R_S=R_L$$

Hence the maximum power will be transferred to the load when load resistance is equal to the source resistance

COUPLED CIRCUITS

It is defined as the interconnected loops of an electric network through the magnetic circuit.

There are two types of induced emf.

(1) Statically Induced emf.

(2) Dynamically Induced emf.

Faraday's Laws of Electro-Magnetic :

Introduction → **First Law :→**

Whenever the magnetic flux linked with a circuit changes, an emf is induced init.

OR

Whenever a conductor cuts magnetic flux an emf is induced in it.

Second Law :→

It states that the magnitude of induced emf is equal to the rate of change of flux linkages.

OR

The emf induced is directly proportional to the rate of change of flux and number of turns

Mathematically :

$$e \propto \frac{d\phi}{dt}$$

$$e \propto N$$

Or $$e = -N\frac{d\phi}{dt}$$

Where e = induced emf

N = No. of turns

ϕ = flux

'- ve' sign is due to Lenz's Law

Inductance :→

It is defined as the property of the substance which opposes any change in Current & flux.

Unit :→ Henry

Fleming's Right Hand Rule:→

It states that "hold your right hand with fore-finger, middle finger and thumb at right angles to each other. If the fore-finger represents the direction of field, thumb represents the direction of motion of the conductor, then the middle finger represents the direction of induced emf."

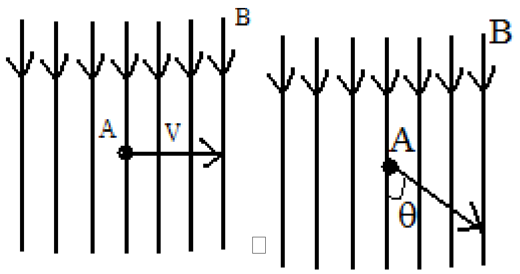
Lenz's Law : →

It states that electromagnetically induced current always flows in such a direction that the action of magnetic field set up by it tends to oppose the vary cause which produces it.

OR

It states that the direction of the induced current (emf) is such that it opposes the change of magnetic flux.

(2) Dynamically Induced emf :→



In this case the field is stationary and the conductors are rotating in an uniform magnetic field at flux density ‘B” Wb/mt^2 and the conductor is lying perpendicular to the magnetic field. Let ‘l’ is the length of the conductor and it moves a distance of ‘dx’ nt in time ‘dt’ second.

The area swept by the conductor = $l.\ dx$

Hence the flux cut = $ldx.\ B$

Change in flux in time ‘dt’ second = $\frac{Bldx}{dt}$

$E = \mathbf{Blv}$

Where $V = \frac{dx}{dt}$

If the conductor is making an angle ‘θ’ with the magnetic field, then

$$\boxed{e = Blv\ \sin\theta}$$

(1) Statically Induced emf :→

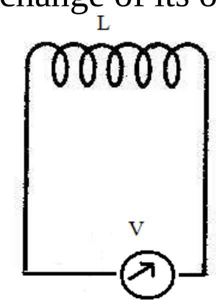
Here the conductors are remain in stationary and flux linked with it changes by increasing or decreasing.

It is divided into two types .

(i) Self-induced emf.

(ii) Mutually-induced emf.

(i) Self-induced emf : → It is defined as the emf induced in a coil due to the change of its own flux linked with the coil.



If current through the coil is changed then the flux linked with its own turn will also change which will produce an emf is called self-induced emf.

Self-Inductance :→

It is defined as the property of the coil due to which it opposes any change (increase or decrease) of current or flux through it.

Co-efficient of Self-Inductance (L) :→

It is defined as the ratio of weber turns per ampere of current in the coil.

OR

It is the ratio of flux linked per ampere of current in the coil

1st Method for 'L' :→

$$L = \frac{N\phi}{I}$$

Where L = Co-efficient of self-induction

N = Number of turns

ϕ = flux

I = Current

2nd Method for L :→

We know that

$$L = \frac{N\phi}{I}$$

$$\Rightarrow LI = N\phi$$

$$\Rightarrow -LI = -N\phi$$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -N\frac{d\phi}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = e_L$$

$$\Rightarrow L\frac{dI}{dt} = -e_L$$

$$\boxed{\Rightarrow L = \frac{-e_L}{\frac{dI}{dt}}}$$

Where L = Inductance

$e_L = -N\frac{d\phi}{dt}$ is known as self-induced emf.

When $\frac{dI}{dt} = 1 amp/\sec.$

e = 1 volt

L = 1 Henry

A coil is said to be a self-inductance of 1 Henry if 1 volt is induced in it. When the current through it changes at the rate of 1 amp/ sec.

3rd Method for L :→

$$L = \frac{M_o M_r A N^2}{l}$$

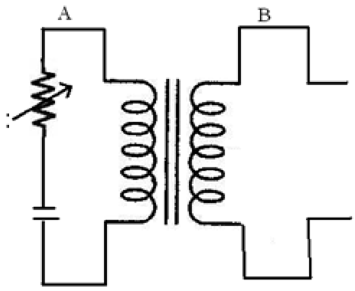
Where A = Area of x-section of the coil

N = Number of turns

L = Length of the coil

(ii) Mutually Induced emf :→

It is defined as the emf induced in one coil due to change in current in other coil. Consider two coils ‘A’ and ‘B’ lying close to each other. An emf will be induced in coil ‘B’ due to change of current in coil ‘A’ by changing the position of the rheostat.



Mutual Inductance :→

It is defined as the emf induced in coil ‘B’ due to change of current in coil ‘A’ is the ratio of flux linkage in coil ‘B’ to 1 amp. Of current in coil ‘A’.

Co-efficient of Mutual Inductance (M)

Coefficient of mutual inductance between the two coils is defined as the weber-turns in one coil due to one ampere current in the other.

1st Method for ‘M’ :→

$$M = \frac{N_2 \phi_1}{I_1}$$

N_2 = Number of turns

M = Mutual Inductance

ϕ_1 = flux linkage

I_1 = Current in ampere

2nd Method for M :→

We know that

$$M = \frac{N_2 \phi_1}{I_1}$$

$$\Rightarrow MI_1 = N_2 \phi_1$$

$$\Rightarrow -MI_1 = N_2 \phi_1$$

$$\Rightarrow -M\frac{dI_1}{dt} = -N_2\frac{d\phi_1}{dt}$$

$$\Rightarrow -M\frac{dI_1}{dt} = e_M$$

$$\Rightarrow M\frac{dI_1}{dt} = -e_M$$

$$\boxed{\Rightarrow M = \frac{-e_M}{\frac{dI_1}{dt}}}$$

Where $e_M = -N_2\frac{d\phi_1}{dt}$ is known as mutually induced emf.

$$e_M = -1 volt$$

Then M = 1 Henry

A coil is said to be a mutual inductance of 1 Henry when 1 volt is induced when the current of 1 amp/sec. is changed in its neighbouring coil.

3rd Method for M :→

$$M = \frac{M_o M_r A N_1 N_2}{l}$$

Co-efficient of Coupling :

Consider two magnetically coupled coils having N_1 and N_2 turns respectively. Their individual co-efficient of self-inductances are

$$L_1 = \frac{M_o M_r A N_2^2}{l}$$

$$L_2 = \frac{M_o M_r A N_2^2}{l}$$

The flux ϕ_1 produced in coil 'A' due to a current of I_1 ampere is

$$\phi_1 = \frac{L_1 I_1}{N_1} = \frac{M_o M_r A N_1^2}{l} \times \frac{I_1}{N_1}$$

$$\phi_1 = \frac{M_o M_r A N_1 I_1}{l}$$

Suppose a fraction of this flux i.e. $K_1\phi_1$ is linked with coil 'B'

Then $M = \frac{K_1\phi_1}{I_1} \times N_2 = \frac{K_1 N_1 N_2}{l / M_o M_r A}$ --------------------------(1)

Similarly the flux ϕ_2 produced in coil 'B' due to I_2 amp. Is

$$\phi_2 = \frac{M_1 M_r A N_2 I_2}{l}$$

Suppose a fraction of this flux i.e. $K_2\phi_2$ is linked with coil 'A'

Then $M = \frac{K_2\phi_2}{I_2} \times N_1 = \frac{K_2 N_{21} N_1}{l / M_o M_r A}$ --------------------------(2)

Multiplying equation (1) & (2)

$$M^2 = \frac{K_1 K_2 N_1^2 N_2^2}{l^2 / M_0^2 M_r^2 A^2} \times N_1$$

$$= K^2 \left(\frac{M_o M_r A N_1^2}{l}\right)\left(\frac{M_o M_r A N_2^2}{l}\right)$$

$$[\because K_1 = K_2 = K]$$

$$M^2 = K^2 . L_1 . L_2$$

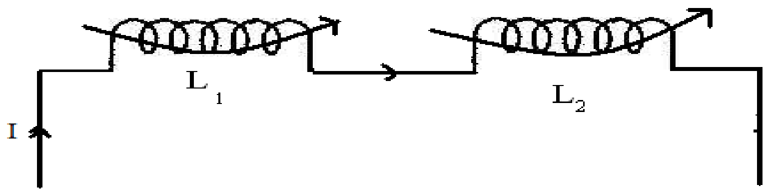
$$K^2 = \frac{M^2 .}{L_1 . L_2}$$

$$\Rightarrow K = \sqrt{\frac{M .}{L_1 . L_2}}$$

Where 'K' is known as the co-efficient of coupling.

Co-efficient of coupling is defined as the ratio of mutual inductance between two coils to the square root of their self- inductances.

Inductances In Series (Additive) :→



Fluxes are in the same durection

Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil.

L_2 = Co-efficient of self-inductance of second coil.

EMF induced in first coil due to self-inductance

$$e_{L_1} = -L_1 \frac{dI}{dt}$$

Mutually induced emf in first coil

$$e_{M_1} = -M \frac{dI}{dt}$$

EMF induced in second coil due to self induction

$$e_{L_2} = -L_2 \frac{dI}{dt}$$

Mutually induced emf in second coil

$$e_{M_2} = -M \frac{dI}{dt}$$

Total induced emf

$$e = e_{L_1} + e_{L_2} + e_{M_1} + e_{M_2}$$

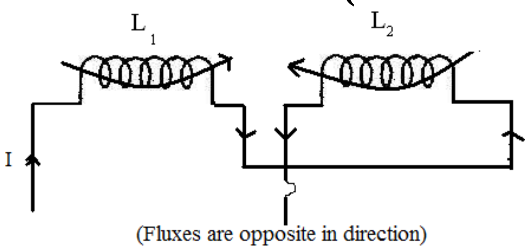
If 'L' is the equivalent inductance, then

$$-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - M\frac{dI}{dt} - L_2\frac{dI}{dt} - M\frac{dI}{dt}$$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 - L_2 - 2M)$$

$$\boxed{\Rightarrow L = L_1 + L_2 + 2M}$$

Inductances In Series (Substnactive) :→



(Fluxes are opposite in direction)

Let M = Co-efficient of mutual inductance

L_1 = Co-efficient of self-inductance of first coil

L_2 -= Co-efficient of self-inductance of second coil

Emf induced in first coil due to self induction,

$$e_{L_1} = -L_1\frac{dI}{dt}$$

Mutually induced emf in first coil

$$e_{M_1} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

Emf induced in second coil due to self-induction

$$e_{L_2} = -L_2\frac{dI}{dt}$$

Mutually induced emf in second coil

$$e_{M_2} = -\left(-M\frac{dI}{dt}\right) = M\frac{dI}{dt}$$

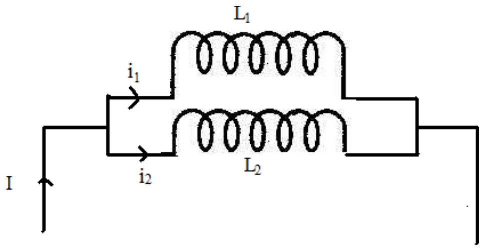
Total induced emf

$$e = e_{L_1} + e_{L_2} + e_{M_1} + e_{M_2}$$

Then $-L\frac{dI}{dt} = -L_1\frac{dI}{dt} - L_2\frac{dI}{dt} + M\frac{dI}{dt} + M\frac{dI}{dt}$

$$\Rightarrow -L\frac{dI}{dt} = -\frac{dI}{dt}(L_1 + L_2 - 2M) \qquad \Rightarrow L = L_1 + L_2 - 2M$$

Inductances In Parallel :→



Let two inductances of $L_1 \& L_2$ are connected in parallel

Let the co-efficent of mutual inductance between them is M.

$$I = i_1 + i_2$$

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \text{--------------------} (1)$$

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$= L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\Rightarrow L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\Rightarrow (L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt}$$

$$\Rightarrow \frac{di_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)} \frac{di_2}{dt} \text{--------------------------------} (2)$$

$$\frac{dI}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$= \frac{(L_2 - M)}{(L_1 - M)} \frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\Rightarrow \frac{dI}{dt} = \left(\frac{L_2 - M}{L_1 - M} + 1 \right) \frac{di_2}{dt} \text{----------------------------} (3)$$

If 'L' is the equivalent inductance

$$e = L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{1}{L} \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) \text{----------------------------} (4)$$

Substituting the value of $\frac{di_1}{dt}$

$$\frac{di}{dt} = \frac{1}{L} \left[L_1 \frac{L_2 - M}{L_1 - M} + M \right] \frac{di_2}{dt} \text{--} (5)$$

Equating equation (3) & (5)

$$\left[\left(\frac{L_2-M}{L_1-M}\right)+1\right]\frac{di_2}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]\frac{di_2}{dt}$$

$$\Rightarrow \frac{L_2-M}{L_1-M}+1 = \frac{1}{L}\left[L_1\left(\frac{L_2-M}{L_1-M}\right)+M\right]$$

$$\Rightarrow \frac{L_2-M+L_1-M}{L_1-M} = \frac{1}{L}\left[\frac{L_1L_2-L_1M+L_1M-M^2}{L_1-M}\right]$$

$$\Rightarrow \frac{L_1+L_2-2M}{L_1-M} = \frac{1}{L}\left[\frac{L_1L_2-M^2}{L_1-M}\right]$$

$$\Rightarrow L_1+L_2-2M = \frac{1}{L}\left[L_1L_2-M^2\right]$$

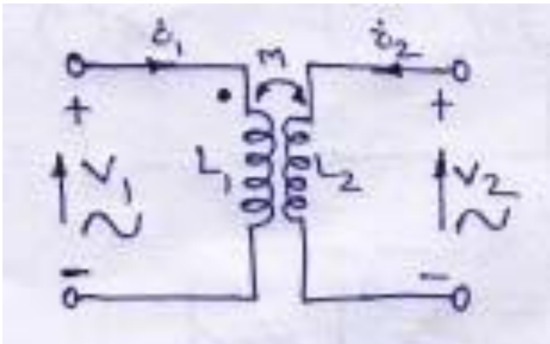
$$\boxed{\Rightarrow L = \frac{L_1L_2-M^2}{L_1+L_2-2M}}$$

When mutual field assist.

$$\boxed{L = \frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

When mutual field opposes.

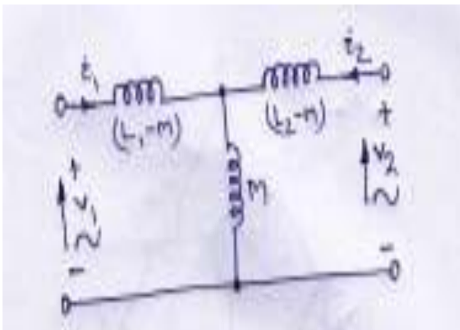
CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS



⇒ The Loop equation are from fig(a)

$$V_1 = L_1\frac{di}{dt}+M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$



⇒ The loop equation are from fig(b)

$$V_1 = (L_1 - M)\frac{di_1}{dt}+M\frac{d}{dt}(i_1+i_2)$$

$$V_2 = (L_2 - M)\frac{di_2}{dt} + M\frac{d}{dt}(i_1 + i_2)$$

Which, on simplification become

$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

So called conductively equivalent of the magnetic circuit . Here we may represent $Z_A = L_1 - M$.

$Z_B = (L_2 - M)$ and $Z_C = M$

In case M is + ve and both the currents then $Z_A = L_1 - M$, $Z_B = L_2 - M$ and $Z_C = M$, also , if is – ve and currents in the common branch opposite to each other $Z_A = L_1 + M$, $Z_B = L_2 + M$ and $Z_C = -M$.

Similarly, if M is –ve but the two currents in the common branch are additive, then also.

$Z_A = L_1 + M$, $Z_B = L_2 + M$ and $Z_C = -M$.

Further Z_A , Z_B and Z_C may also be assumed to be the T equivalent of the circuit.

Exp. -01 :

Two coupled cols have self inductances $L_1 = 10\times10^{-3}$H and $L_2 = 20\times10^{-3}$H. The coefficient of coupling (K) being 0.75 in the air, find voltage in the second coil and the flux of first coil provided the second coils has 500 turns and the circuit current is given by $i_1 = 2\sin 314.1$A.

