Study Material On

STRENGTH OF MATERIAL

3rd Semester, Mechanical Engineering

SIMPLE STRESS & STRAIN

STRENGTH OF MATERIAL:

It is the property of the material due to which the material can resist the external load acting upon it.

LOAD:

It is the external force which acts on a body, machine parts or structural members. It is denoted as P or W. Its SI unit is Newton (N).

- **Types of Load:**
  There are various types of load such as dead/steady load, variable load, suddenly applied or shock load, impact load. It may also be classified as tensile loads, compressive loads, shearing load, bending loads, torsional - loads.

STRESS:

It is the internal resistive force offered by the body against the deformation caused by external load acting upon it. It is denoted as σ or *f*.

If an external force P acts on a body, the body offers resistant force R per unit area of resisting section. Mathematically, $Stress\ (\sigma) = \frac{R}{A} = \frac{P}{A}$ where, P = external load acting on the body = Resistive force (R)

- *Stress is defined as the resistance offered by a body per unit area of resisting section, against the loads producing deformation.*

STRAIN:

It is the measure of deformation of a body per its original dimension. It is denoted as 'e or ϵ'.

Mathematically, $Strain\ (e) = \frac{Deformation}{Original\ \ dimension\ \ of\ the\ body}$

TYPES OF STRESS:

There are two kinds of basic stresses which are

- Normal stresses – always act normal to the stressed surface (either tensile or compressive)
- Shearing stresses – always act parallel to the stressed surface

Tensile Stress:

It is the stress offered by the body against the increase in length caused by the tensile load acting on it. If a tensile load or pulling force (P) acts on the area of cross section (A) of the body, the tensile stress may be written as - $Tensile\ Stress\ (\sigma) = \frac{P}{A}$

Compressive stress:

It is the stress offered by the body against the decrease in length caused by the compressive load acting on it. If a compressive load or pushing force (P) acts on the area of cross section(A) of a body, then the compressive stress may be written as – $Compressive\ Stress\ (\sigma) = \frac{P}{A}$

Shear Stress:

It is the stress offered by the body against the two equal and opposite tangential force acting on its area of cross section which causes the sliding of one part of the body over the other. If the tangential force P acts on a body to cause shear, then shear stress may be written as – *Shear Stress* $(\tau) = \frac{P}{A}$

TYPES OF STRAIN:

Tensile strain:

It is the strain produced in the body when tensile stress induced in it due to the external tensile load. Mathematically, *Tensile strain (e)* $= \frac{\text{deformation of the body due to tensile load}}{\text{original dimension of the body}}$

Compressive strain:

It is the strain produced in the body when compressive stress induces in it due to the external compressive load.

Mathematically, *Compressive strain (e)* $= \frac{\text{deformation of the body due to compressive load}}{\text{original dimension of the body}}$

Shear Strain:

It is the strain produced in the body when shear stress induces in it due to the external tangential load. It is the measure of angle through which the body is deformed by the applied force. Mathematically, *Shear strain* $(ø) = \frac{\text{shear deformation}}{\text{original dimension of the body}}$

Volumetric strain:

It is the ratio of change in volume per original volume of the body.

Mathematically, *Volumetric Strain* $(e_v) = \frac{\text{change in volume of the body}}{\text{original volume of the body}}$

HOOKE'S LAW:

It states that, "*Within elastic limit, stress is directly proportional to strain*".

Mathematically, Stress α Strain $\Rightarrow \sigma \alpha e \quad \Rightarrow \frac{\sigma}{e} = E$ (where E = Young's modulus)

YOUNG'S MODULUS OF ELASTICITY:

It is the ratio of tensile stress to tensile strain or compressive stress to compressive strain, within elastic limit. It is denoted by E. Mathematically, $\frac{\text{normal stress}}{\text{normal strain}} = \frac{\sigma}{e} = E$

MODULUS OF RIGIDITY:

It is the ratio of shear stress to shear strain, within the elastic limit. It is denoted by C /G/ N.

Mathematically, $\frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{ø} = E$

BULK MODULUS:

It is the ratio of normal stress to the volumetric strain, within elastic limit. It is denoted by K.

Mathematically, $\frac{\text{direct stress}}{\text{volumetric strain}} = \frac{\sigma}{e} = K$

LINEAR STRAIN:

It is the strain produced by tensile or compressive forces in longitudinal axis. If a rectangular bar under tension extends along its length, the linear strain or longitudinal strain may be given mathematically, $\text{Linear strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$

LATERAL STRAIN:

It is the strain produced by the tensile or compressive force along the other two mutually perpendicular axes other than the longitudinal axis. It is the ratio of change in lateral dimension to the original lateral dimension of the body. It is also known as the transverse strain.

Mathematically, $\text{Lateral Strain} = \frac{\text{Change in lateral dimension}}{\text{Original lateral dimension}}$

NOTES:

- ♣ If a cylindrical body of length *l* and diameter *d* is subjected to a tensile pull along its length, its length increases and its diameter decreases. This change in length per original length is known as *linear strain* and the change in diameter per original diameter is known as *lateral strain*.

- ♣ If a rectangular body of length *l*, breadth *b* and thickness *t* is subjected to a tensile pull along its length, its length increases and its lateral dimensions such as breadth and thickness decreases. This change in length per original length is known as linear strain and the change in breadth per original breadth or change in thickness per original thickness is known as lateral strain.

POISSON'S RATIO:

It is the ratio of lateral strain to the longitudinal strain. It is denoted by **μ or $\frac{1}{m}$**

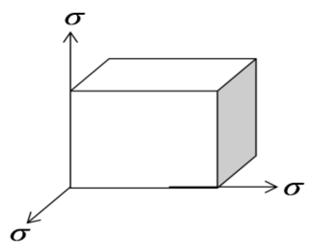
NOTES:

Terms	Symbol	Formula	Units		
Stress	σ or *f*	$\frac{P}{A}$	N/m^2	N/mm^2	N/cm^2
			KN/m^2	MN/m^2	kgf/cm^2
Strain	E	$\frac{\Delta l}{l}$	Unit less		
Young's modulus	E	$\frac{\sigma}{e}$	Same as stress		
Rigidity modulus	C / N / G	$\frac{\tau}{\phi}$	Same as stress		
Bulk modulus	K	$\frac{\sigma}{dv/v}$	Same as stress		
Poisson's ratio	μ or $\frac{1}{m}$	$\frac{\text{Lateral strain}}{\text{Linear strain}}$	Unit less		
Change in length	$\boldsymbol{\Delta} l$ or dl or δl	$\frac{Pl}{AE}$ **or** $e \times l$	mm , cm		
Change in volume	$\boldsymbol{\Delta} V$	$\frac{\Delta l}{l}(1 - \frac{2}{m})\times V$	mm^3, cm^3		

RELATION BETWEEN ELASTIC CONSTANTS:

♣ **Relation between Bulk modulus (K) and Young's modulus (E):**

Consider a cube subjected to direct tensile stresses along three mutually perpendicular directions shown in figure.



Consider the side AB deforms linearly and the other two perpendicular sides undergoes lateral deformations.
So the net tensile strain in any direction may be written as –

$$e = \frac{\sigma}{E} - (\frac{1}{m} \times \frac{\sigma}{E}) - (\frac{1}{m} \times \frac{\sigma}{E}) = \frac{\sigma}{E}(1 - \frac{2}{m}) \qquad \text{-------- (1)}$$

Volume of the body (V) = l^3

Differentiating both sides with respect to l, we get

$$\frac{dV}{dl} = 3l^2 \Rightarrow dV = 3l^2 \times dl = 3l^3 \times \frac{dl}{l} = 3l^3 \times e \qquad (\therefore \text{ e = strain = } \frac{dl}{l})$$

Substituting the value of 'e' from equation-1, we get

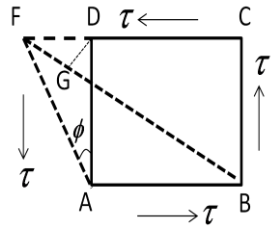
$$dV = 3l^3 \times \frac{\sigma}{E}(1 - \frac{2}{m}) \qquad \Rightarrow \frac{dV}{V} = (\frac{3l^3}{l^3}) \times \frac{\sigma}{E}(1 - \frac{2}{m})$$

We know that, $K = \frac{\sigma}{dV/V} = \frac{\sigma}{(\frac{3l^3}{l^3}) \times \frac{\sigma}{E}(1 - \frac{2}{m})} = \frac{E}{3(1 - \frac{2}{m})}$

$$\Rightarrow E = 3K(1 - \frac{2}{m}) \qquad \text{------------------- (2)}$$

♣ **Relation between Modulus of rigidity (C) and Young's modulus (E):**

Consider a cube of length l subjected to shear stress (τ) as shown in figure.



Consider that one of its diagonal BD elongates and other diagonal AC reduces due to shear stress.
Let after deformation shear strain 'ø' produces.

From figure we get, $\tan\phi = \frac{DF}{AD} \cong \phi$

Consider BD = BG ; as the angle between BD and BG is very small.
Change in length of the diagonal after deformation
= BF – BD = BF – BG = FG

Consider the deformation is very small and $\angle BFC = \angle BDC = 45^0$

$$FG = DF \cos 45^0 = \frac{DF}{\sqrt{2}}$$

Shear strain for the diagonal BD = $\frac{BF - BD}{BD} = \frac{FG}{BD} = \frac{DF \cos 45^0}{AD \times \sqrt{2}} = \frac{DF}{2 \times AD} = \frac{\phi}{2} = \frac{\tau}{2C}$ ….(3)

The diagonal increases due to the direct stresses and also due to the stresses in lateral direction.