Solution :

$$M = K\sqrt{L_1 L_2}$$

$$M = 0.75\sqrt{10\times10^{-3} \times 20\times10^{-3}}$$

$$\Rightarrow M = 10.6\times10^{-3}\,H$$

The voltage induced in second coil is

$$\nu_2 = M\frac{di_1}{dt} = M\frac{di}{dt}$$

$$= 10.6\times10^{-3}\frac{d}{dt}(2\sin 314t)$$

$$= 10.6\times10^{-3} \times 2 \times 314\ \cos\ 314t.$$

The magnetic CKt being linear,

$$M = \frac{N_2\phi_2}{i_1} = \frac{500\times(K\phi_1)}{i_1}$$

$$\phi = \frac{M}{500\times K}\times i_1 = \frac{10.6\times10^{-3}}{500\times0.75}\times 2\sin 314t$$

$$= 5.66\times10^{-5}\sin 314t$$

$\phi_1 = 5.66 \times 10^{-5} \sin s \ 314t.$

Exp. 02

Find the total inductance of the three series connected coupled coils.Where the self and mutual inductances are

$L_1 = 1H, L_2 = 2H, L_3 = 5H$

$M_{12} = 0.5H, M_{23} = 1H, M_{13} = 1H$

Solution:

$L_A = L_1 + M_{12} + M_{13}$

$= 1 + 20.5 + 1$

$= 2.5H$

$L_B = L_2 + M_{23} + M_{12}$

$= 2 + 1 + 0.5$

$= 3.5H$

$L_C = L_3 + M_{23} + M_{13}$

$= 5 + 1 + 1$

$= 7H$

Total inductances are

$L_{ea} = L_A + L_B + L_c$

$= 2.5 + 3.5 + 7$

$= 13H$ (Ans)

Example 03:

Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500A/s in A induces an emf of 11.25 V in B. Calculate the mutual inductance of the arrangement .If the self inductance of each coil is 15mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.

Solution: We know that

$$e_M = \frac{MdI_1}{dt}$$

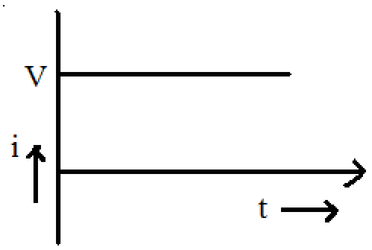
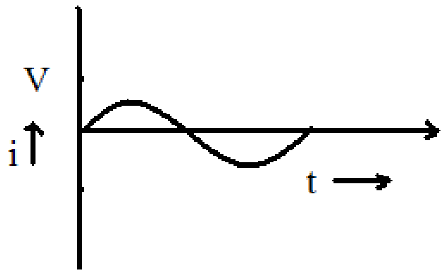
$$M = {}^{e_M}\Big/{}^{dI_1}/_{dt} = \frac{11.25}{1500} = 7.5mH$$

now,

$$L_1 = \frac{N_1\varphi_1}{I_1} = \frac{\varphi_1}{I_1} = \frac{L_1}{N_1} = 15 * \frac{10^{-3}}{750} = 2 * 10^{-5}\text{Wb/A}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{7.5 * 10^{-3}}{15 * 10^{-3}} = 0.5 = 50\%$$

AC CIRCUIT & RESONANCE

	Direct Current		**Alternating Current**
			
(1)	D.C. always flow in one direction and whose magnitude remains constant.	(1)	A.C. is one which reverse periodically in direction and whose magnitude undergoes a definite cycle changes in definite intervals of time.
(2)	High cost of production.	(2)	Low cost of production
(3)	It is not possible by D.C. Because D.C. is dangerous to the transformer.	(3)	By using transformer A.C. voltage can be decreased or increased.
(4)	Its transmission cost is too high.	(4)	A.C. can be transmitted to a long distance economically.

Definition of A.C. terms :-

Cycle : It is one complete set of +ve and –ve values of alternating quality spread over 360° or 2∏ radan.

Time Period : It is defined as the time required to complete one cycle.

Frequency : It is defined as the reciprocal of time period. i.e. $f = 1/T$

Or

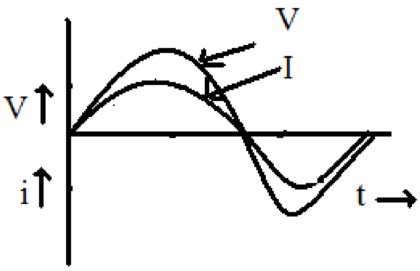
It is defined as the number of cycles completed per second.

Amplitude : It is defined as the maximum value of either +ve half cycle or –ve half cycle.

Phase : It is defined as the angular displacement between two haves is zero.

OR

Two alternating quantity are in phase when each pass through their zero value at the same instant and also attain their maximum value at the same instant in a given cycle.



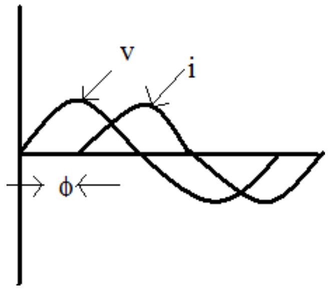
$V = V_m \sin wt$

$i = I_m \sin wt$

Phase Difference :- It is defined as the angular displacement between two alternating quantities.

OR

If the angular displacement between two waves are not zero, then that is known as phase difference. i.e. at a particular time they attain unequal distance.



OR

Two quantities are out of phase if they reach their maximum value or minimum value at different times but always have an equal phase angle between them.

Here $V = V_m \sin wt$

$i = I_m \sin (wt-\phi)$

In this case current lags voltage by an angle ‘ϕ’.

Phasor Diagram:

Generation of Alternating emf :-

Consider a rectangular coil of ‘N” turns, area of cross-section is ‘A’ nt^2 is placed in x-axis in an uniform magnetic field of maximum flux density $Bm\ web/nt^2$. The coil is rotating in the magnetic field with a velocity of w radian / second. At time t = 0, the coil is in x-axis. After interval of time ‘dt’ second the coil make rotating in anti-clockwise direction and makes an angle ‘θ’ with x-direction. The perpendicular component of the magnetic field is $\phi = \phi n \cos wt$

According to Faraday’s Laws of electro-magnetic Induction

$$e = -N \frac{d\phi}{dt}$$
$$= -N \frac{d}{dt}(\phi_m \cos wt)$$
$$= -N(-\phi_m w \cos wt)$$
$$= Nw\phi_m \sin wt$$
$$= 2\pi f N\phi_m \sin wt \quad (\because w = 2\pi f)$$
$$= 2\pi f N B_m A \sin wt$$
$$e = E_m \sin wt$$

Where $E_m = 2\pi f N B_m A$

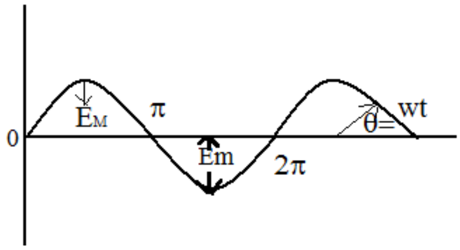
f $\rightarrow$ frequency in Hz

$B_m \rightarrow$ Maximum flux density in Wb/mt^2

Now when θ or wt = 90°

$e = E_m$

i.e. $E_m = 2\pi f N B_m A$



Root Mean Square (R.M.S) Value :→

The r.m.s. value of an a.c. is defined by that steady (d.c.) current which when flowing through a given circuit for a given time produces same heat as produced by the alternating current when flowing through the same circuit for the same time.

Sinuscdial alternating current is

$i = I_m \sin wt = I_m \sin \theta$

The mean of squares of the instantaneous values of current over one complete cycle

$$= \int_0^{2\pi} \frac{i^2.d\theta}{(2\pi - 0)}$$

The square root of this value is

$$= \sqrt{\int_0^{2\pi} \frac{i^2.d\theta}{2\pi}}$$

$$= \sqrt{\int_0^{2\pi} \frac{(I_m \sin\theta)^2}{2\pi} d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\sin^2\theta.d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}\left(1-\cos 2\theta\right)d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\frac{\theta-\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}\left(2\pi-\frac{\sin 4\pi}{2}\right)d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}\left(2\pi-0\right)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

Average Value :→

The average value of an alternating current is expressed by that steady current (d.c.) which transfers across any circuit the same charge as it transferred by that alternating current during the sae time.

The equation of the alternating current is $i = I_m \sin\theta$

$$I_{av} = \int_0^{\pi}\frac{i\ .d\theta}{(\pi-0)}$$

$$= \int_0^{\pi}\frac{I_m.\sin\theta}{\pi}d\theta = \frac{I_m}{\pi}\int_0^{\pi}\sin\theta.\,d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi-(\cos 0^0\right]$$

$$= \frac{I_m}{\pi}\left[1-0(-1)\right]$$

$$I_{av} = \frac{2I_m}{\pi}$$

$$I_{av} = \frac{2\times Maximum\ \ Current}{\pi}$$

Hence, $I_{av} = 0.637 I_m$

The average value over a complete cycle is zero

Amplitude factor/ Peak factor/ Crest factor :- It is defined as the ratio of maximum value to r.m.s value.

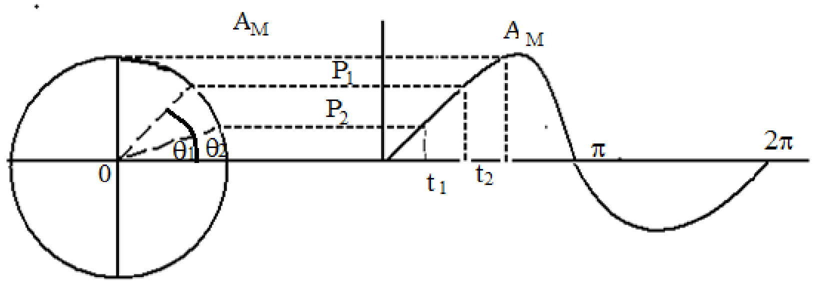
$$Ka = \frac{MaximumValue}{R.M.S.Value} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

Form factor : - It is defined as the ratio of r.m.s value to average value.

$$Kf = \frac{r.m.s.Value}{Average.Value} = \frac{0.707 I_m}{0.637 I_m} = \sqrt{2} = 1.414$$

$Kf = 1.11$

Phasor or Vector Representation of Alternating Quantity: □



An alternating current or voltage, (quantity) in a vector quantity which has magnitude as well as direction. Let the alternating value of current be represented by the equation e = E_m Sin wt. The projection of E_m on Y-axis at any instant gives the instantaneous value of alternating current. Since the instantaneous values are continuously changing, so they are represented by a rotating vector or phasor. A phasor is a vector rotating at a constant angular velocity

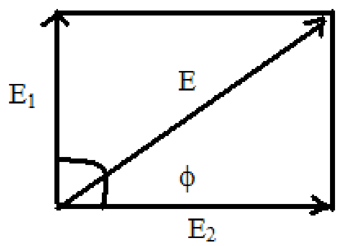
At $t_1, e_1 = E_{m_1} \sin wt_1$

At $t_2, e_2 = E_{m_2} \sin wt_2$

Addition of two alternating Current :→

Let $e_1 = E_{m_1} \sin wt$

$e_2 = E_{m_2} \sin(wt - \phi)$



The sum of two sine waves of the same frequency is another sine wave of same frequency but of a different maximum value and Phase.

$$e = \sqrt{e_1^2 + e_2^2 + 2e_1 e_2 \cos\phi}$$

Phasor Algebra :→

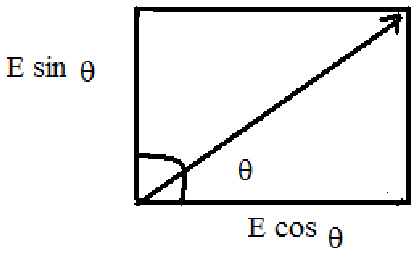
A vector quantity can be expressed in terms of

(i) Rectangular or Cartesian form

(ii) Trigonometric form

(iii) Exponential form

(iv) Polar form



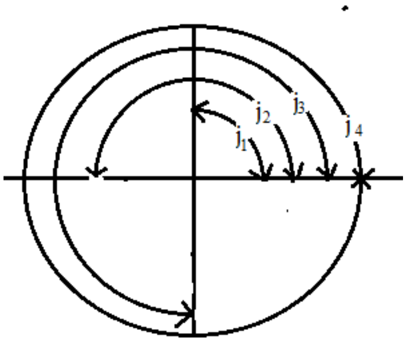
$$E = a + jb$$
$$= E(\cos\theta + j\sin\theta)$$

Where a = E cos θ is the active part

b = E sin θ is the reactive part

$\theta = \tan^{-1}\left(\frac{b}{a}\right) =$ Phase angle

$$j = \sqrt{-1}(90^o)$$
$$j^2 = -1(180^o)$$
$$j^3 = -j(270^o)$$
$$j^4 = 1 \quad (360^o)$$



(i) Rectangular for :-

$$E = a \pm jb$$
$$\tan\theta = b/a$$

(ii) Trigonometric form :-

$$E = E(\cos\theta \pm j\sin\theta)$$

(iii) Exponential form :-

$$E = Ee^{\pm j\theta}$$

(iv) Polar form :-

$$E = E/\pm e \qquad (E = \sqrt{a^2 + b^2})$$

Addition or Subtration :-

$$E_1 = a_1 + jb_1$$
$$E_2 = a_2 + jb_2$$
$$E_1 \pm E_2 = (a_1 + a_2) \pm (b_1 + b_2)$$
$$\phi = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$$

Multiplication : -

$$E_1 \times E_2 = (a_1 + ja_1) \pm (a_1 + jb_2)$$
$$= (a_1a_2 - b_1b_2) + j(a_1a_2 + b_1b_2)$$

$$\phi = \tan^{-1}\left(\frac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$$

$$E_1 = E_1\angle\theta_1$$

$$E_2 = E_2\angle\theta_2$$

$$E_1 \times E_2 = E_1E_2 \quad \angle\phi_1 + \phi_2$$

Division :-

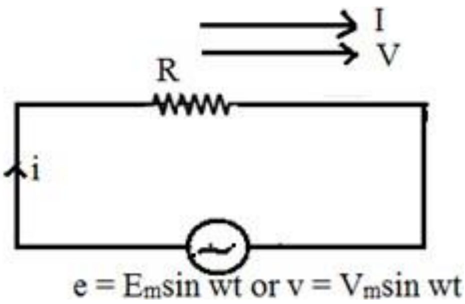
$$E_1 = E_1\angle\theta_1$$

$$E_2 = E_2\angle\theta_2$$

$$\frac{E_1}{E_2} = \frac{E_1\angle\theta_1}{E_2\angle\theta_2} = \frac{E_1}{E_2}\angle\theta_1 - \theta_2$$

A.C. through Pure Resistance :→

Let the resistance of R ohm is connected across to A.C supply of applied voltage



$e = E_m \sin wt$ -------------------------- (1)

Let 'I' is the instantaneous current .

Here $e = iR$

$\Rightarrow i = e/R$

$i = E_m \sin wt / R$----------------------- (2)

By comparing equation (1) and equation (2) we get alternating voltage and current in a pure resistive circuit are in phase

Instantaneous power is given by

P = ei

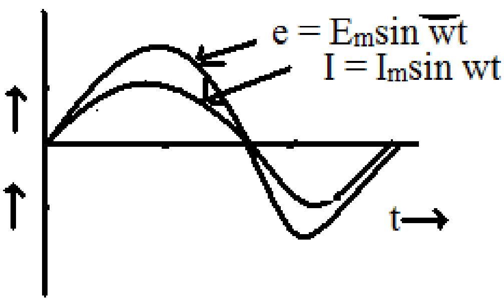
$= E_m \sin wt \,.\, I_m \sin wt$

$= E_m I_m \sin^2 wt$

$= \frac{E_m I_m}{2}.2\sin^2 wt$

$= \frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.(1 - \cos 2wt)$

$P = \frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}} - \frac{E_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$



i.e. $P = \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$

Where $\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}$ is called constant part of power.

$\frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}.\cos 2wt$ is called fluctuating part of power.

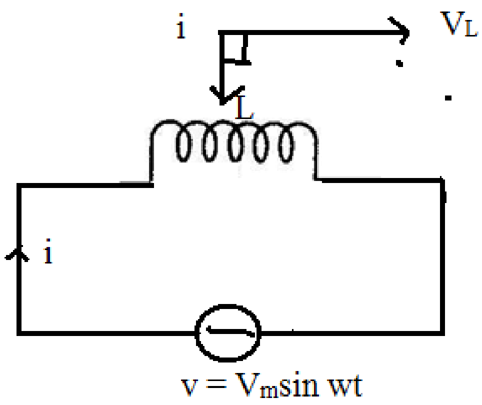
The fluctuating part $\frac{V_m I_m}{2}.\cos 2wt$ of frequency double that of voltage and current waves.