Direct strain = $\frac{\sigma_N}{E}$ Lateral strain = $\frac{\sigma_N}{mE}$

So, the resultant strain = $\frac{\sigma_N}{E} + \frac{\sigma_N}{mE}$ ----------------- (4) Where σ_N = normal stress

From equation -2 we get, $\frac{\sigma_N}{E} + \frac{\sigma_N}{mE} = \frac{\sigma_N}{E}(1+\frac{1}{m})$

From equation 3 and 4 we get, $\frac{\tau}{2C} = \frac{\sigma_N}{E}(1+\frac{1}{m})$

$\Rightarrow E = 2C(1+\frac{1}{m})$ ----------------- (5)

♣ **Relation between E, C and K:**

We know that, $E = 2C(1+\frac{1}{m}) \quad \Rightarrow (1+\frac{1}{m}) = \frac{E}{2C}$ ----------- (6)

We know that, $\Rightarrow E = 3K(1-\frac{2}{m}) \quad \Rightarrow (1-\frac{2}{m}) = \frac{E}{3K}$ ----------- (7)

From equation-6 we get: $\frac{1}{m} = \frac{E}{2C} - 1$

Substituting this value in equation-7 we get: $1 - 2\times(\frac{E}{2C} - 1) = \frac{E}{3K}$

$$\Rightarrow 1 - \frac{E}{C} + 2 = \frac{E}{3K} \quad \Rightarrow \frac{C - E + 2C}{C} = \frac{E}{3K} \quad \Rightarrow \frac{3C - E}{C} = \frac{E}{3K}$$

$$\Rightarrow 9KC - 3EK = EC \quad \Rightarrow EC + 3EK = 9KC$$

$$\Rightarrow E(3K + C) = 9KC \quad \Rightarrow E = \frac{9KC}{3K + C}$$ --------------- (8)

This is the required relation between the three elastic constants.

***** PROBLEMS *****

Q -1) *A metal bar 4 m long and 100 mm × 200 mm in cross section is subjected to a pull of 50 kN. If Young's modulus of the material of the bar is 200 kN/mm², find: (i) stress in the bar ii) Strain produced iii) elongation of the bar.*

Ans) Data Given

Area of cross section (A) = 100 mm × 200 mm = 2×10^4 mm²

E = 200 kN/mm² Length (l) = 4m = 4000 mm Pull/Load (P) = 50 kN

i) stress in the bar (σ)

$\sigma = \frac{P}{A} = \frac{50}{2\times10^4} = 2.5 \times 10^{-3}$ kN/mm²

ii) strain produced (e)

$e = \frac{\sigma}{E} = \frac{2.5 \times 10^{-3}}{200} = 1.25 \times 10^{-5}$

iii) elongation of the bar (δl)

$\delta l = \frac{Pl}{AE} = \frac{50 \times 4000}{2 \times 10^4 \times 200}$ mm = 0.05 mm **(ANS)**

Q -2) *A metal bar 50 mm × 50 mm in section is subjected to an axial compressive load of 500 kN. If the contraction of a 200 mm gauge length is found to be 0.5 mm and the increase in thickness 0.04 mm, find the values of Young's modulus and Poisson's ratio for the bar material.*

Ans) Data Given

Length (l) = 200 mm, width (b) = 50 mm, thickness (t) = 50 mm
Axial compressive load (P) = 500 kN = 500×10^3 N
change in length (δl) = 0.5 mm
change in thickness (δt) = 0.04 mm
Young's modulus**(E)**
We know that, contraction of the bar (δl) = $\frac{Pl}{AE}$

$\therefore$ **E =** $\frac{500 \times 10^3 \times 200}{50 \times 50 \times 0.5}$ = 80×10^3 N/mm^2 = 80 GPa

Poisson's ratio ($\frac{1}{m}$)
linear strain (e) = $\frac{\delta l}{l} = \frac{0.5}{200}$ = 0.0025
lateral strain = $\frac{1}{m}$ × linear strain = $\frac{1}{m}$ × 0.0025
also increase in thickness (δt) = t × lateral strain

$\Rightarrow$ $0.04 = 50 \times \frac{1}{m} \times 0.0025$

$\Rightarrow$ $\frac{1}{m} = \frac{0.04}{50 \times 0.0025} = 0.32$ **(ANS)**

Q – 3) *A steel bar 50 mm × 50 mm in cross section is 1.2 m long. It is subjected to an axial pull of 200 kN. What are the changes in length, width and volume of the bar, if the value of Poisson's ratio is 0.3? Take E = 200 GPa.*

Ans) Data Given

Length (l) = 1.2 m = 1200 mm width (b) = 50 mm thickness (t) = 50 mm
Axial pull (P) = 200×10^3 N Poisson's ratio ($\frac{1}{m}$) = 0.3
modulus of elasticity (E) = 200×10^3 N/mm^2

Change in length $\delta l = \frac{Pl}{AE} = \frac{200 \times 10^3 \times 1.2 \times 10^3}{50 \times 50 \times 200 \times 10^3}$ = 0.48 mm

Change in width (δb)
linear strain (e) = $\frac{\delta l}{l} = \frac{0.48}{1.2 \times 10^3}$ = 0.0004
lateral strain = $\frac{1}{m}$ × e = 0.3 × 0.0004 = 0.00012
$\therefore$ Change in width (δb) = b × lateral strain = 50 × 0.00012 = 0.006 mm

Change in volume (δv)
original volume of the bar (V) = l × b × t
= 1200 × 50 × 50 = 3×10^6 mm^3

We know that, $\frac{\delta v}{V} = \frac{P}{btE}(1 - \frac{2}{m}) = \frac{200 \times 10^3}{50 \times 50 \times 200 \times 10^3}(1 - 2 \times 0.3)$ = 0.00016

$\therefore$ Change in volume (dv) = 0.00016 V = 0.00016 × (3×10^6) = 480 mm^3 **(ANS)**

Q- 4) *In an experiment, a bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039mm. Calculate the Poisson's ratio and the values of the three moduli.*

Ans) Data Given

Diameter (d) = 30 mm   length (l) = 200 mm   extension (δl)= 0.09 mm

Pull (P) = 60×10^3 change in diameter (δd) = 0.0039 mm

Poisson's ratio ($\frac{1}{m}$)

linear strain (e) = $\frac{\delta l}{l}$ = $\frac{0.09}{200}$ = 0.00045

lateral strain = $\frac{\delta d}{d}$ = $\frac{0.0039}{30}$ = 0.00013

$\therefore$ **Poisson's ratio =** $\frac{0.00013}{0.00045}$ = 0.289

Values of three moduli

Area of the bar (A) = $\frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 30^2 = 706.9 \text{ mm}^2$

Extension of the bar (δl) = $\frac{Pl}{AE}$

$$\Rightarrow \quad 0.09 = \frac{60 \times 10^3 \times 200}{706.9\,E} = \frac{17 \times 10^3}{E}$$

$\therefore$ Young's modulus (E) = $\frac{17 \times 10^3}{0.09} = 188.9 \times 10^3$ N/mm^2

From Poisson's ratio, we get m = $\frac{1}{0.289}$ = 3.46

$$\therefore \quad \text{Modulus of rigidity (C)} = \frac{E}{2(1+ 1/m)} = \frac{188.9 \times 10^3}{2 \times (1+0.289)} = 73.3 \times 10^3 \text{ N/mm}^2$$

$$\therefore \quad \text{Bulk modulus (K)} = \frac{E}{3(1- 2/m)} = \frac{188.9 \times 10^3}{3 \times (1-0.289)} = 149.2 \times 10^3 \text{ N/mm}^2$$ **(ANS)**

***** ASSIGNMENT- 01 *****

GROUP – A (2 marks questions)

1. Define stress, strain and Young's modulus.
2. State Hooke's law of elasticity.
3. Define modulus of rigidity and bulk modulus.
4. Write the relation between three elastic constants.
5. What is Poisson's ratio?
6. State the relation between Young's modulus and modulus of rigidity.
7. State the relation between Young's modulus and Bulk modulus.

GROUP – B (5 marks questions)

1. A square steel rod 20 mm ×20 mm in section to carry an axial compressive load of 100 kN. Calculate the shortening in a length of 50 mm. E = 2.14×10^8 kN/mm^2. (**Ans:** *0.0584 mm*)
2. A material has a Young's modulus of 1.25×10^5 N/mm^2 and Poisson's ratio of 0.25. Calculate the modulus of rigidity and the Bulk modulus.

3. Find the change in diameter of the rod 2 cm diameter and 2 m long subjected to a pull of 20 N. Take E = 2×10^6 N/cm^2 and Poisson's ratio = 0.25.
4. For a given material the Young's modulus is 110 GN/m^2 and the modulus of rigidity is 42 GN/m^2. Find the bulk modulus and the lateral contraction of a round bar of 37.5 mm diameter and 2.4 m length when stretched by 2.5 mm. (**Ans:** *96.77 GN/m^2, 0.0121 mm*)
5. Under what axial load the diameter of a steel bar will reduce from 8 cm to 7.995 cm? Take E=200 kN/mm^2 and Poisson's ratio as 0.3 for steel.
6. A hollow steel column of external diameter 250 mm has to support an axial load of 2000 kN. If the ultimate stress for the steel column is 480 N/mm^2, find the internal diameter of the column allowing a load factor of 4.
7. A 20 mm diameter brass rod was subjected to a tensile load of 40 kN. The extension of the rod was found to be 254 divisions in the 200 mm extension meter. If each division is equal to 0.001 mm, find the elastic modulus of brass.

GROUP – C (10 mark questions)

1. Derive the relation between three elastic constants.
2. The following observations were made during the tensile test on a mild steel specimen 40 mm in diameter and 200 mm long.

   Elongation under a load of 40 kN = 0.0304 mm

   Load at yield point = 161 kN

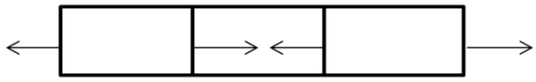
   Maximum load = 242 kN

   Length of specimen at fracture = 249 mm

   Determine the Young's modulus, yield point stress, ultimate stress and percentage elongation.
   *(**Ans:** 2.09×10^8 kN/m^2, 1208×10^4 kN/m^2, 19.2×10^4 kN/m^2, 24.5%)*
3. The following data are related to a bar subjected to a tensile test. Diameter of the bar = 30 mm, tensile load = 54 kN, gauge length = 300 mm, extension of the bar = 0.112 mm, change in diameter = 0.00366 mm. Calculate (i) Poisson's ratio; (ii) the values of three moduli.
   *(**Ans:** 0.327, 2.05×10^5 MN/m^2, 0.77×10^5 MN/m^2, 1.97×10^5 MN/m^2)*

PRINCIPLE OF SUPERPOSITION:

Consider a bar of uniform area A, Young's modulus E and split into three sections 1, 2 and 3 subjected to tensile and compressive forces as shown in figure.

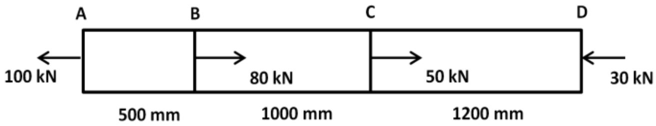


Consider the first section, and determine the tensile/compressive force acting on it. Then find the change in length of the section-1 by using the formula: $\boldsymbol{\Delta} l = \frac{Pl}{AE}$

- Continue the same step for section-2 and section-3 also.
- Add all the values of change in length to determine the total change in length of the bar.
- Consider that the change in length for tensile forces is positive and for compressive forces are negative.
- If area of the bar is given uniform, then the value of area for all three sections is same.
- The value of Young's modulus is same for the same materials.
- If a bar of different section is given then the area of different sections are not same.

***** PROBLEMS *****

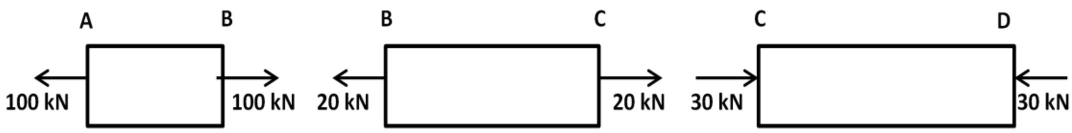
Q-1) *A bar having cross sectional area of 500 mm^2 is subjected to axial forces as shown in figure. Find the total elongation of the bar. Take E = 80 GPa*.



Ans) <u>Data Given</u>

Cross sectional area (A) = 500 mm^2 Young's modulus (E) = 80 kN/mm^2

Consider that AB, BC and CD are three sections. The forces acting on each section are shown individually.



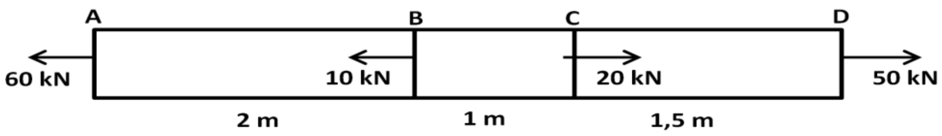
Change in length of section AB = $\delta l_{AB} = \frac{P_{AB}\, L_{AB}}{AE} = \frac{100 \times 500}{500 \times 80}$ = 1.25 mm (tensile)

Change in length of section BC = $\delta l_{BC} = \frac{P_{BC}\, L_{BC}}{AE} = \frac{20 \times 1000}{500 \times 80}$ = 0.5 mm (tensile)

Change in length of section CD = $\delta l_{CD} = \frac{P_{CD}\, L_{CD}}{AE} = \frac{30 \times 1200}{500 \times 80}$ = 0.9 mm (compressive)

Total change in length (δl) = 1 + 0.5 – 0.9 = 0.6 mm **(ANS)**

Q-2) *A steel rod ABCD 4.5m long and 25mm in diameter is subjected to the forces as shown in figure. If the value of Young's modulus for the steel is 200 GPa, determine its deformation.*

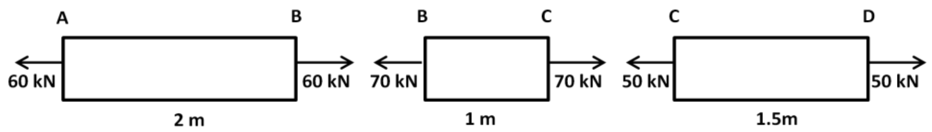


Ans) <u>Data Given</u>

Length of rod (l) = 4.5 m = 4500 mm   diameter (d) = 25 mm   E = 200 kN/mm^2

area of cross section (A) = $\frac{\pi}{4}$ d^2 = $\frac{\pi}{4}$ × (25)^2 = 491 mm^2

Consider that AB, BC and CD are three sections. The forces acting on each section are shown individually.



Change in length of section AB = $\delta l_{AB} = \frac{P_{AB}\, L_{AB}}{AE} = \frac{60 \times 2000}{491 \times 200}$ = 1.22 mm (tensile)

Change in length of section BC = $\delta l_{BC} = \frac{P_{BC}\, L_{BC}}{AE} = \frac{70 \times 1000}{491 \times 200}$ = 0.71 mm (tensile)

Change in length of section CD = $\delta l_{CD} = \frac{P_{CD}\, L_{CD}}{AE} = \frac{50 \times 1500}{491 \times 200}$ = 0.76 mm (tensile)

Total change in length (δl) = 1.22 + 0.71 + 0.76 = 2.69 mm   **(ANS)**

STRESSES IN COMPOSITE SECTION:

- A composite bar is consists of two or more materials.
- The load acting on the composite section is equal to the sum of the loads carried by the sections of different materials.
- The deformation per unit length is constant. i.e. strain = constant
- The extension or contraction of the bar is equal.
- Consider a Composite column of outer tube and inner tube having different materials. For different materials the value of young's modulus is different.

  Let, for outer tube A_1 = area, E_1 = young's modulus

  for inner tube A_2 = area, E_2 = young's modulus

  Let, the length of the column is l and the column is subjected to axial load of P.

  So, we may write,

  Total load on the column = load on outer tube + load on inner tube

  $\therefore \quad P = \sigma_1 A_1 + \sigma_2 A_2$ ------------- (1)

  Where σ_1 and σ_2 are the stresses in the outer and inner tube.

  $\sigma_1 A_1$ and $\sigma_2 A_2$ are the load on outer and inner tube respectively.

  Let dl = decrease in length of the column

  Strain in each tube (e) = $\frac{dl}{l}$

  Strain in outer tube = strain in inner tube $= \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$ ------------ (2)

- $\frac{E_1}{E_2}$ = Modular ratio of two materials = m

***** PROBLEMS *****

Q-3) *A reinforced concrete circular section of 50 000 mm² cross sectional area carries 6 reinforcing bars whose total area is 500 mm². Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.*

Ans) Data Given

Area of the column (A) = 50 000 mm^2 No. of reinforcing bars = 6

Total area of steel bars (A_s) = 500 mm^2 Max. stress in concrete (σ_c) = 3.5 N/mm^2

Modular ratio = $\frac{E_s}{E_c}$ = 18

Area of concrete (A_c)= 50 000 – 500 = 49500 mm^2

We know that, $\frac{\sigma_s}{\sigma_c} = \frac{E_s}{E_c}$

So the stress in steel (σ_s) $= \frac{E_s}{E_c} \times \sigma_c = 18 \times 3.5 = 63$ N/mm^2

For composite section, we can write

Total safe load (P) = load shared by steel (Ps) + load shared by concrete (Pc)

$\Rightarrow$ $P = \sigma_s A_s + \sigma_c A_c = (63 \times 500) + (3.5 \times 49500) = 204750$ N = 204.75 KN **(ANS)**

Q-4) *A reinforced concrete column of 400 mm diameter has 4 steel bars of 20 mm diameter embedded in it. Find the maximum load which the column can carry, if the stresses in steel and concrete are not to exceed 120 MPa and 5 MPa respectively. Take modulus of elasticity of steel as 18 times that of concrete.*

Ans) Data Given

Diameter of concrete column (D) = 400 mm No. of reinforcing bars = 4

diameter of steel bars (d) = 20 mm Max. stress in steel $(\sigma_s)_{max}$ = 120 N/mm^2

Max. stress in concrete $(\sigma_c)_{max}$ = 5 N/mm^2 Modulus of elasticity of steel (E_s) = 18 E_c

Total area of the concrete circular column = $\frac{\pi}{4} D^2 = \frac{\pi}{4} \times (400)^2 = 125660$ mm^2

Area of the steel bars (A_s) = $4 \times \frac{\pi}{4} d^2 = 4 \times \frac{\pi}{4} \times 20^2 = 1257$ mm^2

So, area of the concrete (A_c) = 125660 – 1257 = 124403 mm^2

If the stress in steel is 120 N/mm^2, then

stress in the concrete = $\frac{E_c}{E_s} \times 120 = \frac{1}{18} \times 120 = 6.67$ N/mm^2.

This stress is more than the maximum stress in concrete. So it can’t be considered.