Hence power for the whole cycle is $P = \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}} = V_{rms}.I_{rms}$

$\Rightarrow$ P = VI watts

A.C through Pure Inductance :→

Let inductance of 'L' henry is connected across the A.C. supply

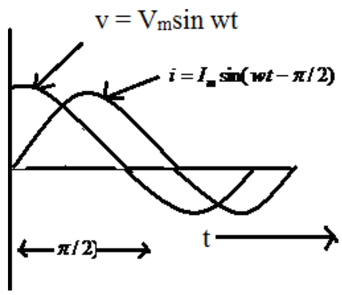


$v = V_m \sin wt$ -------------------------- (1)

According to Faraday's laws of electromagnetic inductance the emf induced across the inductance

$$V = L\frac{di}{dt}$$

$\frac{di}{dt}$ is the rate of change of current



$$V_m \sin wt = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m \sin wt}{L}$$

$$\Rightarrow di = \frac{V_m}{L}\sin wt.dt$$

Integrating both sides,

$$\int di = \int \frac{V_m}{L}\sin wt.dt$$

$$i = \frac{V_m}{L}\left(-\frac{\cos wt}{w}\right)$$

$$i = -\frac{V_m \cos wt}{wL}$$

$$i = -\frac{V_m}{wL}\cos wt$$

$$i = -\frac{V_m}{wL}\sin\left(wt - \frac{\pi}{2}\right)$$

$$= -\frac{V_m}{X_L}\sin\left(wt - \frac{\pi}{2}\right) \quad [\because X_L = 2\pi fL = wL]$$

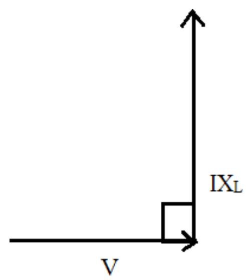
Maximum value of i is $I_m = \frac{V_m}{X_L}$ when $\sin\left(wt - \frac{\pi}{2}\right)$ is unity.

Hence the equation of current becomes $i = I_m \sin(wt - \pi/2)$

So we find that if applied voltage is rep[resented by $v = V_m \sin wt$, then current flowing in a purely inductive circuit is given by

$$i = I_m \sin(wt - \pi/2)$$

Here current lags voltage by an angle π/2 Radian.

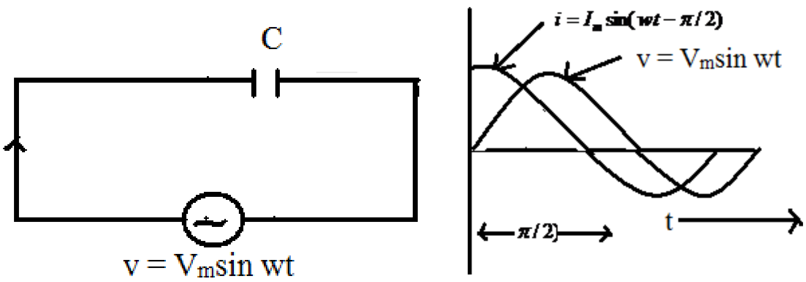


Power factor = cos φ
= cos 90°
= 0

Power Consumed = VI cos φ
= VI × 0
= 0

Hence, the power consumed by a purely Inductive circuit is zero.

A.C. Through Pure Capacitance : →



Let a capacitance of ‘C” farad is connected across the A.C. supply of applied voltage

$$v = V_m \sin wt \text{-----------------------------------} (1)$$

Let ‘q’ = change on plates when p.d. between two plates of capacitor is ‘v’

$q = cv$

$q = cV_m \sin wt$

$$\frac{dq}{dt} = c\frac{d}{dt}(V_m \sin wt)$$

$i = cV_m \sin wt$

$= wcV_m \cos wt$

$$= \frac{V_m}{1/wc} = \cos wt$$

$$= \frac{V_m}{Xc} = \cos wt$$ [$\because X_c = \frac{1}{wc} = \frac{1}{2\pi fc}$ is known as capacitive reactance in ohm.]

$= I_m \cos wt$

$= I_m \sin(wt + \pi / 2)$

Here current leads the supply voltage by an angle $\pi/2$ radian.

Power factor $= \cos \phi$

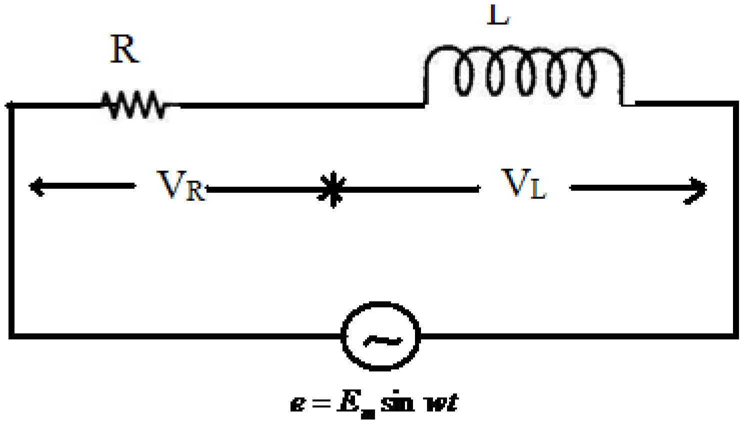
$= \cos 90° = 0$

Power Consumed $= VI \cos \phi$

$= VI \times 0 = 0$

The power consumed by a pure capacitive circuit is zero.

A.C. Through R-L Series Circuit : →



The resistance of R-ohm and inductance of L-henry are connected in series across the A.C. supply of applied voltage

$e = E_m \sin wt$ -----------------------------(1)

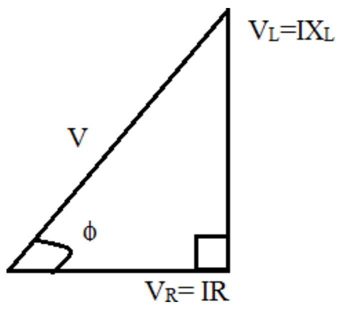
$V = V_R + jV_L$

$= \sqrt{V_R^2 + V_L^2} \angle\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

$= \sqrt{(IR)^2 + (IX_L)^2} \angle\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

$= I\sqrt{R^2 + X_L^2} \angle\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

$V = IZ\angle\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$



Where $Z = \sqrt{R^2 + X_L{}^2}$

$= R + jX_L$ is known as impedance of R-L series Circuit.

$I = \dfrac{V}{Z\angle\phi} = \dfrac{E_m \sin wt}{Z\angle\phi}$

$I = I_m \sin(wt - \phi)$

Here current lags the supply voltage by an angle ϕ.

Power Factor :$\rightarrow$ It is the cosine of the angle between the voltage and current.

OR

It is the ratio of active power to apparent power.

OR

It is the ratio of resistance to inpedence .

Power :$\rightarrow$

$= v.i$

$= V_m \sin wt . I_m \sin(wt - \phi)$

$= V_m I_m \sin wt . \sin(wt - \phi)$

$= \frac{1}{2} V_m I_m 2 \sin wt . \sin(wt - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos 2(wt - \phi)]$

Obviously the power consists of two parts.

(i) a constant part $\frac{1}{2} V_m I_m \cos\phi$ which contributes to real power.

(ii) a pulsating component $\frac{1}{2} V_m I_m \cos(2wt - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence average power consumed

$$= \frac{1}{2} V_m I_m \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos\phi$$

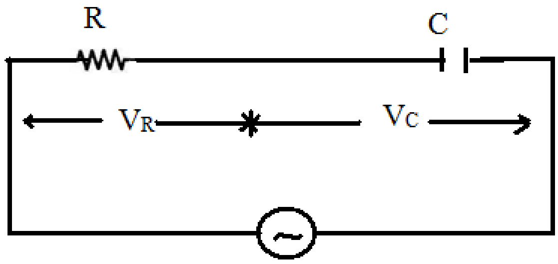
$$= VI \cos\phi$$

Where V & I represents the r.m.s value.

A.C. Through R-C Series Circuit : $\rightarrow$

The resistance of ‘R’-ohm and capacitance of ‘C’ farad is connected across the A.C. supply of applied voltage

$e = E_m \sin wt$ -----------------------------(1)



$V = V_R + (-jV_C)$

$= IR + (-jIX_C)$

$= I(R - jX_C)$

$V = IZ$

Where $Z = R - jX_C = \sqrt{R^2 + X_C^2}$ is known as impedance of R-C series Circuit.

$Z = R - jX_C$

$= \sqrt{R^2 + X_C^2}$

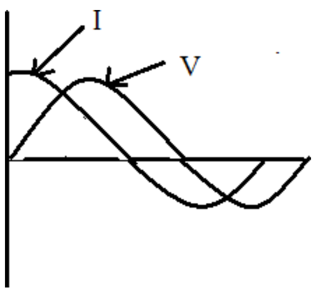
$\angle -\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$V = IZ\angle -\phi$

$\Rightarrow I = \frac{V}{Z\angle -\phi}$

$= \frac{E_m \sin wt}{Z\angle -\phi}$

$= \frac{E_m}{Z\angle}\sin(wt + \phi)$

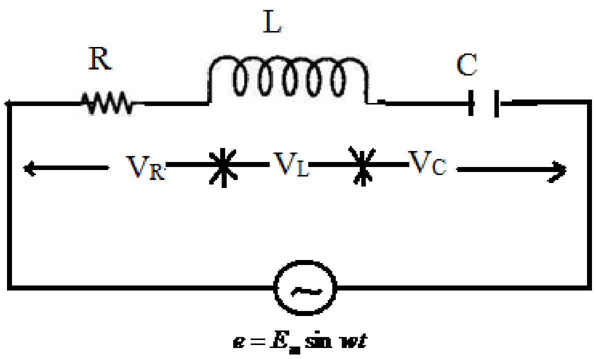


$\Rightarrow I = I_m \sin(wt + \phi)$

Here current leads the supply voltage by an angle ‘ϕ’.

A.C. Through R-L-C Series Circuit : →

Let a resistance of ‘R’-ohm inductance of ‘L’ henry and a capacitance of ‘C’ farad are connected across the A.C. supply in series of applied voltage



$e = E_m \sin wt$ --------------------------------- (1)

$$e = \vec{V_R} + \vec{V_L} + \vec{V_C}$$
$$= V_R + jV_L - jV_C$$
$$= V_R + j(V_L - V_C)$$
$$= I_R + j(IX_L - IX_C)$$
$$= I[R + j(X_L - X_C)]$$
$$= I\sqrt{R^2 + (X_L - X_C)^2} \quad \angle \pm \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$
$$= IZ\angle \pm \phi$$

Where $Z = I\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of R-L-C Series Circuit.

If $X_L > X_C$, then the angle is +ve.

If $X_L < X_C$, then the angle is -ve.

Impedance is defined as the phasor sum of resistance and net reactance

$$e = IZ\angle \pm \phi$$
$$\Rightarrow I = \frac{e}{Z\angle \pm \phi} IZ\angle \pm \phi \quad = \frac{E_m \sin wt}{Z\angle \pm \phi} = I_m \sin(wt \pm \phi)$$

(1) If $X_L > X_C$, then P.f will be lagging.

(2) If $X_L < X_C$, then, P.f will be leading.

(3) If $X_L = X_C$, then, the circuit will be resistive one. The p.f. becomes unity and the resonance occurs.

REASONANCE

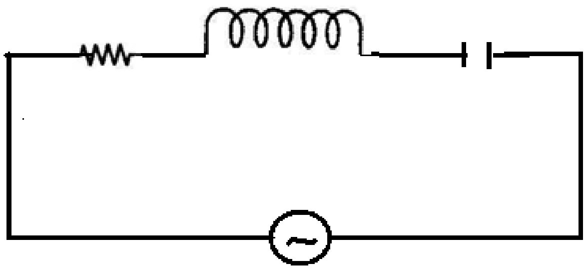
It is defined as the resonance in electrical circuit having passive or active elements represents a particular state when the current and the voltage in the circuit is maximum and minimum with respect to the magnitude of excitation at a particular frequency and the impedances being either minimum or maximum at unity power factor

Resonance are classified into two types.

(1) Series Resonance

(2) Parallel Resonance

(1) Series Resonance :- Let a resistance of 'R' ohm, inductance of 'L' henry and capacitance of 'C' farad are connected in series across A.C. supply



$e = E_m \sin wt$

The impedance of the circuit

$Z = R + j(X_L - X_C)]$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

The condition of series resonance:

The resonance will occur when the reactive part of the line current is zero

The p.f. becomes unity.

The net reactance will be zero.

The current becomes maximum.

At resonance net reactance is zero

$X_L - X_C = 0$

$\Rightarrow X_L = X_C$

$\Rightarrow W_o L = \frac{1}{W_o C}$

$\Rightarrow W_o^2 LC = 1$

$\Rightarrow W_o^2 = \frac{1}{LC}$

$\Rightarrow W_o = \frac{1}{\sqrt{LC}}$

$\Rightarrow 2\pi f_o = \frac{1}{\sqrt{LC}}$

$\Rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$

Resonant frequency $(f_o) = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$

Impedance at Resonance

$Z_0 = R$

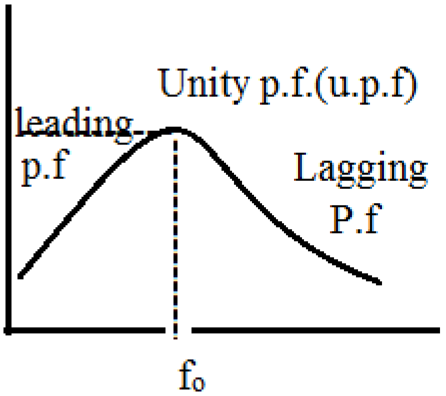
Current at Resonance

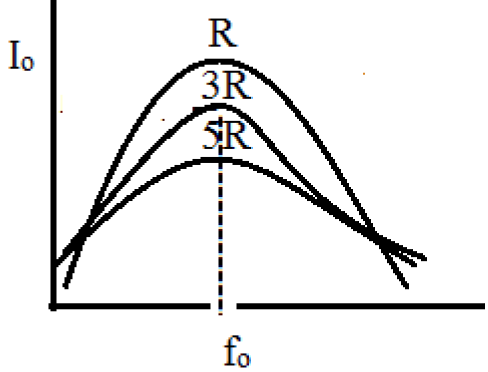
$I_o = \frac{V}{R}$

Power factor at resonance

$p.f. = \frac{R}{Z_o} = \frac{R}{R} = 1$ $\quad [\because Z_o = R]$

Resonance Curve :-





At low frequency the X_c is greater and the circuit behaves leading and at high frequency the X_L becomes high and the circuit behaves lagging circuit.

If the resistance will be low the curve will be stiff (peak).

- If the resistance will go oh increasing the current goes on decreasing and the curve become flat.

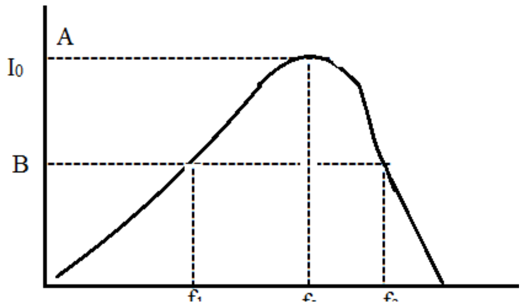
Band Width :→

At point 'A' the power loss is I_0^2R.

The frequency is f_0 which is at resonance.

At point 'B' the power loss is $\frac{I_0^2R}{2}$.

The power loss is 50% of the power loss at point 'A"/



Hence the frequencies corresponding to point 'B' is known as half power frequencies f_1 & f_2.

f_1 = Lower half power frequency

$$f_1 = f_0 - \frac{R}{4\pi L}$$

F_2 = Upper half power frequency

$$f_2 = f_0 + \frac{R}{4\pi L}$$

Band width (B.W.) is defined as the difference between upper half power frequency ad lower half power frequency.

$$\text{B.W.} = f_2 - f_1 = \frac{R}{2\pi L}$$

Selectivity : →

Selectivity is defined as the ratio of Band width to resonant frequency

Selectivity = $\frac{B.W.}{f_0} = \frac{R}{2\pi L}$ $\quad Selectivity = \frac{R}{2\pi f_o L}$

Quality Factor (Q-factor) :→

It is defined as the ratio of 2π × Maximum energy stored to energy dissipated per cycle

$$\text{Q-factor} = \frac{2\pi \times \frac{1}{2} L I_0^2}{I^2 R T}$$

$$= \frac{\pi L\left(\sqrt{2} I\right)^2}{I^2 R T}$$

$$= \frac{\pi L . 2 I^2}{I^2 R T}$$

$$= \frac{\pi L . 2 I^2}{I^2 R T}$$

$$= \frac{2\pi L.}{R T}$$

$$\boxed{\text{Quality factor} = \frac{2\pi f_0 L.}{R}} \qquad \left[Q \frac{1.}{I} = f_0 \right]$$

Quality factor is defined as the reciprocal of power factor.

$$\boxed{\text{Q factor} = \frac{1.}{\cos\phi}}$$

It is the reciprocal of selectivity.

$$\text{Q-factor Or Magnification factor} = \frac{\text{Voltage across Inductor.}}{\text{Voltage across resistor}}$$

$$= \frac{I_0 X_L}{I_0 R}$$

$$= \frac{X_L}{R}$$

$$= \frac{2\pi f_0 L}{R} = \frac{W_0 L}{R}$$

$$\boxed{\text{Q- factor} = \frac{W_0 L}{R}}$$

$$\text{Q-factor factor} = \frac{\text{Voltage across Capacotor.}}{\text{Voltage across resistor}}$$

$$= \frac{I_0 X_c}{I_0 R}$$

$$= \frac{X_C}{R}$$

$$= \frac{1}{2\pi f_0 C} = \frac{1}{2\pi f_0 CR}$$

$$\text{Q-factor} = \frac{1}{W_0 CR}$$

$$Q^2 = \frac{W_0 L}{R} \times \frac{1}{W_0 CR}$$

$$Q^2 = \frac{1}{R^2 C}$$

$$Q = \sqrt{\frac{1}{R^2 C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

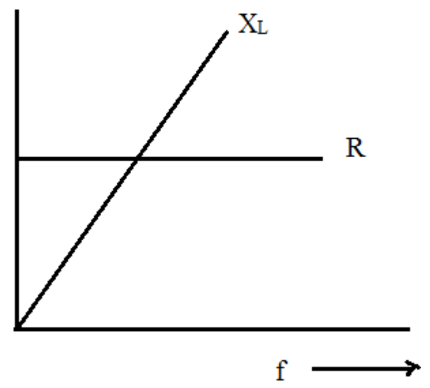
Graphical Method :→

(1) Resistance is independent of frequency It represents a straight line.