If the stress in concrete is 5 N/mm^2, then

stress in steel = $\frac{E_s}{E_c} \times 5 = 18 \times 5 = 90$ N/mm^2

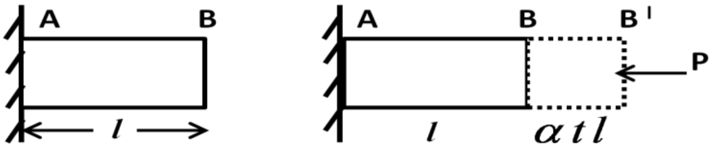
This stress is less than the maximum stress in steel, therefore stresses in steel and concrete may be taken as 90 N/mm^2 & 5 N/mm^2.

$\therefore$ Maximum load (P) = load shared by steel (Ps) + load shared by concrete (Pc)

$\Rightarrow$ $P = \sigma_s A_s + \sigma_c A_c = (90 \times 1297) + (5 \times 124403) = 735\ 150$ N = 735.15 kN **(ANS)**

TEMPERATURE / THERMAL STRESS:

- When the temperature of a body increases or decreases, some stresses develop in the body to resist the deformation of the body due to change in temperature. This stress is known as temperature or thermal stress.
- Consider a bar AB of length 'l' fixed at one end A, deformed due to change in temperature as shown in figure.



- Let, t = change in temperature
  α = coefficient of linear expansion.
  Let, the body expands due to rise in temperature 't' = $BB^{/} = \alpha t l$
  Let, the load 'P' is applied at the free end to prevent expansion.
  Then compressive stress induce in the body $\{\sigma_c\} = \frac{P}{A}$
- Strain in bar = $\frac{change\ in\ length}{original\ \ length} = \frac{\sigma t l}{l + \sigma t l} = \frac{\sigma t l}{l} = l$ (the value αtl is very small, it is neglected)
  We know that, $\frac{stress}{strain} = E \quad \Rightarrow \frac{\sigma}{\sigma t} = E$
  or **σ = αtE = thermal stress**
  e = αt = temperature strain
  P = σA = αTEA = load on the body

***** PROBLEMS *****

Q-5) *Two parallel walls 6 m apart are stayed together by a steel rod 25 mm diameter passing through metal plates and nuts at each end. The nuts are tightened when the rod is at a temperature of 100^0C. Determine the stress in the rod, when the temperature falls down to 60^0 C, if a) the ends don't yield b) The ends yield by 1 mm*
Take E = 200 GPa and $\alpha = 12 \times 10^{-6}$ per ^{0}C.

Ans) <u>Data Given</u>

Length of rod (l) = 6m = 6000 mm
Diameter of rod (d) = 25 mm
Change in temperature (t) = $100^0 - 60^0 = 40^0$ C
Modulus of elasticity (E) = 200 N/mm^2
Coefficient of linear expansion (α) = 12×10^{-6} per ^{0}C

a) Stresses in the rod when the ends don't yield –
$\sigma_1 = \alpha.t.E = (12 \times 10^{-6}) \times 40 \times (200 \times 10^3) = 96\ N/mm^2 = 96\ MPa$

b) Stresses in the rod when the ends yield by 1 mm –
$\sigma_2 = [\ \alpha t - \frac{amount\ \ of\ yield\ \ in\ ends}{l}\ ] \times E$
$= [(12 \times 10^{-6}) \times 40 - \frac{1}{6 \times 10^3}\ ] \times 200 \times 10^3 = 62.6\ N/mm^2 = 62.6\ MPa$ **(ANS)**

Q-6) A gun metal rod 20 mm diameter, screwed at the ends, passes through a steel tube 25 mm and 30 mm internal and external diameters respectively. The nuts on the rod are screwed tightly home on the ends of the tube. Find the intensity of stress in each metal, when the common temperature rises by 200^0F. Take Coefficient of expansion for steel = 6×10^{-6} per ^{0}F, Coefficient of expansion for gun metal = 10×10^{-6} per ^{0}F, Modulus of elasticity for steel = 200 GPa, Modulus of elasticity for gun metal = 100 GPa.

Ans) Data Given

Diameter of gun metal rod = 20 mm Internal diameter of steel tube = 25 mm
External diameter of steel tube = 30 mm Rise in temperature (t) = 200^0F
Coefficient of expansion for steel (α_s) = 6×10^{-6} per ^{0}F
Coefficient of expansion for gun metal (α_G) = 10×10^{-6} per ^{0}F
Modulus of elasticity for steel (E_S) = 200 GPa = 200×10^3 N/mm^2
Modulus of elasticity for gun metal (E_G) = 100 GPa = 100×10^3 N/mm^2

Let, σ_G = stress in gun metal rod and σ_s = stress in steel tube
area of gun metal rod (A_G) = $\frac{\pi}{4} \times 20^2 = 100 \times \pi$ mm^2
area of steel tube (A_s) = $\frac{\pi}{4}$ [$(30)^2 - (25)^2$] = $68.75\ \pi$ mm^2

As α_g is greater than α_s , the expansion of gun metal will be more than steel tube.
Since the gun metal will be subjected to compressive stress and the steel tube will be subjected to tensile stress.
Tensile load in steel tube = Compressive load in Gun metal rod
i.e. $\sigma_s \times A_s = \sigma_G \times A_G$

Stress in steel tube (σ_s) = $\frac{A_g}{A_s} \times \sigma_G = \frac{100\ \pi}{68.75\ \pi} \times \sigma_G = 1.45\ \sigma_G$
Strain in steel tube (e_s) = $\frac{\sigma_s}{E_s} = \frac{\sigma_s}{200 \times 10^3}$
Strain in gun metal (e_G) = $\frac{\sigma_G}{E_G} = \frac{\sigma_G}{100 \times 10^3}$
We know that, Strain in steel = Strain in Gun metal

$\Longrightarrow$ $e_s + \alpha_s\, t = \alpha_G\, t - e_G$

$\Longrightarrow$ $e_s + e_G = (\alpha_G - \alpha_s) \times t$

$\Longrightarrow$ $\frac{\sigma_s}{200 \times 10^3} + \frac{\sigma_G}{100 \times 10^3} = [\,(10 \times 10^{-6}) - (6 \times 10^{-6})] \times 200$

$\Longrightarrow$ $\frac{1.45\ \sigma G}{200 \times 10^3} + \frac{\sigma_G}{100 \times 10^3} = 200 \times (4 \times 10^{-6})$

$\Longrightarrow$ $\frac{3.45\ \sigma G}{200 \times 10^3} = 800 \times 10^{-6}$

$\Longrightarrow$ $3.45\ \sigma_G = 800 \times 10^{-6} \times 200 \times 10^3 = 160$

$\therefore$ $\sigma_G = \frac{160}{3.45} = 46.4$ N/mm^2 = 46.4 MPa

$\therefore$ $\sigma_s = 1.45\ \sigma_G = 1.45 \times 46.4 = 67.3$ N/mm^2 = 67.3 MPa **(ANS)**

STRAIN ENERGY:

It is the energy stored in a deformed elastic body when external load acts on it. This energy is released when load is removed.

RESILIENCE:

It is the total strain energy stored by the body when external load acts on it within elastic limit. It is measured by the amount of energy absorbed per unit volume of the material within elastic limit.

PROOF RESILIENCE:

It is the maximum strain energy stored by the body per unit volume when the external load acts on it within the elastic limit.

EXPRESSION FOR STRAIN ENERGY:

Consider a bar of area 'A', length l subjected to load W.

Let, the bar extends by dl due to the load and produces maximum stress σ.

Let U = strain energy stored in the bar = work done by the load

so we may write, $U = \frac{1}{2} \times W \times dl$

we know that, $dl = \frac{\sigma\, l}{E}$ substituting the value of dl, we get

$U = \frac{W}{2} \times \frac{\sigma\, l}{E} = \frac{\sigma\, A}{2} \times \frac{\sigma\, l}{E}$ (∵ $W = \sigma \times A$)

$\mathbf{U = \frac{\sigma^2 A\, l}{2E} = \frac{\sigma^2 V}{2E}}$ (∵ $V = A \times l$)

STRESSES DUE TO GRADUALLY APPLIED LOAD:

When a load acts upon a body stepwise starting from zero to its last value, then it is known as gradually applied load.

Let, W = gradually applied load on the body

dl = change in length

σ = maximum stress

Energy due to external load = $\frac{1}{2} \times \sigma \times A \times dl$

Work done on the body = $\frac{1}{2} \times W \times dl$

But strain energy stored = work done on the body

⇒ $\frac{1}{2} \times \sigma \times A \times dl = \frac{1}{2} \times W \times dl$

⇒ $\boldsymbol{\sigma = \frac{W}{A}}$ **------------ (1)**

Stress due to suddenly applied load:

When a load acts suddenly on a body it is known as suddenly applied load.

Let, W = suddenly applied load

σ_s = suddenly applied load

dl = increase in length

Strain energy stored = External load acting on the body

$$W \times dl = \frac{1}{2} \times \sigma_s \times A \times dl$$

$$\Rightarrow \quad \boldsymbol{\sigma_s = \frac{2W}{A}} \quad \text{-------------- (2)}$$

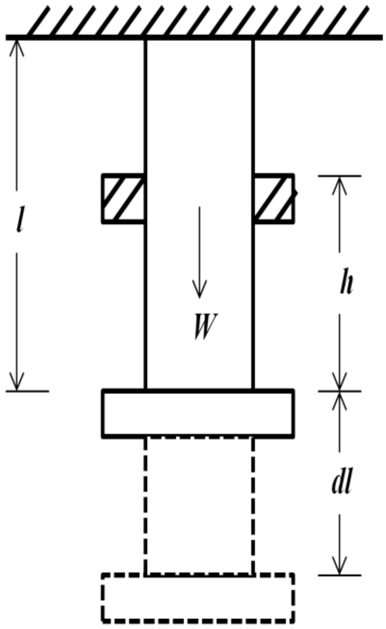
Thus stress due to suddenly applied load is double than stress due to gradually applied load.

STRESS DUE TO IMPACT LOAD:

When a load falls from a height and strikes the body with some momentum, it is known as impact load. Consider a weight W is falling from height ‘h’ on the collar fitted on the rod.

Let:

l = length of the rod
A = cross sectional area of rod
dl = change in length
σ = maximum stress
External work done on the load = Energy stored in the rod



$$\Rightarrow \quad W(h + dl) = \frac{1}{2} \times \sigma \times A \times dl$$

$$\Rightarrow \quad W\left[h + \frac{\sigma\, l}{E}\right] = \frac{1}{2} \times \sigma \times A \times \frac{\sigma\, l}{E} \qquad \left(\because \quad dl = \frac{\sigma\, l}{E}\right)$$

$$\Rightarrow \quad W\left[h + \frac{\sigma\, l}{E}\right] = \frac{\sigma^2 A l}{2E}$$

$$\Rightarrow \quad \frac{\sigma^2 A l}{2E} - \frac{\sigma W l}{E} - Wh = 0$$

$$\Rightarrow \quad \sigma = \frac{\frac{W l}{E} \pm \sqrt{\frac{W^2 l^2}{E^2} + \frac{2WhA\, l}{E}}}{\frac{Al}{E}} = \frac{\frac{W l}{E} \pm \frac{W l}{E}\sqrt{1 + \frac{2WhA\, l}{E} + \frac{E^2}{W^2 l^2}}}{\frac{Al}{E}}$$

$$\Rightarrow \quad \sigma = \frac{W + W\sqrt{1 + \frac{2hA\,E}{Wl}}}{A}$$

Taking positive sign we get,

$$\boldsymbol{\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hA\,E}{Wl}}\right]} \text{ ------ (3)}$$

***** ASSIGNMENT- 02 *****

GROUP – A (2 mark questions)

1. What do mean by composite section?
2. Define temperature/thermal stress and strain.
3. Define strain energy.
4. Define resilience.

GROUP – B (6 mark questions)

1. Find the expression for temperature stress for a rise in temperature of t^0C when the ends don't yield. Take α as coefficient of expansion and *l* as original length.
2. A reinforces concrete column is 300 mm × 300 mm in section. The column is provided with 8 steel bars of 20 mm diameter. The column carries a load of 360 KN. find the stresses in concrete and steel bars. Take $E_s = 2.1 \times 10^5$ N/mm² and $E_b = 0.14 \times 10^5$ N/mm².
3. A steel rod of 20 m long at a temperature 20^0C is subjected to rise in temperature to 65^0C. Find the temperature stress produced, i) when the expansion of the rod is prevented; ii) when the rod is permitted to expand by 5.8 mm. Take $\alpha = 12 \times 10^{-6}$ per ^0C and $E = 2 \times 10^5$ N/mm².
4. Derive the expressions for stresses due to gradually applied load, suddenly applied load and impact load.

GROUP – C (8 mark questions)

1. A compound tube consists of a steel tube 150 mm internal diameter and 10 mm thickness and an outer brass tube 170 mm internal diameter and 10 mm thickness. The two tubes are of the same length. The compound tubes carries an external load of 1000 KN. Find the stresses and load carried by each tube and the amount it reduced. Length of each tube is 100mm.
   Take $E_s = 2 \times 10^5$ N/mm² and $E_b = 1 \times 10^5$ N/mm².
2. A 15 mm diameter steel rod passes centrally through a copper tube 50 mm external diameter and 40 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting parts of the rod. If the temperature of the assembly is raised by 60^0C, calculate the stresses developed in copper and steel.
3. A steel bar is placed between two copper bars each having the same area and length as the steel bar at 15^0C. The bars are rigidly connected at both ends when the temperature rises to 315^0C. The length of the bars increases by 1.50 mm. Determine the original length and the final stresses in the bars. Take $E_s = 2.1 \times 10^5$ N/mm² $E_c = 1 \times 10^5$ N/mm²
   $\alpha_s = 0.000012$ per^0C $\alpha_c = 0.0000175$ per^0C
4. A solid steel bar 50 cm long and 7 cm in diameter is placed inside an aluminium tube having 7.5 cm inside diameter and 10 cm outside diameter. The aluminium tube is 0.015 cm longer than the steel bar. An axial load of 600 kN is applied to the bar and tube through rigid cover plates. Find the stresses developed in the steel bar and aluminium tube. E for steel = 220 GPa. E for aluminium = 70 GPa. (**Ans:** $\sigma_s = 106.82\ MN/m^2$, $\sigma_{al} = 54.99\ MN/m^2$)

THIN CYLINDRICAL SHELL

THIN CYLINDER:

If the thickness of the wall of the cylinder is less than 1/20 of its diameter then the cylinder is said to be thin cylinder.

ASSUMPTION FOR THIN SHELLS:

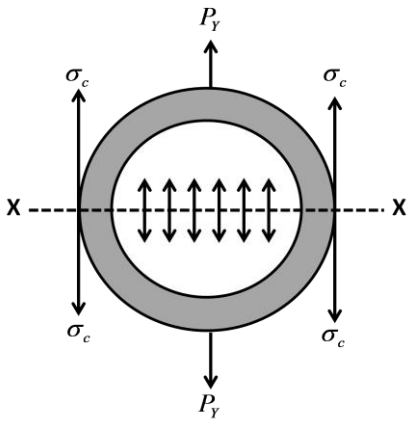
- A thin cylinder may fail along longitudinal seam or across a transverse section.
- Hoop or circumferential stresses acting across longitudinal section.
- Longitudinal or axial stresses acting across at right angles to the longitudinal axis of the cylinder.
- Radial stresses are neglected.

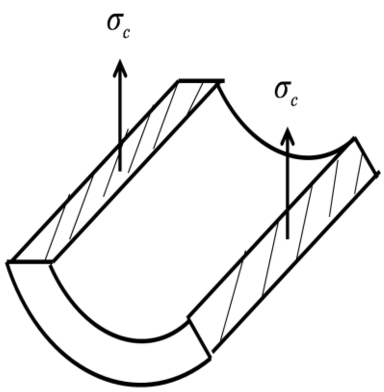
HOOP OR CIRCUMFERENTIAL STRESS:

When a thin cylindrical shell is subjected to an internal pressure, tensile stresses develop in a tangential direction to its circumference, which may split the shell into two troughs, such stresses are known as hoop or circumference stresses.

Determination of hoop stress:

Consider a thin cylinder subjected to an internal pressure (p). Consider any section X-X which divides the cylinder into two parts. Due to internal pressure, force P_Y will act normal to the X-X axis. This force P_Y will develop the tensile stress σ_c which is known as *hoop or circumferential stress.*





Let l = length of the shell　　d = internal diameter of the shell
t = thickness of the shell　　p = intensity of internal pressure

Pressure force acting along each side of X-X plane of the shell (P_Y) = internal pressure × area on X-X plane = p × (d × l)

Area resisting the bursting force P_Y = 2 $t\,l$

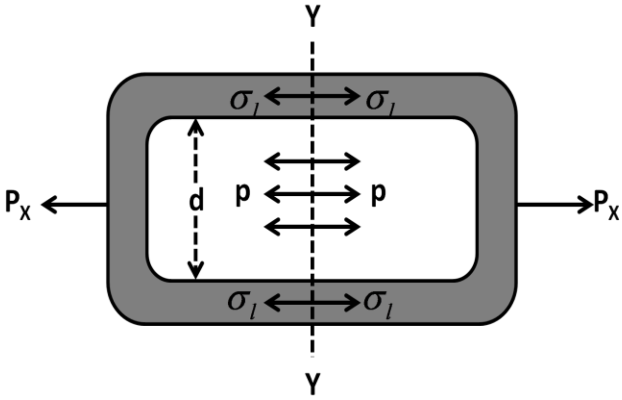
Circumferential stress in the shell = $\sigma_c = \dfrac{\text{Pressure force}}{\text{area resisting the internal pressure}} = \dfrac{pd\,l}{2tl} = \dfrac{\mathbf{pd}}{\mathbf{2t}}$

∴ $\boxed{\boldsymbol{\sigma c = \frac{pd}{2t}}}$ This tensile stress across X-X axis is also known as Hoop stress.

LONGITUDINAL STRESS:

When a thin cylindrical shell is subjected to an internal pressure, tensile stresses may also develop in longitudinal direction which may split the shell into two halves or into two cylinders, such stresses are known as longitudinal stresses.

Consider a thin cylinder subjected to an internal pressure (p). Consider any section Y-Y at right angles to the length of the cylinder which divides the cylinder into two half cylinders. Due to internal pressure, force P_X will act normal to the Y-Y axis. This force P_Y will develop the tensile stress σ_l parallel to the length of the cylinder. It is known as *longitudinal stress*.



Let l = length of the shell; d = diameter of the shell

t = thickness of the shell; p = intensity of internal pressure

Total pressure force along longitudinal direction = P_X = internal pressure × area on Y-Y plane

$$= p \times \frac{\pi}{4} d^2$$

Area resisting the bursting force $P_X = \pi dt$

$$\text{Longitudinal stress in the shell} = \sigma_l = \frac{\text{total pressure force}}{\text{area resisting the internal pressure}} = \frac{p \times \pi/4\, d^2}{\pi dt} = \frac{pd}{4t}$$

∴ $\boxed{\sigma_l = \frac{\mathbf{pd}}{\mathbf{4t}}}$ This tensile stress across Y-Y axis is known as longitudinal stress.