(2) Inductive Reactance $X_L = 2\pi fL$

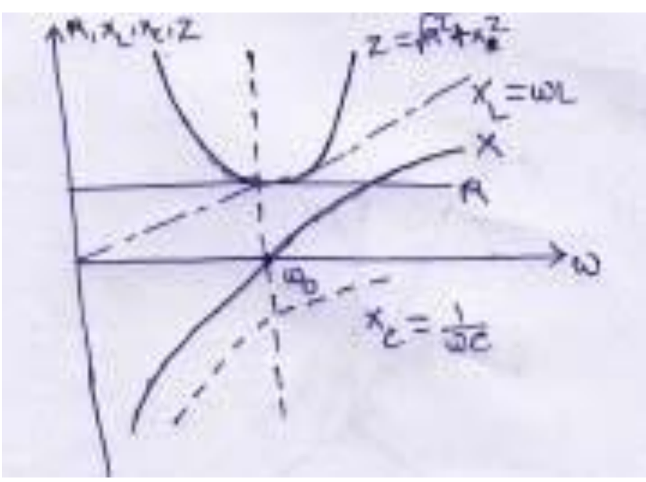
It is directly proportional to frequency. As the frequency increases , X_L increases

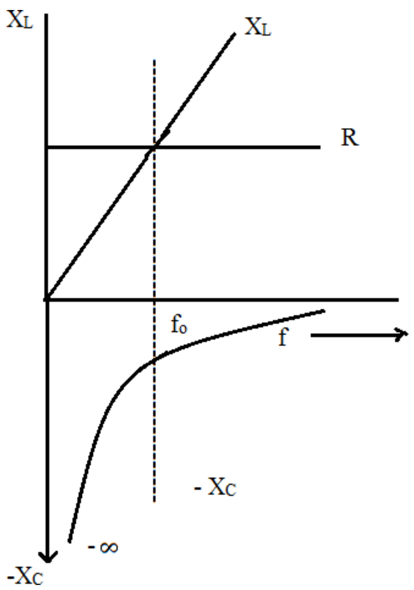
(3) Capacitive Reactance $X_C = = \frac{1}{2\pi fC}$



It is inversely proportional to frequency. As the frequency increases, X_C decreases.

When frequency increases, X_L increases and X_C decreases from the higher value.



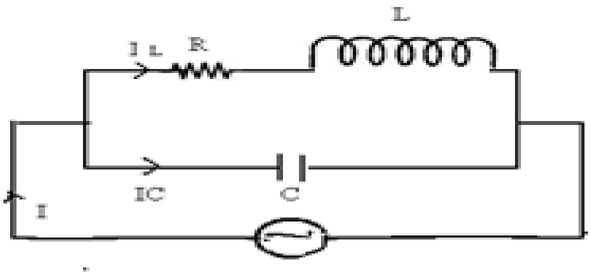


At a certain frequency. $X_L = X_C$

That particular frequency is known as Resonant frequency.

Variation of circuit parameter in series resonance:

(2) Parallel Resonance :- Resonance will occur when the reactive part of the line current is zero.



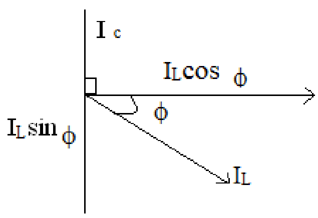
At resonance,

$I_C - I_L \sin\phi = 0$

$I_C = I_L \sin\phi$

$\Rightarrow \dfrac{V}{X_C} = \dfrac{V}{\sqrt{R^2 + X_L^2}} \sin\phi$

$\Rightarrow \dfrac{V}{X_C} = \dfrac{V}{\sqrt{R^2 + X_L^2}} \times \dfrac{X_L}{\sqrt{R^2 + X_L^2}}$



$\Rightarrow \dfrac{1}{X_C} = \dfrac{X_L}{R^2 + X_L^2}$

$\Rightarrow R^2 + X_L^2 = X_L . X_C$

$\Rightarrow Z^2 == X_L . X_C = W_0 L \times \dfrac{1}{W_0 C}$

$$Z^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + X_L{}^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + (2\pi f_0 L)^2 = \frac{L}{C}$$

$$\Rightarrow R^2 + 4\pi^2 f_0{}^2 L^2 = \frac{L}{C}$$

$$\Rightarrow 4\pi^2 f_0{}^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow f_0{}^2 = \frac{1}{4\pi^2 f_0{}^2 L^2} = \left(\frac{L}{C} - R^2\right)$$

$$\Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

f_0 = Resonant frequency in parallel circuit.

Current at Resonance = $I_L \cos\phi$

$$= \frac{V}{\sqrt{R^2 + X_L{}^2}} \cdot \frac{R}{\sqrt{R^2 + X_L{}^2}}$$

$$= \frac{VR}{R^2 + X_L^2}$$

$$= \frac{VR}{Z^2}$$

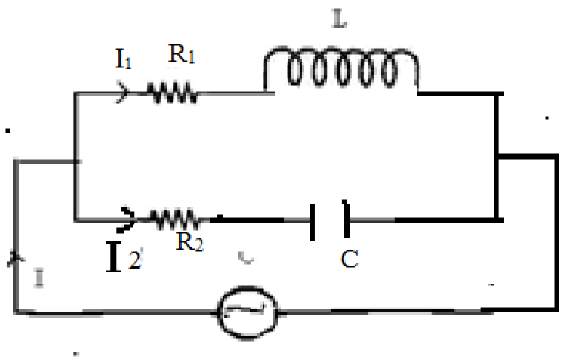
$$= \frac{VR}{L/C} = \frac{V}{L/RC}$$

$$= \frac{\text{V}}{\text{Dynamic Impedence}}$$

$L/RC \rightarrow$ Dynamic Impedance of the circuit.

or, dynamic impedances is defined as the impedance at resonance frequency in parallel circuit.

Parallel Circuit :→



The parallel resonance condition:

When the reactive part of the line current is zero.
The net reactance is zero.
The line current will be minimum.
The power factor will be unity

Impedance $Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

Admittance $Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L}$

$= \frac{(R_1 + jX_L)}{(R_1 + jX_L)(R_1 - jX_L)}$

$= \frac{R_1 + jX_L}{R_1^2 + X_L^2}$

$Y_1 = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}$

Admittance $Y_2 = \frac{1}{Z_2} = \frac{1}{R_1 + jX_C}$

$= \frac{(R_2 + jX_C)}{(R2_1 - jX_C)(R_2 + jX_C)}$

$= \frac{R_2 + jX_L}{R_2^2 + X_C^2}$

$Y_2 = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

Total Admittance Admittance $\left(\frac{1}{Z}\right) = \frac{1}{Z_1} + \frac{1}{Z_2}$

$\Rightarrow Y = Y_1 + Y_2$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow Y = \frac{R_1}{R_1^2 + X_L^2} + \frac{R2}{R_2^2 + X_C^2} - j\left(\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right)$

At Resonance,

$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$

$\Rightarrow \frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$

$\Rightarrow X_L\left(R_2^2 + X_C^2\right) = X_C\left(R_1^2 + X_L^2\right)$

$\Rightarrow 2\pi fL\left(R_2^2 + \frac{1}{4\pi^2 f^2 C^2}\right) = \frac{1}{2\pi fC}\left(R_1^2 + 4\pi^2 f^2 L^2\right)$

$\Rightarrow 2\pi fLR_2^2 + \frac{L}{2\pi fC^2} = \frac{R_1^2}{2\pi fC} + \frac{2\pi fL^2}{C}$

$$\Rightarrow \frac{L}{2\pi f C^2} - \frac{R_1^2}{2\pi f C} = \frac{2\pi f L^2}{C} - 2\pi f L R_2^2$$

$$\Rightarrow \frac{1}{2\pi f C}\left(\frac{L}{C} - R_1^2\right) = 2\pi f L\left(\frac{L}{C} - R_2^2\right)$$

$$\Rightarrow 4\pi^2 f^2 LC = \frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2} = \frac{L - CR_1^2}{L - CR_2^2}$$

$$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2 LC}\left(\frac{L - CR_1^2}{L - CR_2^2}\right)$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{L - CR_1^2}{L - CR_2^2}\right)}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left(\frac{L - CR_1^2}{L^2C - LC^2R_2^2}\right)}$$

f is called Resonant frequency.

If $R^2 = 0$

Then $f = \frac{1}{2\pi}\sqrt{\frac{L - CR_1^2}{L^2C}}$

$$= \frac{1}{2\pi L}\sqrt{\frac{L - CR_1^2}{C}}$$

$$= \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_1^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{L}{L^2C} - \frac{R_1^2}{L^2}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{L}{LC} - \frac{R_1^2}{L^2}}}$$

If R_1 and $R_2 = 0$, then

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{L^2C}}$$

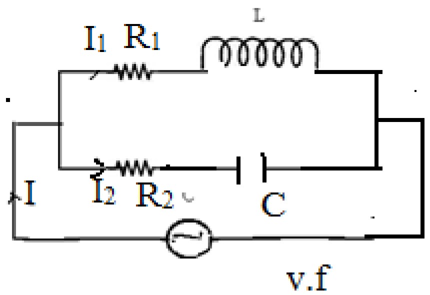
$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

Comparison of Series and Parallel Resonant Circuit :→

Item	**Series ckt (R-L-C)**	**Parallel ckt (R– L and C)**

	Series	Parallel
❖ Impedance at Resonance	Minimum	Maximum
❖ Current at Resonance	Maximum= $\frac{V}{R}$	Minimum= $\frac{V}{(L/CR)}$
❖ Effective Impedance	R	$\frac{L}{CR}$
❖ P.f. at Resonance	Unity	Unity
❖ Resonant Frequency	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$
❖ It Magnifies	Voltage	Current
❖ Magnification factor	$\frac{WL}{R}$	$\frac{WL}{R}$

Parallel circuit :→



$$Z_1 = R_1 + jX_L = \sqrt{R_1^2 + X_L^2}\angle\phi_1$$

$$Z_2 = R_1 - jX_C = \sqrt{R2_1^2 + X_C^2}\angle -\phi_2$$

$$I_1 = \frac{V}{Z_1\angle\phi_1} = \frac{V}{Z_1}\angle -\phi_1 = I_1\angle -\phi_1$$

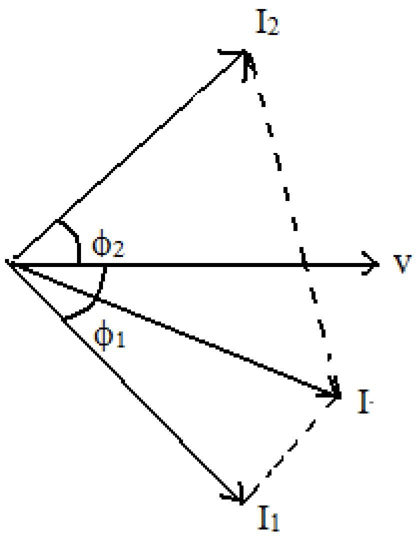
Where $\frac{V}{Z_1} = VY_1$

Here Y_1 → Admittance of the circuit

Admittance is defined as the reciprocal of impedence.

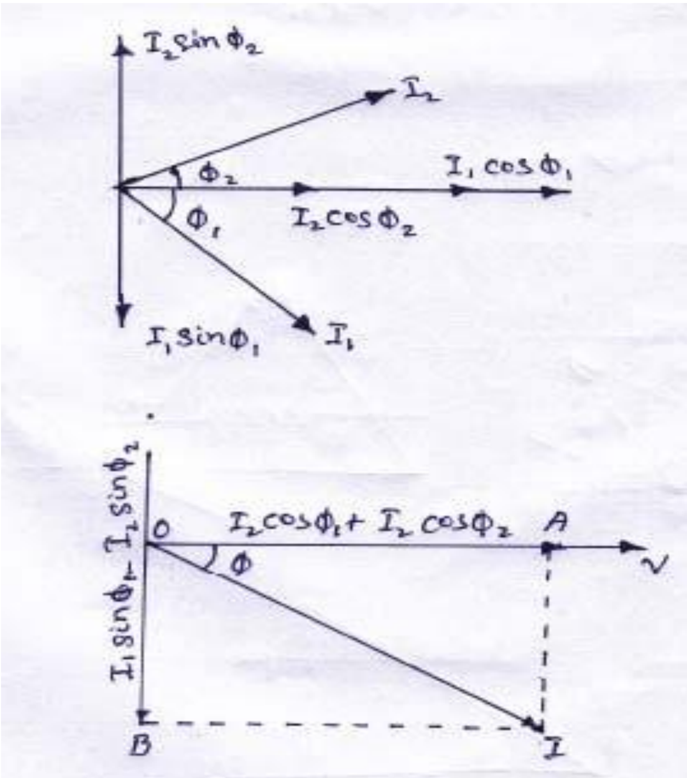
$$I_1 = VY_1 = \frac{v}{R_1 + jX_L}$$

$$I_2 = \frac{V}{Z_2\angle -\phi 2_1} = \frac{V}{Z_2}\angle\phi_1 = VY_2\angle\phi = I_2\angle\phi_2$$



$$I = \sqrt{I_1^2 + I_2^2} + 2I_1 I_2 \cos(\phi_1 + \phi_2)$$

$$I = I_1\angle -\phi_1 + I_2\angle\phi_2$$



The resultant current "I" is the vector sum of the branch currents I_1 & I_2 can be found by using parallelogram low of vectors or resolving I_2 into their X

– and Y- components (or active and reactive components respectively) and then by combining these components.

Sum of active components of I_1 and I_2 = $I_1 \cos \phi_1 + I_2 \cos \phi_2$
Sum of the reactive components of I_1 and I_2 = $I_2 \sin \phi_2 - I_1 \sin \phi_1$

EXP – 01 :

A 60Hz voltage of 230 V effective value is impressed on an inductance of 0.265 H

(i) Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0.
(ii) Show the voltage and current on a phasor diagram.
(iii) Find the maximum energy stored in the inductance.

Solution :-

$V_{max} = \sqrt{2}V = \sqrt{2} \times 230V$

$f = 60Hz, \quad W = 2\pi f = 2\pi \times 60 = 377 rad/s.$

$x_l = wl = 377 \times 0.265 = 100\Omega$.

(i) The time equation for voltage is $V(t) = 230\sqrt{2} \sin 377t.$

$I_{max} = V_{max} / x_l = 230\sqrt{2} /100. = 2.3\sqrt{3}$

$\phi = 90^o (lag).$

∴ Currente quation is.

$i(t) = 2.3\sqrt{2} \sin(377t - \pi / 2)$

or $= 2.3\sqrt{2} \cos 377t$

(ii) Iti

(iii) or $E_{max} = \frac{1}{2} LI^2_{max} = \frac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2 = 1.4J$

Example -02 :

The potential difference measured across a coil is 4.5 v, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9A at 25 Hz, the potential difference is 24 v. Find the power and the power factor when it is supplied by 50 v, 50 Hz supply.

Solution :

Let R be the d.c. resistance and L be inductance of the coil.

$R = V / I = 4.5 / 9 = 0.5\Omega$

With a.c. current of 25Hz, z = V/1.

$$\frac{24}{9} = 2.66\Omega$$

$$x_l = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2}$$

$$= 2.62\Omega$$

$$x_l = 2\pi \times 25 \times L$$

$$x_l = 0.0167\Omega$$

At 50Hz

$$x_l = 2.62 \times 2 = 5.24\Omega$$

$$Z = \sqrt{0.5^2 + 5.24^2}$$

$$= 5.06\,\Omega$$

I = 50/5.26 = 9.5 A

P = I^2/R = $9.5^2 \times 0.5$ = 45 watt.

Example – 03 :

A 50- μf capacitor is connected across a 230-v, 50 – Hz supply. Calculate

(a) The reactance offered by the capacitor.

(b) The maximum current and

(c) The r.m.s value of the current drawn by the capacitor.

Solution :

(a) $x_l = \frac{1}{wc} = \frac{1}{2\pi\,\text{fe}} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.6\Omega$

(c) Since 230 v represents the r.m.s value

$\because I_{rms} = 230 / x_l = 230 / 63.6 = 3.62A$

(b) $I_m = I_{r.m.s} \times \sqrt{2} = 3.62 \times \sqrt{2} = 5.11A$

Example – 04 :

In a particular R – L series circuit a voltage of 10v at 50 Hz produces a current of 700 mA. What are the values of R and L in the circuit ?

Solution :

(i) $Z = \sqrt{R^2 + (2\pi \times 50L)^2}$

$$= \sqrt{R^2 + 98696L^2}$$

$$V = 1z$$

$$10 = 700 \times 10^{-3} \sqrt{(R^2 + 98696L^2)}$$

$$\sqrt{(R^2 + 98696L^2)} = 10 / 700 \times 10^{-3} = 100 / 7$$

$$R^2 + 98696L^2 = 10000/49 \text{-----------------------(I)}$$

(ii) In the second case $Z = \sqrt{R^2 + (2\pi \times 75L)^2}$

$$\because 10 = 500 \times 10^{-3} \sqrt{R^2 + 222066L^2)} = 20$$

$$\sqrt{R^2 + 222066L^2)} = 20$$

$$R^2 + 222066L^2 = 400 \text{------------------------------------} (II)$$

Subtracting Ea.(I) from (ii), we get,

$$222066L^2 - 98696L^2 = 400 - (10000/49)$$

$$\Rightarrow 123370L^2 = 196$$

$$\Rightarrow L^2 = \frac{196}{123370}$$

$$\Rightarrow L = \sqrt{\frac{196}{123370}} = 0.0398H = 40 \text{ mH.}$$

Substituting this value of L in equation (ii) we get $R^2 + 222066L^2(0.398)^2 = 400$

$$\Rightarrow R = 6.9\Omega .$$

Example – 04 :

A 20Ω resistor is connected in series with an inductor, a capacitor and an ammeter across a 25 –v, variable frequency supply. When the frequency is 400Hz, the current is at its Max^{m} value of 0.5 A and the potential difference across the capacitor is 150v. Calculate

(a) The capacitance of the capacitor.

(b) The resistance and inductance of the inductor.

Solution :

Since current is maximum, the circuit is in resonance.

$x_l = V_C/1 = 150/0.5 = 300\Omega$

(a) $x_l = 1/2\pi fe \Rightarrow 300 = 1/2\pi \times 400 \times c$

$\Rightarrow c = 1.325 \times 10^{-6} f = 1.325\mu f$.

(b) $x_l = x_l = 150/0.5 = 300\Omega$

$2\pi \times 400 \times L = 300$

$\Rightarrow L = 0.49H$

(c) At resonance,

Circuit resistance = 20+R

$\Rightarrow$ V/Z = 2510.5

$\Rightarrow$ R = 30Ω

Exp.-05

An R-L-C series circuits consists of a resistance of 1000Ω, an inductance of 100MH an a capacitance of wμ μf or 10PK

(ii) The half power points.

Solution :

i) $fo = \frac{1}{2\pi\sqrt{10^{-1} \times 10^{-4}}} = \frac{10^6}{2\pi} = 159KHz$

ii) $\phi = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{1000}\times\sqrt{\frac{10^{-1}}{10^{-11}}} = 100$

iii) $f_1 = fo - \frac{R}{4\pi l} = 159\times10^{3} - \frac{1000}{4\pi\times10^{-1}} = 158.2KHz$

$f_2 = fo - \frac{R}{4\pi l} = 159\times10^{-3} + \frac{1000}{4\pi\times10^{-1}} = 159.8KHz.$

Exp. -06

Calculate the impedance of the parallel –turned circuit as shown in fig. 14.52 at a frequency of 500 KHz and for band width of operation equal to 20 KHz. The resistance of the coil is 5Ω.

Solution :

At resonance, circuit impedance is L/CR. We have been given the value of R but that of L and C has to be found from the given the value of R but that of L and C has to be found from the given data.

$BW = \frac{R}{2\pi l}, 20\times10^{3} = \frac{5}{2\pi\times l} or\ l = 39\mu H$

$fo - \frac{1}{2\pi} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6}C} - \frac{5^2}{(39\times10^{-6})^2}}$

C = 2.6×10^{-9}

Z = L/CR = $39\times10^{-6} / 2.6\times10^{-9}\times5$

= $3\times10^{3}\Omega$

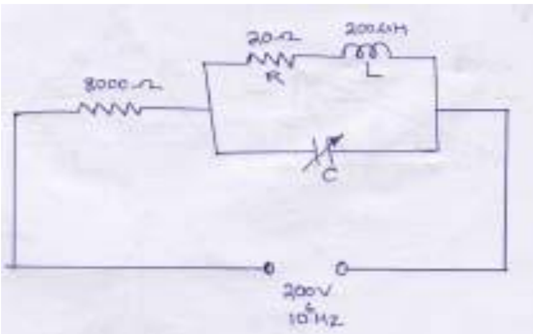
Example: A coil of resistance 20Ω and inductance of 200µH is in parallel with a variable capacitor. This combination is series with a resistor of 8000Ω.The voltage of the supply is 200V at a frequency of $10^6 H_Z$.Calculate

i) the value of C to give resonance

ii) the Q of the coil

iii) the current in each branch of the circuit at resonance

Solution:



$X_L = 2\pi fL = 2\pi * 10^6 * 200 * 10^{-6} = 1256\Omega$

The coil is negligible resistance in comparison to reactance.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$10^6 = \frac{1}{2\pi\sqrt{200 * C * 10^{-6}}}$$

ii) Q=$\frac{2\pi fL}{R} = 2\pi * 10^6 * 200 * \frac{10^{-4}}{20}$=62.8

iii) dynamic impedance of the circuit Z=L/CR=$200*10^{-6}/(125*10^{-12}*20)$=80000Ω

total Z=80000+8000=88000Ω

I=200/88000=2.27mA

p.d across tuned circuit=$2.27*10^{-3}*80000$=181.6V

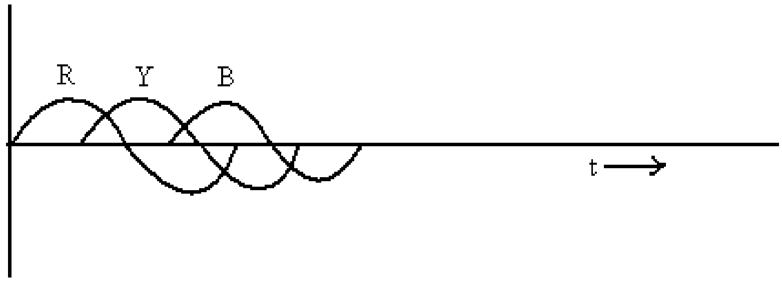
current through inductive branch=$\frac{181.6}{\sqrt{10^2+1256^2}} = 144.5mA$

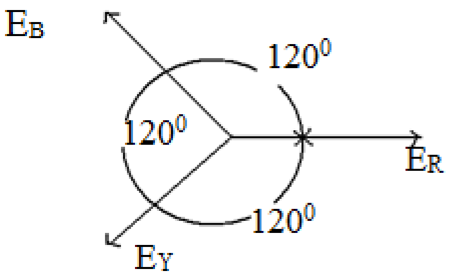
current through capacitor branch= ωVC

=$181.6*2\pi*10^6*125*10^{-12}$=142.7mA

POLY-PHASE CIRCUIT

Three-phase circuits consists of three windings i.e. R.Y.B





$E_R = E_m \sin wt = E_m \angle 0$

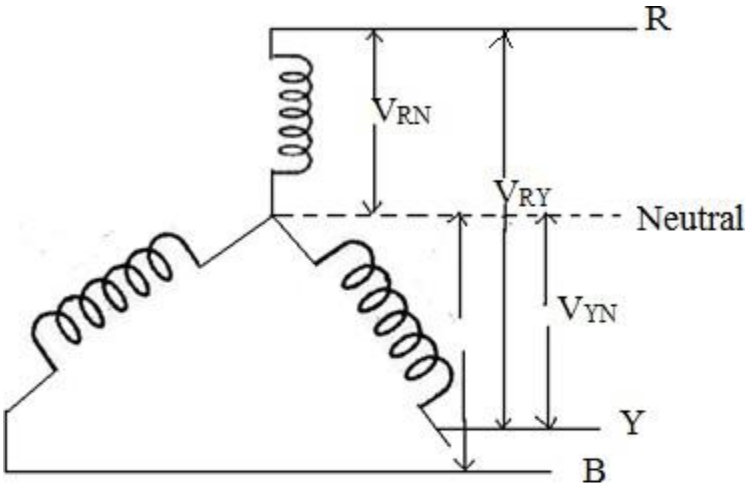
$E_Y = E_m \sin(wt - 120) = E_m \angle -120$

$E_B = E_m \sin(wt - 240) = E_m \angle -240 = E_m \angle 120$

3 - ϕ Circuit are divided into two types

- Star Connection
- Delta Connection

Star Connection: □



If three similar ends connected at one point, then it is known as star connected system.

The common point is known as neutral point and the wire taken from the neutral point is known as Neutral wire.

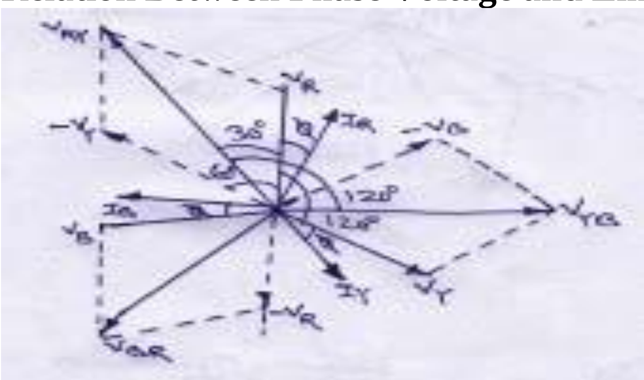
Phase Voltage :→

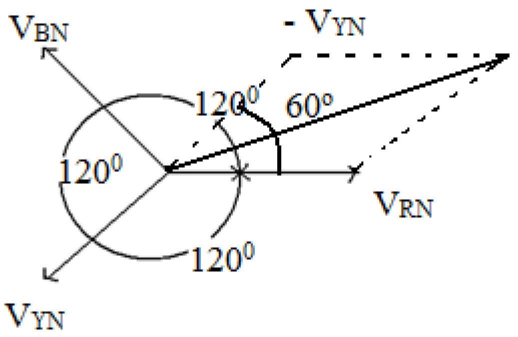
It is the potential difference between phase and Neutral.

Line Voltage : →

It is It is the potential difference between two phases.

Relation Between Phase Voltage and Line Voltage :→





Line Volatage $\vec{V}_{RY} = \vec{V}_{RN} - \vec{V}_{YN}$

$$V_L = \sqrt{V_{RN} + V_{YN} - 2V_{RN}V_{YN}Cos60^o}$$

$$= \sqrt{V_{Ph}^2 + V^2_{ph} - 2V_{ph}V_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3V_{Ph}^2} \quad = \sqrt{3}V_{Ph}$$

$$V_L = \sqrt{3}V_{Ph}$$

Since in a balanced B –phase circuit $V_{RN} = V_{YN} = V_{BN} = V_{ph}$

Relation Between Line current and Phase Current :-

In case of star connection system the leads are connected in series with each phase

Hence the line current is equal to phase current

$I_L = I_{ph}$

Power in 3- Phase circuit:-

$P = V_{ph} I_{ph} \cos\phi$ per phase

$= 3V_{ph} I_{ph} \cos\phi$ for 3 phase

$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi \; (\because V_L = \sqrt{3}V_{ph}$

$P = \sqrt{3}V_L I_L \cos\phi$

Summaries in star connection:

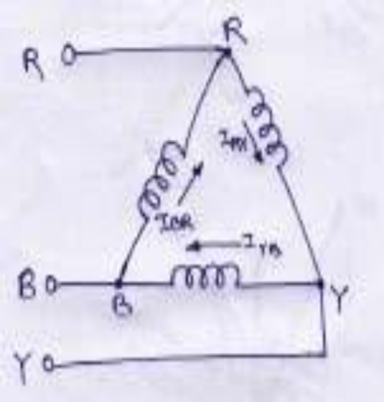
i) The line voltages are 120^0 apart from each other.

ii) Line voltages are 30^0 ahead of their respective phase voltage.

iii) The angle between line currents and the corresponding line voltage is 30+φ

iv) The current in line and phase are same.

Delta Connection :-



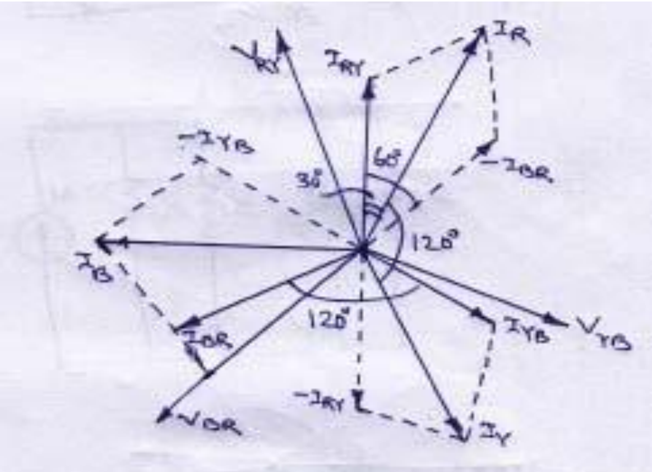
If the dissimilar ends of the closed mesh then it is called a Delta Connected system

Relation Between Line Current and Phase Current :-

Line Current in wire – 1 = $\vec{i}_R - \vec{i}_Y$

Line Current in wire -2 = $\vec{i}_Y - \vec{i}_B$

Line Current in wire – 3 = $\vec{i}_B - \vec{i}_R$



$$I_L = \vec{I}_R - \vec{I}_Y$$

$$= \sqrt{I_R^{\,2} + I_Y^{\,2} - 2 I_R I_Y \cos 60^0}$$

$$= \sqrt{I_{ph}^{\,2} + I_{ph}^{\,2} - 2 I_{ph} I_{ph} \times \frac{1}{2}}$$

$$= \sqrt{3 I_{ph}^{\,2}}, \; I_L = \sqrt{3 I_{ph}^{\,2}}$$

$$I_L = \sqrt{3} I_{ph}$$

Relation Between Line Voltage & Phase Voltage : →

$$V_L = V_{ph}$$

Power = $= \sqrt{3} V_L I_L \cos\phi$

Summaries in delta:

i) Line currents are 120^0 apart from each other.

ii) Line currents are 30^0 behind the respective phase current.

iii) The angle between the line currents and corresponding line voltages is 30+φ

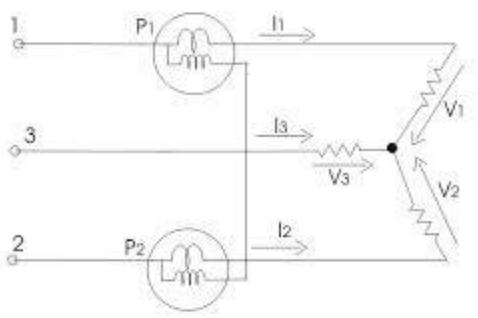
Measurement of Power : →

(1) By single watt-meter method

(2) By Two-watt meter Method

(3) By Three-watt meter Method

Measurement of power By Two Watt Meter Method :-



Phasor Diagram :-

Let V_R, V_Y,V_B are the r.m.s value of 3-ϕ voltages and I_R,I_Y,I_B are the r.m.s. values of the currents respectively.

Current in R-phase which flows through the current coil of watt-meter

$W_1 = I_R$

And $W_2 = I_Y$

Potential difference across the voltage coil of $W_1 = \vec{V}_{RB} = \vec{V}_R - \vec{V}_B$

And $W_2 = \vec{V}_{YB} = \vec{V}_Y - \vec{V}_B$

Assuming the load is inductive type watt-meter W_1 reads.