CIRCUMFERENTIAL STRAIN AND LONGITUDINAL STRAIN:

Let δd = change in diameter of the shell

δl = change in length of the shell

$\frac{1}{m}$ = Poisson's ratio

σ_c = circumferential stress and σ_l = longitudinal stress

$$\textit{Circumferential strain } (e_c) = \frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{\sigma_l}{mE} = \frac{pd}{2tE} - \frac{pd}{4tEm} = \boxed{\frac{\mathbf{pd}}{\mathbf{2tE}}\left(1 - \frac{1}{2m}\right)}$$

$$\textit{Longitudinal strain } (e_l) = \frac{\delta l}{l} = \frac{\sigma_l}{E} - \frac{\sigma_c}{mE} = \frac{pd}{4tE} - \frac{pd}{2tEm} = \boxed{\frac{\mathbf{pd}}{\mathbf{2tE}}\left(\frac{1}{2} - \frac{1}{m}\right)}$$

CHANGE IN DIMENSION OF THIN CYLINDRICAL SHELL:

We know that, $e_c = \frac{\delta d}{d} = \frac{pd}{2tE}(1- \frac{1}{m})$

∴ *Change in diameter* (δd) = $e_c \times d = \frac{pd}{2tE}(1- \frac{1}{m}) \times d = \mathbf{\frac{pd^2}{2tE}(1- \frac{1}{2m})}$

We know that, $e_l = \frac{\delta l}{l} = \frac{pd}{2tE}(\frac{1}{2} - \frac{1}{m})$

∴ *Change in length* (δ*l*) = $e_l \times l = \frac{pd}{2tE}(\frac{1}{2} - \frac{1}{m}) \times l = \mathbf{\frac{pDl}{2tE}(\frac{1}{2} - \frac{1}{m})}$

Internal volume of the cylinder (V) = $\frac{\pi}{4} \times d^2 \times l$

Final volume of the cylinder = V + δV = $\frac{\pi}{4} \times (d+ \delta d)^2 \times (l + \delta l)$

Change in Volume = δV = final volume – initial volume = $\frac{\pi}{4} \times (d+ \delta d)^2 \times (l + \delta l) - \frac{\pi}{4} \times d^2 \times l$

Neglecting the smaller terms we get:

Change in volume (δV) = $\frac{\pi}{4} \times (d^2 . \delta l + 2dl . \delta d)$

∴ *Volumetric strain* (e_v) $= \frac{\delta V}{V} = \frac{\frac{\pi}{4} \times (d^2 . \delta l + 2dl . \delta d)}{\frac{\pi}{4} \times d^2 \times l} = \frac{\delta l}{l} + \frac{2\,\delta d}{d} = \mathbf{e_l + 2\times e_c}$

∴ *Change in volume* (δV) = $\mathbf{V (e_l + 2\times e_c)}$

THIN SPHERICAL SHELL:

Stress in spherical shell (σ) = $\frac{pD}{4t}$ ----------------- (10)

Strain in any section of spherical section (e) = $\frac{\sigma_c}{E} - \frac{\sigma_l}{mE} = \frac{\sigma_c}{E}(1 - \frac{1}{m}) = \mathbf{\frac{pD}{4t\,E}(1 - \frac{1}{m})}$ --------- (10)

NOTES:

Terms	Symbol	Formula	Units
Hoop stress	σ_c or f_y	$\frac{p \times d}{2 \times t}$	Same as Stress
Longitudinal stress	σ_l or f_x	$\frac{p \times d}{4 \times t}$	Same as Stress
Change in diameter	Δd	$\frac{pd^2}{2tE}(1- \frac{1}{2m})$	mm or cm
Change in length	Δ*l*	$\frac{pdl}{2tE}(\frac{1}{2} - \frac{1}{m})$	mm or cm
Change in volume	Δv	$V (e_l + 2 e_c) = \frac{pD}{2tE}(\frac{5}{2} - \frac{2}{m})$	mm or cm

***** PROBLEM *****

Q-1) *A mild steel cylinder contains some fluid under pressure and its diameter is 1.5 m. If the thickness of the cylinder wall is 4 mm, determine the safe pressure inside the cylinder. Assume the maximum allowable tensile stress in M.S as 80 N/mm^2.*

Ans) <u>Data Given</u>

Diameter (d) = 1.5 m = 1500 mm thickness (t) = 4 mm
Maximum allowable stress (σ) = 80 N/mm^2
Let, p = safe pressure inside the cylinder
Hoop stress is given by -

$$\sigma c = \frac{p\,d}{2t} = \frac{p \times 1500}{2 \times 4} \quad \Rightarrow \quad 80 = \frac{p \times 1500}{2 \times 4} \quad \Rightarrow \quad p = \frac{2 \times 4 \times 80}{1500} = \mathbf{0.4267\ N/mm^2}$$

longitudinal stress is given by -

$$\sigma_l = \frac{p\,d}{4\,t} = \frac{p \times 1500}{4 \times 4} \quad \Rightarrow \quad 80 = \frac{p \times 1500}{4 \times 4} \quad \Rightarrow \quad p = \frac{4 \times 4 \times 80}{1500} = \mathbf{0.8534\ N/mm^2}$$

∴ The required safe pressure inside the cylinder is p = 0.4267 N/mm^2. **(ANS)**

Q-2) *A cylinder is made of a material whose maximum allowable tensile stress is 60 N/mm^2. The diameter of the cylinder is 100 mm. If pressure of a fluid contained in the cylinder is 20 bar, determine the safe thickness of the cylinder wall.*

Ans) <u>Data Given</u>

Maximum tensile stress = 60 N/mm^2 Diameter (d) = 100 mm
Pressure (p) = 20 bar = 2 N/mm^2
Hoop stress is given by –

$$\sigma c = \frac{p\,d}{2t} = \frac{2 \times 100}{2 \times t} \quad \Rightarrow \quad 60 = \frac{2 \times 100}{2 \times t} \quad \Rightarrow \quad t = \frac{2 \times 100}{2 \times 60} \quad \therefore \quad \mathbf{t = 1.667\ mm}$$

longitudinal stress is given by -

$$\sigma_l = \frac{p\,d}{4\,t} = \frac{2 \times 100}{4 \times t} \quad \Rightarrow \quad 60 = \frac{2 \times 100}{4 \times t} \quad \Rightarrow \quad t = \frac{2 \times 100}{4 \times 60} \quad \therefore \quad \mathbf{t = 0.833\ mm}$$

∴ The required safe thickness is 1.667 mm. **(ANS)**

Q-3) *A boiler is 1.5 m in diameter having thickness of plate as 10 mm. The efficiencies of the longitudinal and circumferential joints are respectively 60% and 80%. If the maximum allowable tensile stress in plate be 70 N/mm^2, calculate the safe steam pressure in the boiler.*

Ans) <u>Data Given</u>

Diameter (d) = 1.5 m = 1500 mm thickness (t) = 10 mm
efficiency η_l = 0.60 & η_c = 0.80
Maximum allowable tensile stress = 70 N/mm^2

Hoop stress is given by –

$$\sigma_c = \frac{p\,d}{2t \times \eta_l} \quad \Rightarrow \quad 70 = \frac{p \times 1500}{2 \times 10 \times 0.60} \quad \Rightarrow \quad p = 0.56\ N/mm^2$$

Longitudinal stress is given by –

$\sigma_l = \frac{p\,d}{4\,t \times \eta_l}$ $\Rightarrow$ $70 = \frac{p \times 1500}{4 \times 10 \times 0.80}$ $\Rightarrow$ p = 1.439 N/mm^2

$\therefore$ The required safe pressure is 0.56 N/mm^2. **(ANS)**

Q-4) *The internal diameter of a cylindrical shell is 1 m and its length is 3 m, the plates being 1.5 cm thick. Determine the circumferential and longitudinal stresses set up and changes in dimensions of the shell when a fluid is introduced in it at a pressure of 1.5 N/mm^2. Take E = 200 kN/mm^2 and poison's ratio 0.3.*

Ans) Data Given

Diameter (d) = 1 m = 1000 mm
length (l) = 3 m = 3000 m
thickness (t) = 1.5 cm = 15 mm
pressure (p) = 1.5 N/mm^2
E = 200 kN/mm^2
Poisson's ratio ($\frac{1}{m}$) = 0.3

Circumferential/hoop stress (σc) = $\frac{p\,d}{2t} = \frac{1.5 \times 1000}{2 \times 15}$ = 50 N/mm^2

longitudinal stress (σl) = $\frac{p\,d}{4\,t} = \frac{1.5 \times 1000}{4 \times 15}$ = 25 N/mm^2

Change in diameter (δd) = $\frac{pd^2}{2tE}(1 - \frac{1}{2m}) = \frac{1.5 \times 1000^2}{2 \times 15 \times 2 \times 10^5}\ (1 - \frac{1}{2 \times 0.3})$ = 0.2125 mm

change in length (δl) = $\frac{pD\,l}{2tE}(\frac{1}{2} - \frac{1}{m}) = \frac{1.5 \times 1000 \times 3000}{2 \times 15 \times 2 \times 10^5}\ (\frac{1}{2} - 0.3)$ = 0.15 mm **(ANS)**

***** ASSIGNMENT – 03 *****

GROUP - A (2 mark questions)

1. Define thin cylindrical shell.
2. Define hoop and longitudinal stress.
3. State the expression of hoop and longitudinal stress.

GROUP - B (6 mark questions)

1. Write the assumptions for thin cylindrical shell.
2. Derive the expression for hoop and longitudinal stresses for a thin cylindrical shell.
3. A steam boiler of 800 mm diameter is made up of 10 mm thick plates. If the boiler is subjected to an internal pressure of 2.5 MPa, find the circumferential and longitudinal stresses induced in the boiler plates.
4. A cylindrical air receiver for a compressor is 2 m in internal diameter and made of plates 12 mm thick. If the hoop stress is not to exceed 60 N/mm^2, find the maximum safe air pressure.
5. A boiler is subjected to an internal steam pressure of 2 MPa. The thickness of the boiler plate is 2 cm and the permissible tensile stress is 120 MPa. Find the maximum diameter, when the efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

GROUP - C (8 marks questions)

1. A closed vessel made of steel plates 4 mm thick with plane carries fluid under pressure of 3 N/mm^2. The diameter of the cylinder is 25 cm and length 75 cm. Calculate the longitudinal and hoop stress. Determine the change in diameter, length and volume of cylinder.
   Take $E = 2.1 \times 10^5$ N/mm^2 and $1/m = 0.28$.

2. A thin cylindrical shaft 2 m long, 20 cm diameter and 1 cm thick is subjected to an internal pressure of 200 kg/cm^2. Take Poisson's ratio as 0.25 and $E = 2 \times 10^6$ kg/cm^2.
   Find i) hoop stress ii) longitudinal stress iii) change in its dimensions and volume.

3. A hollow cylindrical drum 600 mm in diameter has a thickness of 10 mm. If the drum is subjected to an internal air pressure of 3 N/mm^2, determine the increase in volume of the drum. Take $E = 2 \times 10^5$ N/mm^2 and $1/m = 0.3$.

4. A cylindrical thin shell, 80 cm in diameter and 3 m long is having 1 cm metal thickness. If the shell is subjected to an internal pressure of 2.5 MPa, determine (i) change in diameter (ii) change in length (iii) change in volume. Take E = 200 GPa and Poisson's ratio = ¼.

TWO DIMENSIONAL STRESS SYSTEM

PRINCIPAL PLANE:

The three mutually perpendicular planes along which the stresses at a certain point in a body can be resolved at right angle to these planes are known as principal planes. These planes carry only normal stresses but no shear stress.

Out of three stresses one is maximum, one is minimum and the third one is lying between these two stresses.

The plane carrying the maximum normal stress is known as Major principal plane and the plane carrying the minimum normal stress is known as the minor principal plane.

PRINCIPAL STRESS:

The normal stresses across the principal planes are known as principal stresses.

The stress across the major principal plane is known as major principal plane and the stress across the minor principal plane is known as minor principal plane.

DETERMINATION OF PRINCIPAL STRESSES:

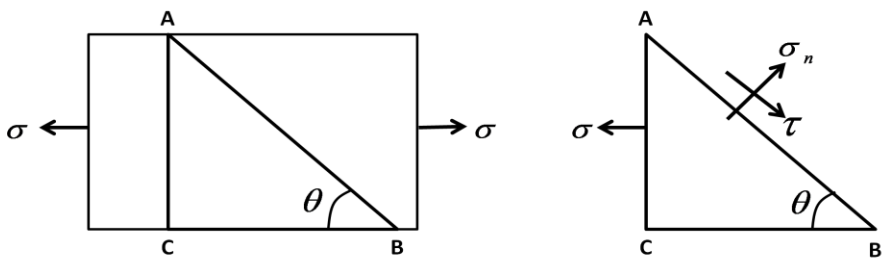
<u>Sign convention</u>

- All tensile stress and strain are positive.
- All compressive stress and strain are negative.
- The shear stress which tends to rotate the body in clockwise direction is positive.
- The shear stress which tends to rotate the body in anti clockwise direction is negative.
- The shear stresses on the vertical faces are positive and the shear stresses on the horizontal faces are negative.

Case-I: **<u>Stresses on an oblique section of a body subjected to direct stress in one plane:</u>**

Consider a rectangular body of unit thickness and uniform area of cross section subjected to direct stress along X-X axis as shown in figure.

Consider an oblique section AB inclined at angle Θ with the X-X axis.



Let σ = tensile stress across the face AC

Θ = Angle between AB and BC.

Consider the equilibrium of ABC.

The horizontal force acting on the face AC = P = σ × AC

Resolving the forces normal to the section AB –

$P_n = P \sin \Theta = \sigma \times AC \times \sin \Theta$

Resolving the forces tangentially to the section AB –

$P_t = P \cos \Theta = \sigma \times AC \cos \Theta$

Normal stress across the section AB $= \sigma_n = \frac{P_n}{AB} = \frac{\sigma \times AC \times \sin\Theta}{AB} = \frac{\sigma \times AC \times \sin\Theta}{\frac{AC}{\sin\Theta}}$

$= \sigma \sin^2\Theta = \frac{\sigma}{2} \times (1 - \cos 2\Theta)$

$= \frac{\sigma}{2} - \frac{\sigma}{2} \times (1 - \cos 2\Theta)$ ------- (1)

Shear stress across the section AB $= \tau = \frac{P_t}{AB} = \frac{\sigma \times AC \times \cos\Theta}{AB} = \frac{\sigma \times AC \times \cos\Theta}{\frac{AC}{\sin\Theta}}$

$= \sigma \sin\Theta \cos\Theta$

$= \frac{\sigma}{2} \sin 2\Theta$ ------------------ (2)

Normal stress across the section AB will be maximum, if $\sin^2\Theta = 1$ or $\Theta = 90^0$

Shear stress across the section AB will be maximum, if $\sin 2\Theta = 1$ or $\Theta = 45^0$ or 135^0.

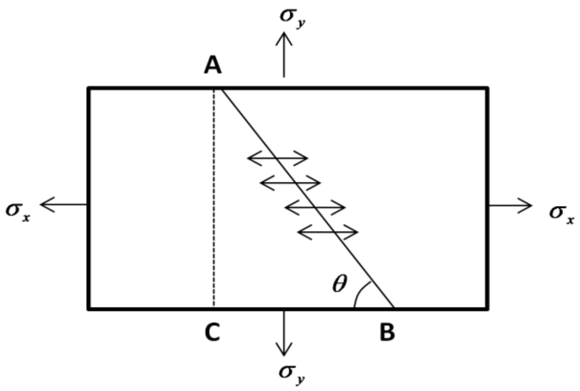
Thus maximum shear stress, when $\Theta = 45^0$ or 135^0 $= \tau_{max} = \sigma/2$

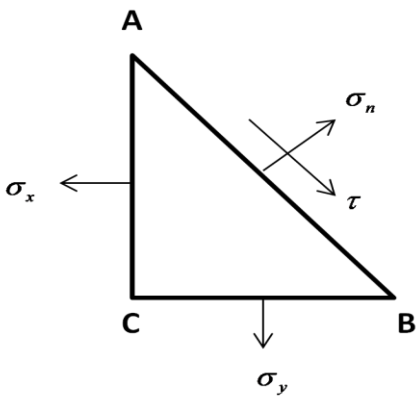
The resultant stress may be obtain from the relation, $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$ ----- (3)

Case-II: **Stresses on an oblique section of a body subjected to direct stress in two mutually perpendicular stresses:**

Consider a rectangular body of unit thickness and uniform area of cross section subjected to direct tensile stresses along X-X axis and Y-Y axis as shown in figure.

Consider an oblique section AB inclined at angle Θ with the X-X axis.





Let σ_x = tensile stress along X-X axis

σ_y = tensile stress along Y-Y axis

Θ = angle made by section AB with X-X axis

Consider the equilibrium of the section ABC

Horizontal force acting on the face AC $= P_x = \sigma_x \times AC$

Vertical force acting on the face AC $= P_y = \sigma_y \times BC$

Resolving the forces normal to the section AB

$P_n = P_x \sin \Theta + P_y \cos \Theta = (\sigma_x \times AC \times \sin \Theta) + (\sigma_y \times BC \times \cos \Theta)$

Resolving the forces tangentially to the section AB –

$P_t = P_x \cos \Theta - P_y \sin \Theta = (\sigma_x \times AC \times \cos \Theta) - (\sigma_y \times BC \times \sin \Theta)$

Normal stress across the section AB $= \sigma_n$

$= \frac{P_n}{AB} = \frac{(\sigma_x \times AC \times \sin\Theta) + (\sigma_y \times BC \times COS\ \Theta)}{AB}$

$$= \frac{(\sigma_x \times AC \times \sin\theta)}{AB} + \frac{(\sigma_y \times BC \times COS\ \theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \sin\theta)}{\frac{AC}{\sin\theta}} + \frac{(\sigma_y \times BC \times COS\ \theta)}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta = \frac{\sigma_x}{2}(1-\cos 2\theta) + \frac{\sigma_y}{2}(1+\cos 2\theta)$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta$$

$$\therefore \quad \boldsymbol{\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta} \quad \text{--------------------------- (1)}$$

Shear stress across the section AB = τ

$$= \frac{P_t}{AB} = \frac{(\sigma_x \times AC \times \cos\ \theta) - (\sigma_y \times BC \times \sin\ \theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \cos\ \theta)}{AB} - \frac{(\sigma_y \times BC \times \sin\ \theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \cos\ \theta)}{\frac{AC}{\sin\theta}} - \frac{(\sigma_y \times BC \times \sin\ \theta)}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta$$

$$= (\sigma_x - \sigma_y)\sin\theta\cos\theta$$

$$\therefore \quad \boldsymbol{\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta} \quad \text{------------------------------------ (2)}$$