$W_1 = V_{RB} I_R \cos(30-\phi)$

$W_1 = V_L I_L \cos(30-\phi)$ --------------------------- (1)

Wattmeter W_2 reads

$W_2 = V_{YB} I_Y \cos(30+\phi)$

$W_2 = V_L I_L \cos(30+\phi)$ --------------------------- (2)

$W_1 + W_2 = V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$

$= V_L I_L [\cos(30-\phi) + V_L I_L \cos(30+\phi)]$

$= V_L I_L (2\cos 30^o \cos\phi)$

$= V_L I_L (2 \times \frac{\sqrt{3}}{2} \cos\phi)$

$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$------------------------------- (3)

$W_1 - W_2 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$

$$= V_L I_L (2 \sin 30^o \sin \phi)$$
$$= V_L I_L (2 \times \frac{1}{2} \times \sin \phi)$$
$$W_1 - W_2 = V_L I_L \sin\phi$$
$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos\phi}$$
$$\frac{1}{\sqrt{3}} = \tan\phi$$
$$\Rightarrow \tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$
$$\Rightarrow \phi = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Variation in wattmeter reading with respect to p.f:

Pf	W_1 reading	W_2 reading
φ=0,cos φ=1	+ve equal	+ve equal
φ=60,cos φ=0.5	0	+ve
φ=90,cos φ=0	-ve, equal	+ve equal

Exp. : 01

A balanced star – connected load of (8+56). Per phase is connected to a balanced 3-phase 100-v supply. Find the cone current power factor, power and total volt-amperes.

Solution :

$$Z_{ph} = \sqrt{8^2 + 6^2} = 10\Omega$$
$$V_{ph} = 400 / \sqrt{3} = 23 / v$$
$$I_{ph} = V_{ph} / Z_{ph} = 231/10 = 23.1A$$

i) $I_L = Z_{ph} = 23.1A$

ii) P.f. = cosθ = R_{ph}/z_{ph} = 8/10 = 0.8 (lag)

iii) $Power P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

= 12, 800 watt.

iv) Total volt ampere s =$\sqrt{3}$ V_L I_L

= $\sqrt{3} \times 400 \times 23.1$

= 16, 000 VA.

Exp. -02

Phase voltage and current of a star-connected inductive load is 150V and 25A. Power factor of load as 0.707 (Lag). Assuming that the system is 3-wire and power is measured using two watt meters, find the readings of watt meters.

Solution :

$V_{ph} = 150V$

$V_L = \sqrt{3} \times 150$

$I_{ph} = I_L = 25A$

Total power = $\sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707 = 7954$ watt.

$W_1 + W_2 = 7954.00$, $\cos\phi = 0.707$

$\phi = \cos^{-1}(0.707) = 45°$, $\tan 45° = 1$

Now for a lagging power factor,

$$\tan\phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$$

$$\Rightarrow 1 = \sqrt{3}\left\{\frac{(W_1 - W_2)}{7954}\right\}$$

$$\therefore (W_1 - W_2) = 4592w$$

From (i) and (ii) above, we get

$W_1 = 6273w$ $W_2 = 1681w$

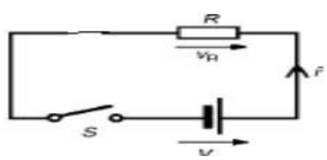
TRANSIENTS

Introduction:

For higher order differential equation, the number of arbitrary constants equals the order of the equation. If these unknowns are to be evaluated for particular solution, other conditions in network must be known. A set of simultaneous equations must be formed containing general solution and some other equations to match number of unknown with equations.

We assume that at reference time t=0, network condition is changed by switching action. Assume that switch operates in zero time. The network conditions at this instant are called initial conditions in network.

1. Resistor:



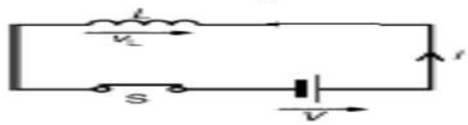
$$V = i \times R \longrightarrow (1)$$

Eq.1 is linear and also time dependent. This indicates that current through resistor changes if applied voltage changes instantaneously. Thus in resistor, change in current is instantaneous as there is no storage of energy in it.

2. Inductor:

If dc current flows through inductor, dil/dt becomes zero as dc current is constant with respect to time. Hence voltage across inductor, VL becomes zero. Thus, as for as dc quantities are considered, in steady stake, inductor acts as short circuit

$$V_L = L \bullet \frac{diL}{dt} \longrightarrow (2)$$



$$iL = \frac{1}{L}\int V_L\,dt$$

In above eqn. The limits of integration is from $-\infty$ to t

Assuming that switching takes place at t=0, we can split limits into two intervals as $-\infty$ to t

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L\,dt$$

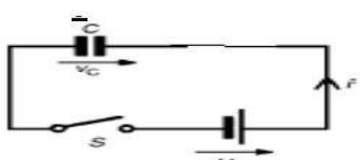
$$i_L = \frac{1}{L}\int_{-\infty}^{0^-} V_L\,dt + \frac{1}{L}\int_{0^-}^{t} V_L\,dt$$

$$i_L = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} V_L\,dt$$

at $t = 0^+$ we can write $i_L(0^+) = i_L(0^-) + \frac{1}{L}\int V_L\,dt$

$i_L(0^+) = i_L(0^-)$

3. Capacitor:



$$i_C = C\frac{dV_C}{dt}$$

If dc voltage is applied to capacitor, dVC / dt becomes zero as dc voltage is constant with respect to time.

Hence the current through capacitor iC becomes zero, Thus as far as dc quantities are considered capacitor acts as open circuit.

$$V_C = \frac{1}{C}\int i_C\,dt$$

$$V_C = \frac{1}{C}\int_{-\infty}^{t} i_C\,dt$$

Splitting limits of integration

$$V_C = \frac{1}{C}\int_{-\infty}^{0-} i_C\,dt + \frac{1}{C}\int_{0-}^{t} i_C\,dt$$

At $t(0^+)$, equation is given by

$$V_C(0^+) = V_C(0^-) + \frac{1}{C}\int_{0-}^{0+} i_C\,dt$$

$$V_C(0^+) = V_C(0^-)$$

Thus voltage across capacitor cannot change instantaneously.

Initial Condition for (DC steady state solution)

- Initial condition: response of a circuit before a switch is first activated.

– Since power equals energy per unit time, finite power requires continuous change in energy.

• Primary variables: capacitor voltages and inductor currents-> energy storage elements

$$W_L(t) = \frac{1}{2} L i_L^2(t) \quad W_C(t) = \frac{1}{2} C v_C^2(t)$$

- Capacitor voltages and inductor currents cannot change instantaneously but

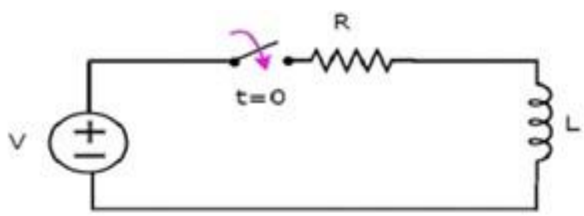
Should be continuous -> Continuity of capacitor voltages and inductor currents

- The value of an inductor current or a capacitor voltage just prior to the closing (or opening) of a switch is equal to the value just after the switch has been closed (or opened).

$$v_C(t = 0^-) = v_C(t = 0^+)$$
$$i_L(t = 0^-) = i_L(t = 0^+)$$

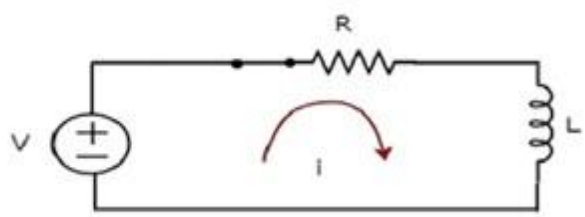
1. TRANSIENT RESPONSE OF RL CIRCUITS:

Consider the following series RL circuit given below



In the above circuit, the **switch** was kept **open** up to t = 0 and it was closed at t = 0. So, the DC voltage source having V volts is not connected to the series RL circuit up to this instant. Therefore, there is **no initial current** flows through inductor.

The circuit diagram, when the **switch** is in **closed** position is shown in the following figure.



Now, the current ***i*** flows in the entire circuit, since the DC voltage source having ***V*** volts is connected to the series RL circuit.

Now, apply **KVL** around the loop.

$$V = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{V}{L} \qquad \textbf{Equation 1}$$

The above equation is a first order differential equation and it is in the form of

$$\frac{dy}{dt} + Py = Q \qquad \textbf{Equation 2}$$

By **comparing** Equation 1 and Equation 2, we will get the following relations.

$$x = t$$

$$y = i$$

$$P = \frac{R}{L}$$

$$Q = \frac{V}{L}$$

The **solution** of Equation 2 will be

$$ye^{\int p dx} = \int Qe^{\int p dx}\, dx + k \qquad \textbf{Equation 3}$$

Where, **k** is the constant.

Substitute, the values of x, y, P & Q in Equation 3.

$$ie^{\int \left(\frac{R}{L}\right)dt} = \int \left(\frac{V}{L}\right)\left(e^{\int \left(\frac{R}{L}\right)dt}\right)dt + k$$

$$\Rightarrow ie^{\left(\frac{R}{L}\right)t} = \frac{V}{L}\int e^{\left(\frac{R}{L}\right)t}dt + k$$

$$\Rightarrow ie^{\left(\frac{R}{L}\right)t} = \frac{V}{L}\left\{\frac{e^{\left(\frac{R}{L}\right)t}}{\frac{R}{L}}\right\} + k$$

$$\Rightarrow i = \frac{V}{R} + ke^{-\left(\frac{R}{L}\right)}t \qquad \textbf{Equation 4}$$

We know that there is no initial current in the circuit. Hence, substitute, $t = 0$ and $i = 0$ in Equation 4 in order to find the value of the constant ***k***.

$$0 = \frac{V}{R} + ke^{-\left(\frac{R}{L}\right)(0)}$$

$$0 = \frac{V}{R} + k(1)$$

$$k = -\frac{V}{R}$$

Substitute, the value of k in Equation 4.

$$i = \frac{V}{R} + \left(-\frac{V}{R}\right)e^{-\left(\frac{R}{L}\right)t}$$

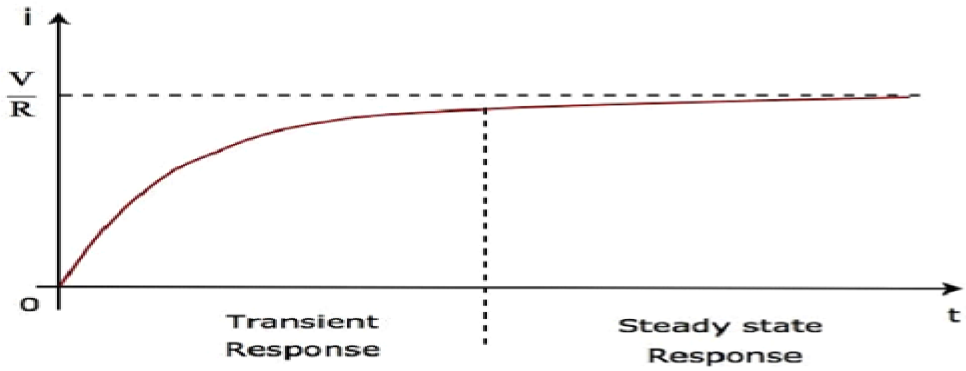
$$i = \frac{V}{R} - \frac{V}{R}e^{-\left(\frac{R}{L}\right)t}$$

Therefore, the **current** flowing through the circuit is

CI $$i = -\frac{V}{R}e^{-\left(\frac{R}{L}\right)t} + \frac{V}{R} \qquad \textbf{Equation 5}$$

So, the response of the series RL circuit, when it is excited by a DC voltage source, has the following two terms.

- The first term $-\frac{V}{R}e^{-\left(\frac{R}{L}\right)t}$ corresponds with the **transient response.**
- The second term $\frac{V}{R}$ corresponds with the **steady state response.** These two responses are shown in the following figure.



We can re-write the Equation 5 as follows −

$$i = \frac{V}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

$$\Rightarrow i = \frac{V}{R}\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right) \qquad \textbf{Equation 6}$$

Where, τ is the **time constant** and its value is equal to $\frac{L}{R}$.

Both Equation 5 and Equation 6 are same. But, we can easily understand the above waveform of current flowing through the circuit from Equation 6 by substituting a few values of **t** like 0, τ, 2τ, 5τ, etc.

In the above waveform of current flowing through the circuit, the transient response will present up to five time constants from zero, whereas the steady state response will present from five time constants onwards.

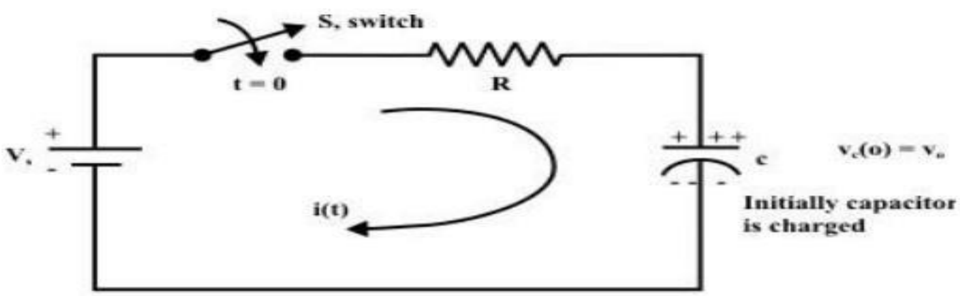
Actual time (t) in sec	Growth of current in inductor (Eq.10.15)
$t=0$	$i(0)=0$
$t=\tau\left(=\frac{L}{R}\right)$	$i(\tau)= 0.632 \times \frac{V_s}{R}$
$t=2\tau$	$i(2\tau)= 0.865 \times \frac{V_s}{R}$
$t=3\tau$	$i(3\tau)= 0.950 \times \frac{V_s}{R}$
$t=4\tau$	$i(4\tau)= 0.982 \times \frac{V_s}{R}$
$t=5\tau$	$i(5\tau)= 0.993 \times \frac{V_s}{R}$

2. TRANSIENT RESPONSE OF RC CIRCUITS:

Ideal and real capacitors: An ideal capacitor has an infinite dielectric resistance and plates (made of metals) that have zero resistance. However, an ideal capacitor does not exist as all dielectrics have some leakage current and all capacitor plates have some resistance. A capacitor's of how much charge (current) it will allow to leak through the dielectric medium. Ideally, a charged capacitor is not supposed to allow

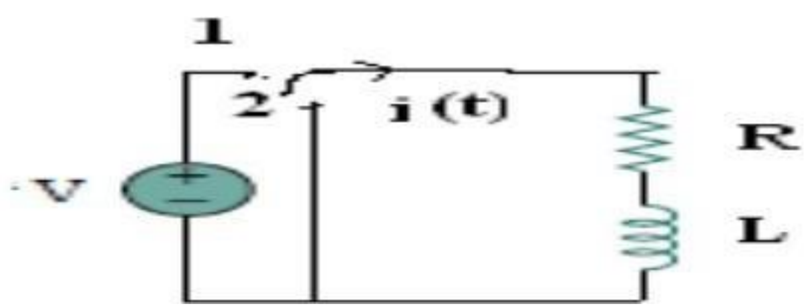
leaking any current through the dielectric medium and also assumed not to dissipate any power loss in capacitor plate's resistance. Under this situation, the model as shown in fig. 10.16(a) represents the ideal capacitor. However, all real or practical capacitor leaks current to some extend due to leakage resistance of dielectric medium. This leakage resistance can be visualized as a resistance connected in parallel with the capacitor and power loss in capacitor plates can be realized with a resistance connected in series with capacitor. The model of a real capacitor is shown in fig.

Let us consider a simple series RC−circuit shown in fig. 10.17(a) is connected through a switch 'S' to a constant voltage source .



The switch 'S' is closed at time t =0' It is assumed that the capacitor is initially charged with a voltage and the current flowing through the circuit at any instant of time '' after closing the switch is

3. Current decay in source free series RL circuit:



t = 0- , , switch k is kept at position 'a' for very long time. Thus, the network is in steady state. Initial current through inductor is given as,

$$i_L(0^-) = I_0 = \frac{V}{R} = i_L(0^+) \text{ -------------------------- } 1$$

Because current through inductor cannot change instantaneously

Assume that at t = 0 switch k is moved to position 'b'.
Applying KVL,

$$L\frac{di}{dt} + iR = 0 \quad \text{-----------------------------} \; 2$$

$$\therefore L\frac{di}{dt} = -iR$$

Rearranging the terms in above equation by separating variables

$$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides with respect to corresponding variables

$$\therefore \ln(i) = -\frac{R}{L} \quad \text{--------------------------} \; 3$$

Where k’ is constant of integration.

To find- k’:

Form equation 1, at t=0, i=I0

Substituting the values in equation 3

Where k’ is constant of integration.

To find- k’:Form equation 1, at t=0, i=I0
Substituting the values in eq

$$\ln(i) = -\frac{R}{L}t + \ln(I_0)$$

$$\ln(i) - \ln(I_0) = -\frac{R}{L}t$$

$$\frac{i}{i_0} = e^{-\frac{R}{L}t}$$

$$\therefore i = I_0 \cdot e^{-\frac{R}{L}t} \quad \text{-----------------------} 5$$

From the graph, H is clear that current is exponentially decaying. At point P on graph. The current value is (0.363) times its maximum value. The characteristics of decay are determined by values R and L which are two parameters of network.

The voltage across inductor is given by

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left[I_0 \cdot e^{-\frac{R}{L}t}\right] = L \cdot I_0\left(-\frac{R}{L}\right) \cdot e^{-\frac{R}{L}t}$$

$$\therefore V_L = -I_0 \cdot \mathrm{Re}^{-\frac{R}{L}t}$$

But $I_0 \cdot R = V$

$$\therefore V_L = -V \cdot e^{-\frac{R}{L}t} \text{ Volts}$$

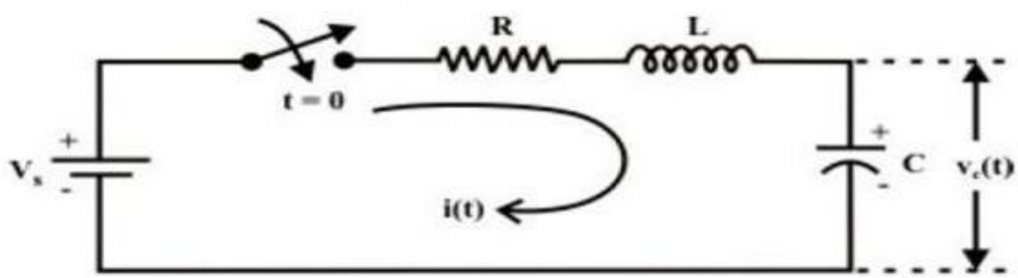
4. TRANSIENT RESPONSE OF RLC CIRCUITS

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor or capacitor (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.

5. **Response of a series R-L-C circuit:**

   Consider a series RL circuit as shown in fig.11.1, and it is excited with a dc voltage source C−−sV.
   Applying around the closed path for,

$$L\frac{di(t)}{dt} + R\,i(t) + v_c(t) = V_s$$



The current through the capacitor can be written as Substituting the current ‘’expression in eq.(11.1) and rearranging the terms,

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

The above equation is a 2nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

$$v_c(t)=v_{cn}(t)+v_{cf}(t) = \left(A_1\, e^{\alpha_1 t} + A_2\, e^{\alpha_2 t}\right)+A$$

$$LC\frac{d^2v_c(t)}{dt^2}+RC\frac{dv_c(t)}{dt}+v_c(t) = 0 \Rightarrow \frac{d^2v_c(t)}{dt^2}+\frac{R}{L}\frac{dv_c(t)}{dt}+\frac{1}{LC}v_c(t) = 0$$

$$a\,\frac{d^2v_c(t)}{dt^2}+b\,\frac{dv_c(t)}{dt}+c\;v_c(t) = 0\;\;(\text{where } a=1,\, b=\frac{R}{L} \text{ and } c=\frac{1}{LC})$$

Since the system is linear, the nature of steady state response is same as that of forcing function (input voltage) and it is given by a constant value. Now, the first part of the total response is completely dies out with time while and it is defined as a transient or natural response of the system. The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$\alpha^2+\frac{R}{L}\alpha+\frac{1}{LC}=0 \quad \Rightarrow a\,\alpha^2+b\,\alpha+c=0\;\;(\text{where } a=1,\, b=\frac{R}{L} \text{ and } c=\frac{1}{LC})$$

And solving the roots of this equation (11.5) on that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$\alpha_1=\left(-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)=\left(-\frac{b}{2a}+\frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2-ac}\right);$$

$$\alpha_2=\left(-\frac{R}{2L}-\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\right)=\left(-\frac{b}{2a}-\frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2-ac}\right)$$

where, $b=\frac{R}{L}$ and $c=\frac{1}{LC}$.

The roots of the characteristic equation are classified in three groups depending upon the values of the parameters, Rand of the circuit Case-A (over damped response): That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$v_{cn}(t) = A_1\, e^{\alpha_1 t} + A_2\, e^{\alpha_2 t}$$

and each term of the above expression decays exponentially and ultimately reduces to zero as and it is termed as over damped response of input free system. A system that is over damped responds slowly to any change in excitation. It may be noted that the exponential term t→∞11tAeαtakes longer time to decay its value to zero than the term21tAeα. One can introduce a factor ξ that provides information about the speed of system response and it is defined by damping ratio

$$(\xi)=\frac{Actual\ damping}{critical\ damping}=\frac{b}{2\sqrt{ac}}=\frac{R/L}{2/\sqrt{LC}}>1$$

TWO PORT NETWORK

Introduction:

A pair of terminals through which a current may enter or leave a network is known as a *port*. Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks. Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in Figure 2(a). We have considered the voltage across or current through a single pair of terminals—such as the two terminals of a resistor, a capacitor, or an inductor. We have also studied four-terminal or two-port circuits involving op amps, transistors, and transformers, as shown in Figure 2(b). In general, a network may have *n* ports. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

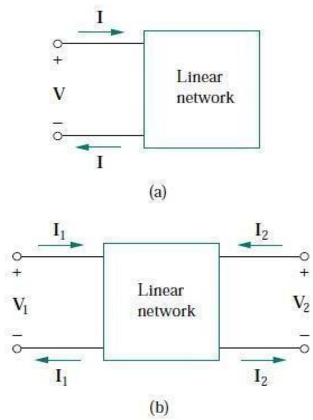
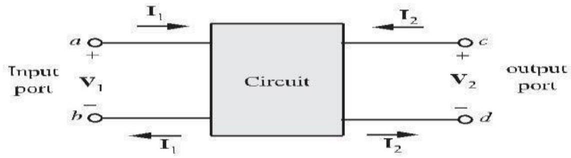


Figure 2: (a) One-port network, (b) two-port network.

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. There are several reasons why we should study two-ports and the parameters that describe them. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describe its behavior in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-port parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other

IMPEDANCE PARAMETERS:



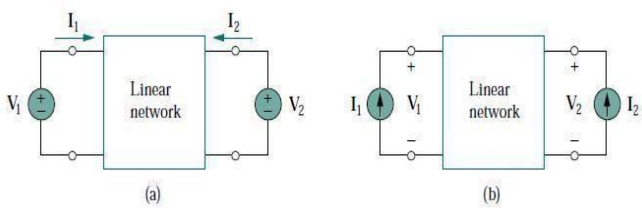
Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. We discuss impedance parameters in this section and admittance parameters in the next section.

A two-port network may be voltage-driven as in Figure 3 (a) or current-driven as in Figure 3(b). From either Figure 3(a) or (b), the terminal voltages can be related to the terminal currents as

$$V1=Z_{11}I_1+Z_{12}I_2$$

$$V2=Z_{21}I_1+Z_{22}I_2$$

Where the z terms are called the impedance parameters, or simply z parameters, and have units of ohms.



The values of the parameters can be evaluated by setting $\mathbf{I}_1 = 0$ (input port open-circuited) or $\mathbf{I}_2 = 0$ (output port open-circuited).

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the open-circuit impedance parameters. Specifically,

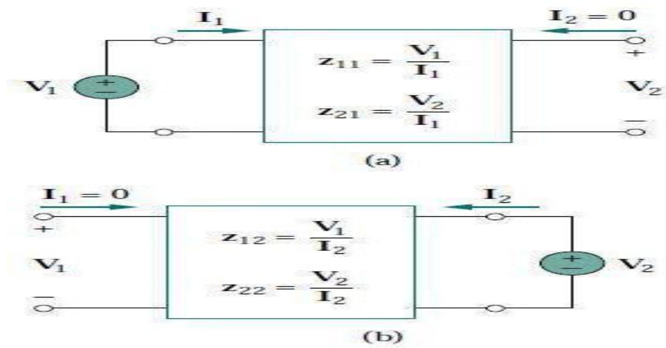
$\mathbf{z}_{11}$ = Open-circuit input impedance

$\mathbf{z}_{12}$ = Open-circuit transfer impedance from port 1 to port 2 $\mathbf{z}_{21}$

= Open-circuit transfer impedance from port 2 to port 1 $\mathbf{z}_{22}$ =

Open-circuit output impedance

We obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ by connecting a voltage $\mathbf{V}_1$ (or a current source $\mathbf{I}_1$) to port 1 with port 2 open-circuited as in Figure 4 and finding $\mathbf{I}_1$ and $\mathbf{V}_2$; we then get



Determination of the *z* parameters: (a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$ (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

$Z_{11}=V_1/I_1$, $Z_{21}=V_2/I_1$

We obtain $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$ by connecting a voltage $\mathbf{V}_2$ (or a current source $\mathbf{I}_2$) to port 2 with port 1 open-circuited as in Figure 4) and finding $\mathbf{I}_2$ and $\mathbf{V}_1$; we then get

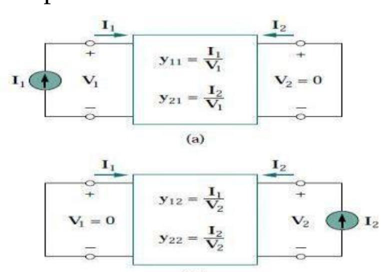
$Z_{12}=V_1/I_2$, $Z_{21}=V_2/I_2$

The above procedure provides us with a means of calculating or measuring the *z* parameters. Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called driving-point impedances, while $\mathbf{z}_{21}$ and $\mathbf{z}_{12}$ are called *transfer impedances*. A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, $\mathbf{z}_{11}$ is the input driving-point impedance with the output port open-circuited, while $\mathbf{z}_{22}$ is the output driving-point impedance with the input port open circuited.

When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be symmetrical. This implies that the network has mirror like symmetry about some center line; that is, a line can be found that divides the network into two similar halves. When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($\mathbf{z}_{12} = \mathbf{z}_{21}$), and the two-port is said to be reciprocal. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same. A two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

ADMITTANCE PARAMETERS:

In the previous section we saw that impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Figure 5(a) or (b), the terminal currents can be expressed in terms of the terminal voltages as



Determination of the *y* parameters: (a) finding y_{11} and y_{21}, (b) finding y_{12} and y_{22}.

$$I_1=Y_{11}V_1+Y_{12}V_2$$

$$I_2=Y_{21}V_1+Y_{22}V_2$$

The **Y** terms are known as the admittance parameters (or, simply, y parameters) and have units of Siemens

The values of the parameters can be determined by setting **V**1 = 0 (input port short-circuited) or **V**2 = 0 (output port short-circuited). Thus,

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$
$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

Since the *y* parameters are obtained by short-circuiting the input or output port, they are also called the short-circuit admittance parameters. Specifically,

$\mathbf{y}_{11}$ = Short-circuit input admittance

$\mathbf{y}_{12}$ = Short-circuit transfer admittance from port 2 to port 1

$\mathbf{y}_{21}$ = Short-circuit transfer admittance from port 1 to port 2

$\mathbf{y}_{22}$ = Short-circuit output admittance

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by connecting a current $\mathbf{I}_1$ to port 1 and short-circuiting port 2 and finding V1And $\mathbf{I}_2$.

Similarly, we obtain $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$ by connecting a current source $\mathbf{I}_2$ to port 2 and short-circuiting port 1 and finding $\mathbf{I}_1$ and $\mathbf{V}_2$, and then getting

This procedure provides us with a means of calculating or measuring the *y* parameters. The impedance and admittance parameters are collectively referred to as admittance parameters

HYBRID PARAMETERS:

The *z* and *y* parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. This third set of parameters is based on making **V**1 and **I**2 the dependent variables. Thus, we obtain

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

Or in matrix form,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

The **h** terms are known as the hybrid parameters (or, simply, h parameters) because they are a hybrid Combination of ratios. They are very useful for describing electronic devices such as transistors; it is much easier to measure experimentally the *h* parameters of such devices than to measure their *z* or *y* parameters. The hybrid parameters are as follows.

It is evident that the parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ represent impedance, a voltage gain, a current gain, and admittance, respectively. This is why they are called the hybrid parameters. To be specific,

$\mathbf{h}_{11}$ = Short-circuit input impedance

$\mathbf{h}_{12}$ = Open-circuit reverse voltage gain

$\mathbf{h}_{21}$ = Short-circuit forward current gain

$\mathbf{h}_{22}$ = Open-circuit output admittance

The procedure for calculating the *h* parameters is similar to that used for the *z* or *y* parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis.

TRANSMISSION PARAMETERS:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port.

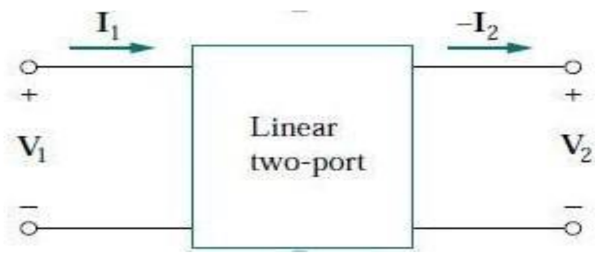
$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

Thus,

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

The above Equations are relating the input variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) to the output variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). Notice that in computing the transmission parameters, $-\mathbf{I}_2$ is used rather than $\mathbf{I}_2$, because the current is considered to be

leaving the network, as shown in Figure 6. This is done merely for conventional reasons; when you cascade two-ports (output to input), it is most logical to think of $\mathbf{I}_2$ as leaving the two-port. It is also customary in the power − industry to consider $\mathbf{I}_2$ as leaving the two-port.



Terminal variables used to define the **ABCD** parameters.

The two-port parameters in above equations provide a measure of how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables ($\mathbf{V}_1$ and $\mathbf{I}_1$) in terms of the receiving-end variables ($\mathbf{V}_2$ and $-\mathbf{I}_2$). For this reason, they are called transmission parameters. They are also known as **ABCD** parameters. They are used in the design of telephone systems, microwave networks, and radars.

The transmission parameters are determined as

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{B} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

$$\mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{D} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$$

Thus, the transmission parameters are called, specifically,

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

A and **D** are dimensionless, **B** is in ohms, and **C** is in Siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

Condition of symmetry:

A two port network is said to be symmetrical if the ports can be interchanged without port voltages and currents

Condition of reciprocity:

A two port network is said to be reciprocal, if the rate of excitation to response is invariant to an interchange of the position of the excitation and response in the network. Network containing resistors, capacitors and inductors are generally reciprocal

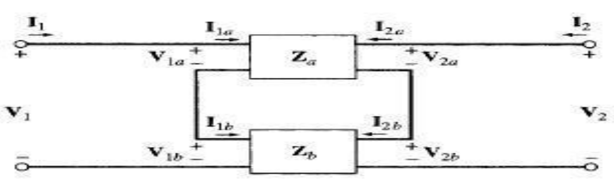
Condition for reciprocity and symmetry in two port parameters:

In Z parameters a network is termed to be reciprocal if the ratio of the response to the excitation remains unchanged even if the positions of the response as well as the excitation are interchanged.
A two port network is said to be symmetrical it the input and the output port can be interchanged without altering the port voltages or currents.

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{22} = Z_{11}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h	$h_{11}\ h_{21}$	$h_{11}\ h_{21}$
ABCD	**AD - BC = 1**	**A = D**

Interconnecting Two-Port Networks:

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connection, resulting in new two-port networks. For each configuration, certain set of parameters may be more useful than others to describe the network. A series connection of two two-port networks a and b with open-circuit impedance parameters Za and Zb, respectively. In this configuration, we use the Z-parameters since they are combined as a series connection of two impedances.

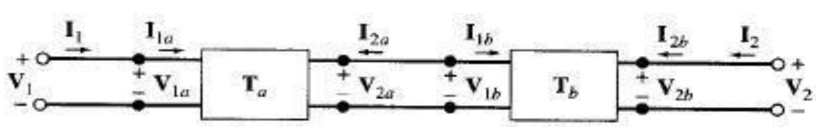


The Z-parameters of the series connection are Z 11= Z11A + Z11B
Or in the matrix form [Z]=[ZA]+[ZB]

Parallel Connection

[Y] = [YA] + [YB]

Cascade Connection



RELATIONSHIPS BETWEEN PARAMETERS:

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated. If two sets of parameters exist, we can relate one set to the other set. Let us demonstrate the process with two examples.

Given the *z* parameters, let us obtain the *y* parameters.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}]^{-1} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

	z		y		h		g	
z	$\mathbf{z}_{11}$	$\mathbf{z}_{12}$	$\frac{\mathbf{y}_{22}}{\Delta_y}$	$-\frac{\mathbf{y}_{12}}{\Delta_y}$	$\frac{\Delta_h}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{g}_{11}}$	$-\frac{\mathbf{g}_{12}}{\mathbf{g}_{11}}$
	$\mathbf{z}_{21}$	$\mathbf{z}_{22}$	$-\frac{\mathbf{y}_{21}}{\Delta_y}$	$\frac{\mathbf{y}_{11}}{\Delta_y}$	$-\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$	$\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{11}}$
y	$\frac{\mathbf{z}_{22}}{\Delta_z}$	$-\frac{\mathbf{z}_{12}}{\Delta_z}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$-\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}$	$\frac{\Delta_g}{\mathbf{g}_{22}}$	$\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$
	$-\frac{\mathbf{z}_{21}}{\Delta_z}$	$\frac{\mathbf{z}_{11}}{\Delta_z}$	$\mathbf{y}_{21}$	$\mathbf{y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{\Delta_h}{\mathbf{h}_{11}}$	$-\frac{\mathbf{g}_{21}}{\mathbf{g}_{22}}$	$\frac{1}{\mathbf{g}_{22}}$
h	$\frac{\Delta_z}{\mathbf{z}_{22}}$	$\frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{y}_{11}}$	$-\frac{\mathbf{y}_{12}}{\mathbf{y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$	$\frac{\mathbf{g}_{22}}{\Delta_g}$	$-\frac{\mathbf{g}_{12}}{\Delta_g}$
	$-\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{z}_{22}}$	$\frac{\mathbf{y}_{21}}{\mathbf{y}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$	$-\frac{\mathbf{g}_{21}}{\Delta_g}$	$\frac{\mathbf{g}_{11}}{\Delta_g}$
g	$\frac{1}{\mathbf{z}_{11}}$	$-\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$	$\frac{\Delta_y}{\mathbf{y}_{22}}$	$\frac{\mathbf{y}_{12}}{\mathbf{y}_{22}}$	$\frac{\mathbf{h}_{22}}{\Delta_h}$	$-\frac{\mathbf{h}_{12}}{\Delta_h}$	$\mathbf{g}_{11}$	$\mathbf{g}_{12}$
	$\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}$	$\frac{\Delta_z}{\mathbf{z}_{11}}$	$-\frac{\mathbf{y}_{21}}{\mathbf{y}_{22}}$	$\frac{1}{\mathbf{y}_{22}}$	$-\frac{\mathbf{h}_{21}}{\Delta_h}$	$\frac{\mathbf{h}_{11}}{\Delta_h}$	$\mathbf{g}_{21}$	$\mathbf{g}_{22}$

FILTERS

PASSIVE FILTERS:

Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, Hv (jω) = Vo/Vi. As Hv (jω) is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals. To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the ‖open-loop‖ transfer functions, Hvo, and denote this simply by H(jω).

Low-Pass Filters:

An ideal low-pass filter‘s transfer function is shown. The frequency between the pass- and-stop bands is called the cut-off frequency (ωc). All of the signals with frequencies below ωc are transmitted and all other signals are stopped.

In practical filters, pass and stop bands are not clearly defined, |H(jω)| varies continuously from its maximum toward zero. The cut-off frequency is, therefore, defined as the frequency at which |H(jω)| is reduced to 1/√2 =0.7 of its maximum value. This corresponds to signal power being reduced by 1/2 as P a V 2.

Band-pass filters:

A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

Constant – K Low Pass Filter:

A network, either *T* or \[\pi\], is said to be of the constant-*k* type if Z_1 and Z_2 of the network satisfy the relation

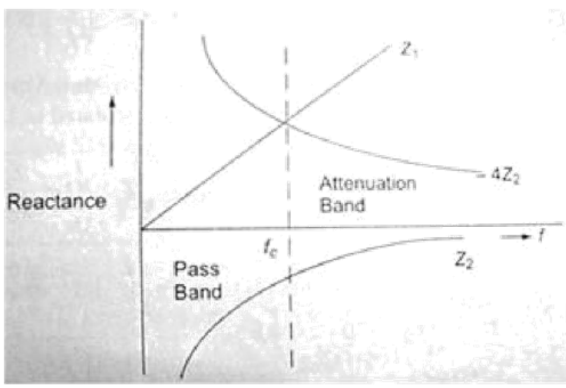
$$Z_1 Z_2 = k^2$$

Where Z_1 and Z_2 are impedance in the T and [pi] sections as shown in Fig. Equation 17.20 states that Z_1 and Z_2 often termed as design impedance or nominal impedance of the constant *k*-filter.

The constant *k*, *T* or π type filter is also known as the prototype because other more complex networks can be derived from it. Where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1 Z_2 = {L \over C} = {k^2}$ which is independent of frequency

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves Z_1 and $4Z_2$ is indicated as f_c. On the X-axis as

$Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1 = -4Z_2$.



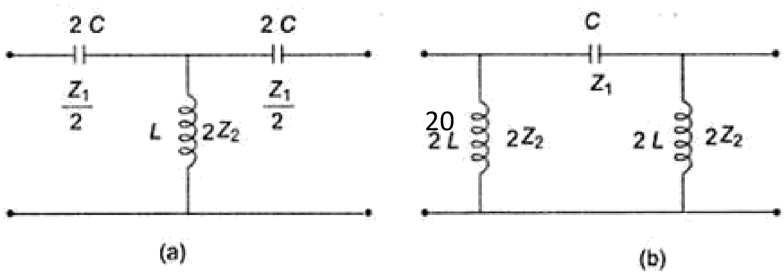
All the frequencies above f_c lie in a stop or attenuation band

The characteristic impedance of a π-network is given by

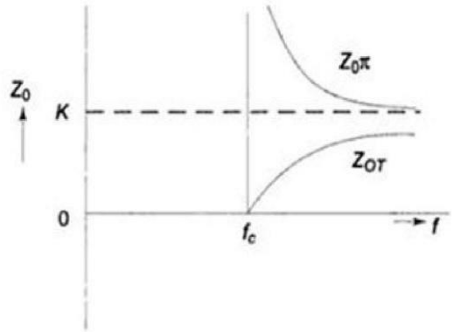
$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30.5)$$

Constant K-High Pass Filter:

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.

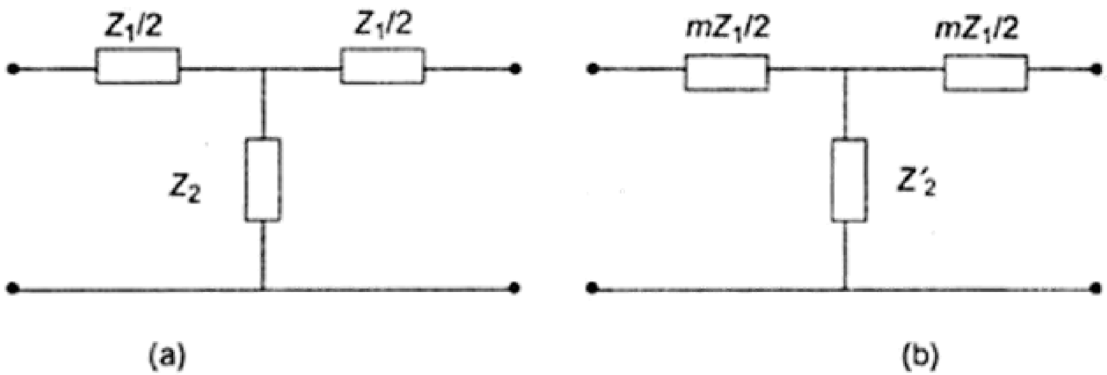

2 C
2 C
$\frac{Z_1}{2}$
L
$2Z_2$
$\frac{Z_1}{2}$
(a)
C
Z_1
20
2 L
$2Z_2$
2 L
$2Z_2$
(b)

Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. The plot of characteristic impedance with respect to frequency is shown



m-Derived T-Section:

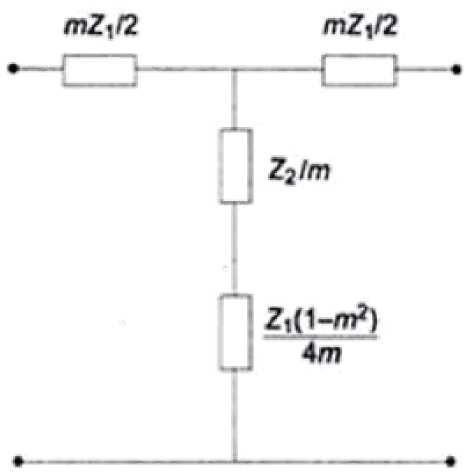
It is clear from previous chapter Figs 30.3 & 30.7 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of ‗a'in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in us has



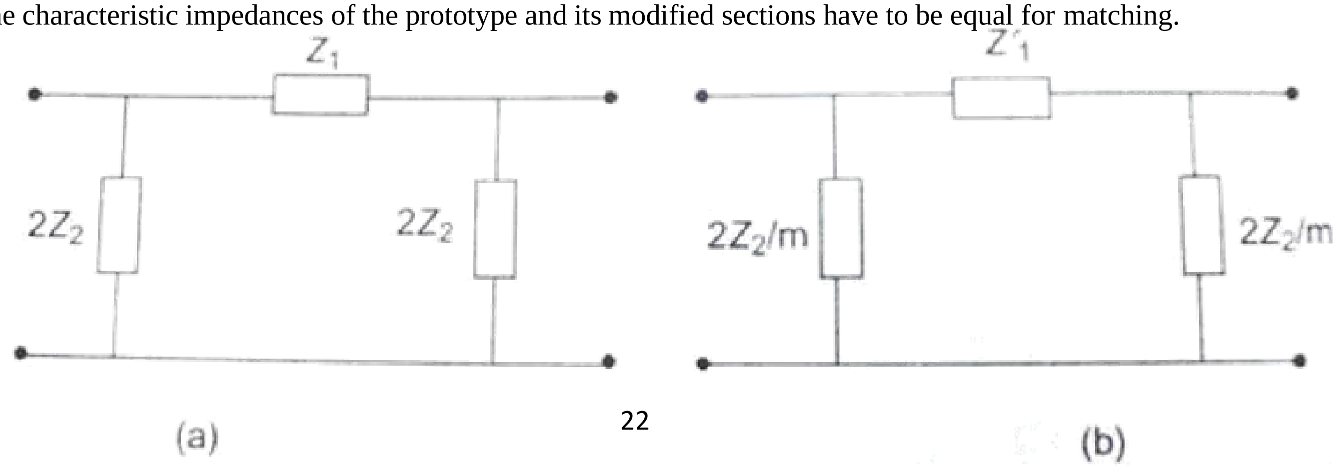
$Z_{0T} = Z_{0T}$

Where $Z_{0T'}$ is the characteristic impedance of the modified (m-derived) T-network.

Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to \[\pi\] section network. Suppose a prototype p-network shown in Fig.31.3 (a) has the shunt arm modified as shown in Fig.31.3 (b).



The characteristic impedances of the prototype and its modified sections have to be equal for matching.



The characteristic impedance of the modified (m-derived) \[\pi\]-network

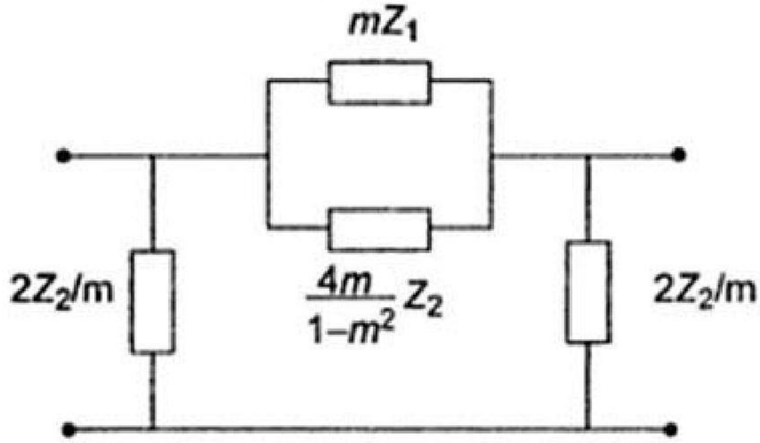
$$\therefore \qquad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1' \frac{Z_2}{m}}{1+\frac{Z_1'}{4.Z_2/m}}}$$

Or

$$Z_1' = \frac{Z_1 Z_2}{\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{mZ_1}{4}}$$

$$= \frac{Z_1 Z_2}{\frac{Z_2}{m} + \frac{Z_1}{4m}\left(1-m^2\right)}$$

$$Z_1' = \frac{Z_1 Z_2 \frac{4m^2}{\left(1-m^2\right)}}{\frac{Z_2 4m_2}{m\left(1-m_2\right)} + Z_1 m} = \frac{mZ_1 \frac{Z_2 4m}{\left(1-m^2\right)}}{mZ_1 + \frac{Z_2 4m}{\left(1-m^2\right)}} \quad \text{.......................................(31.2)}$$

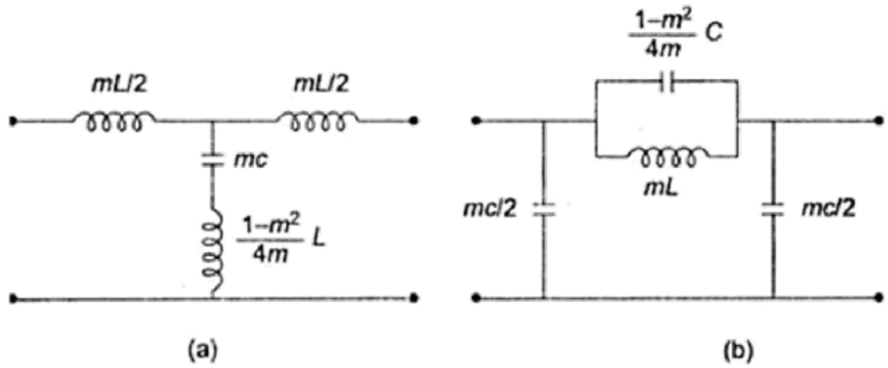


The series arm of the m-derived \[\pi\] section is a parallel combination of mZ_1 and $4mZ_2/1\text{-}m^2$

m-Derived Low Pass Filter:

In Fig.31.5, both m-derived low pass T and \[\pi\] filter sections are shown. For the T-section shown Fig.31.5

(a) The shunt arm is to be chosen so that it is resonant at some frequency f_X above cut-off frequency f_C its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency.



$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

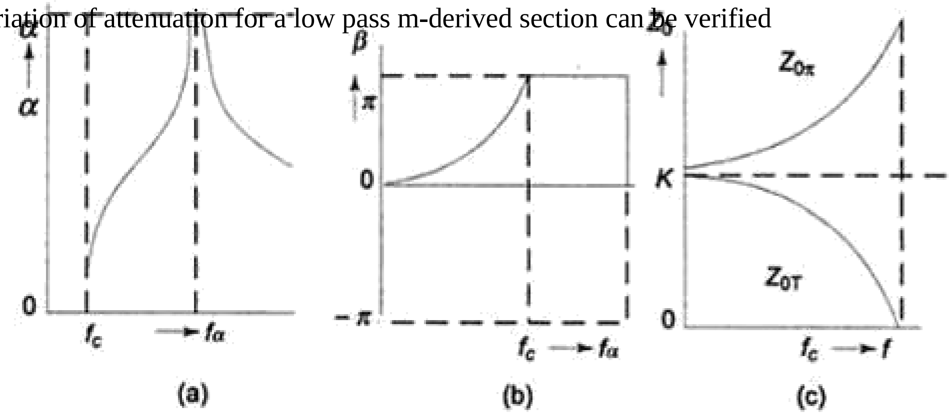
$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_\alpha}\right)^2}}$$

And $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\dfrac{f}{f_c}}{\sqrt{1-\left(\dfrac{f}{f_c}\right)^2(1=m)^2}}$$

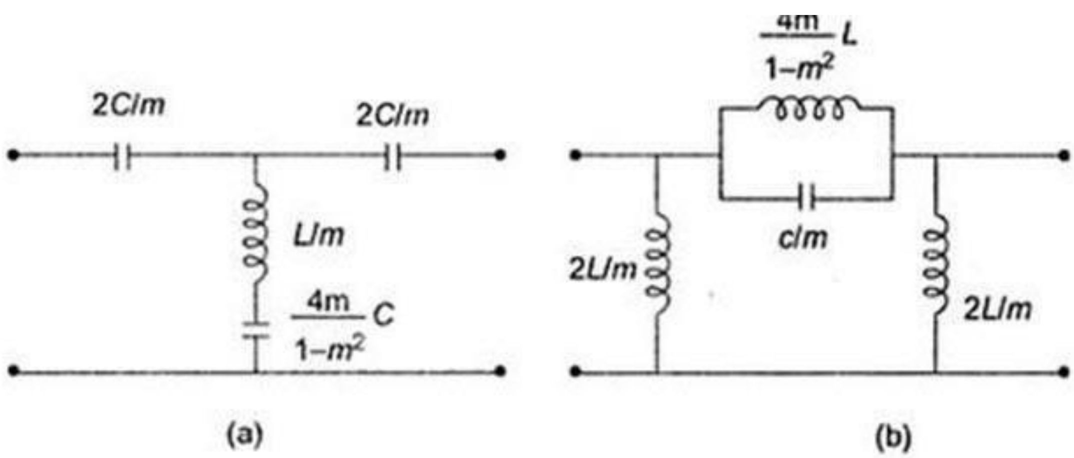
The variation of attenuation for a low pass m-derived section can be verified



m-derived High Pass Filter:

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$



the m-derived \[\pi\]-section, the resonant circuit is constituted by the series arm inductance and capacitance

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m} C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \quad or \quad f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$