Shear stress across the section AB wil be maximum, if $\Theta = 45^0$

Thus maximum shear stress, $\boldsymbol{\tau_{max} = \frac{\sigma_x - \sigma_y}{2}}$

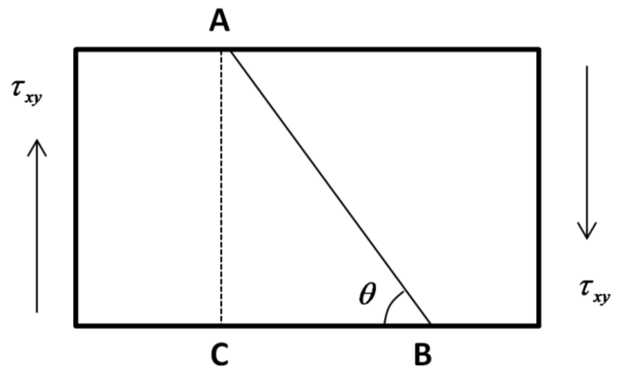
Resultant stress may be obtain from the relation, $\boldsymbol{\sigma_R = \sqrt{\sigma_n^2 + \tau^2}}$ ----- (3)

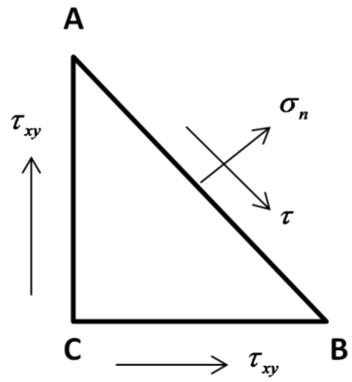
Case-III: **<u>Stresses on an oblique section of a body subjected to shear stress</u>:**

Consider a rectangular body of unit thickness and uniform area of cross section subjected to a shear stress along X-X axis.

Consider an oblique section AB at an angle Θ with the axis X-X as shown in figure.

Let τ_x = shear stress along X-X axis





Consider the equilibrium of the section ABC

The vertical force acting on the face BC = $P_1 = \tau_x \times AC$

The horizontal force acting on the face BC = $P_2 = \tau_x \times BC$

Resolving the forces normal to the section AB

$P_n = P_1 \cos\Theta + P_2 \sin\Theta = \tau_x \cdot AC \cos\Theta + \tau_x \cdot BC \sin\Theta$

Resolving the forces tangential to the section AB

$P_t = P_2 \sin\Theta - P_1 \cos\Theta = \tau_x \cdot BC \sin\Theta - \tau_x \cdot AC \cos\Theta$

Normal stress across the section AB = σ_n

$$= \frac{P_n}{AB} = \frac{(\tau_x \times AC \times \cos\ \Theta) + (\tau_x \times BC \times \sin\ \Theta)}{AB}$$

$$= \frac{(\tau_x \times AC \times \cos\ \Theta)}{AB} + \frac{(\tau_x \times BC \times \sin\ \Theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \cos\ \Theta)}{\frac{AC}{\sin\Theta}} + \frac{(\tau_x \times BC \times \sin\ \Theta)}{\frac{AC}{\sin\Theta}}$$

$$= \tau_x \sin\ \Theta.\cos\Theta + \tau_x \sin\Theta.\cos\Theta$$

$$= 2 \times \tau_x \sin\Theta.\cos\Theta$$

$$= \boldsymbol{\tau_x \sin 2\Theta} \quad \text{-------------------- (1)}$$

Shear stress across the section AB = τ

$$= \frac{P_t}{AB} = \frac{(\tau_x \times BC \times \sin\ \Theta) - (\tau_x \times AC \times \mathrm{COS}\ \Theta)}{AB}$$

$$= \frac{(\tau_x \times BC \times \sin\ \Theta)}{AB} - \frac{(\tau_x \times AC \times \cos\ \Theta)}{AB}$$

$$= \frac{(\sigma_x \times BC \times \sin\ \Theta)}{\frac{BC}{\sin\Theta}} - \frac{(\tau_x \times AC \times \cos\ \Theta)}{\frac{AC}{\sin\Theta}}$$

$$= \tau_x \sin^2\Theta - \tau_x \cos^2\Theta$$

$$= \frac{\tau_x}{2}(1 - \cos 2\Theta) - \frac{\tau_x}{2}(1 + \cos 2\Theta)$$

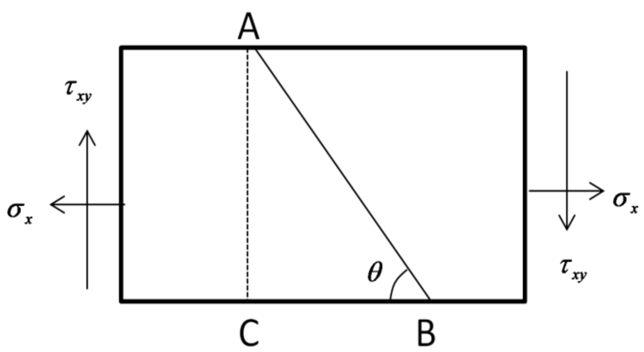
$$= \frac{\tau_x}{2} - \frac{\tau_x}{2}\cos 2\Theta - \frac{\tau_x}{2} - \frac{\tau_x}{2}\cos 2\Theta$$

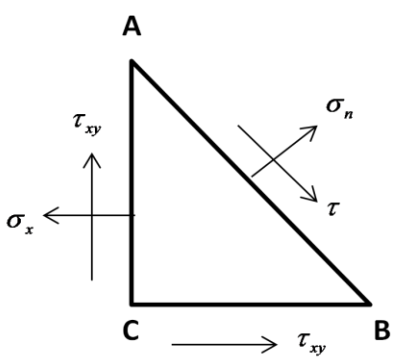
$$= -\ \boldsymbol{\tau_x \cos 2\Theta} \quad \text{---------------------- (2)}$$

Maximum or minimum shear stress may be obtain by equating $-\tau_x \cos 2\Theta = 0$ when $\Theta = 45^0$ or 135^0.

Case-IV: **<u>Stresses on an oblique section of a body subjected to normal stress in one plane and shear stress</u>**

Consider a rectangular body of unit thickness and uniform area of cross section subjected to direct tensile stress along X-X axis and followed by a shear stress along X-X axis as shown in figure.
Consider an oblique section AB at an angle Θ with the X-X axis.





Let σ_x = tensile stress along X-X axis

τ_x = shear stress along X-X axis

Consider the equilibrium of the section ABC

Horizontal force acting on the face AC = $P_x = \sigma_x \times AC$

The vertical force acting on the face AC = $P_y = \tau_x \times AC$

The horizontal force acting on the face BC = $P = \tau_x \times BC$

Resolving the forces normal to the section AB

$$P_n = P_x \sin\Theta - P_y \cos\Theta - P \sin\Theta$$

$$= \sigma_x \times AC \sin\Theta - \tau_x . AC \cos\Theta - \tau_x . BC \sin\Theta$$

Resolving the forces tangential to the section AB

$$P_t = P_x \cos\Theta + P_y \sin\Theta - P \cos\Theta$$

$$= \sigma_x \times AC \cos\Theta + \tau_x . AC \sin\Theta - \tau_x . BC \cos\Theta$$

Normal stress across the section AB = σ_n

$$= \frac{P_n}{AB} = \frac{(\sigma_x \times AC \times \sin\Theta) - (\tau_x \times AC \times \cos\Theta) - (\tau_x \times BC \times \sin\Theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \sin\Theta)}{AB} - \frac{(\tau_x \times AC \times \cos\Theta)}{AB} - \frac{(\tau_x \times BC \times \sin\Theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \sin\Theta)}{\frac{AC}{\sin\Theta}} - \frac{(\tau_x \times AC \times \cos\Theta)}{\frac{AC}{\sin\Theta}} - \frac{(\tau_x \times BC \times \sin\Theta)}{\frac{BC}{\cos\Theta}}$$

$$= \sigma_x \sin^2\Theta - \tau_x \sin\Theta . \cos\Theta - \tau_x \sin\Theta . \cos\Theta$$

$$= \frac{\sigma_x}{2}(1 - \cos 2\Theta) - 2\tau_x \sin\Theta . \cos\Theta$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\Theta - \tau_x \sin 2\Theta \quad \text{------------------------- (1)}$$

Shear stress across the section AB

$$= \frac{P_t}{AB} = \frac{(\sigma_x \times AC \times \cos\Theta) + (\tau_x \times AC \times \sin\Theta) - (\tau_x \times BC \times \cos\Theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \cos\Theta)}{AB} + \frac{(\tau_x \times AC \times \sin\Theta)}{AB} - \frac{(\tau_x \times BC \times \cos\Theta)}{AB}$$

$$= \frac{(\sigma_x \times AC \times \cos\Theta)}{\frac{AC}{\sin\Theta}} + \frac{(\tau_x \times AC \times \sin\Theta)}{\frac{AC}{\sin\Theta}} - \frac{(\tau_x \times BC \times \cos\Theta)}{\frac{BC}{\cos\Theta}}$$

$$= \sigma_x \sin\Theta \cos\Theta + \tau_x \sin^2\Theta - \tau_x \cos^2\Theta$$

$$= \frac{\sigma_x}{2}\sin 2\Theta + \frac{\tau_x}{2}(1 - \cos 2\Theta) - \frac{\tau_x}{2}(1 + \cos 2\Theta)$$

$$= \frac{\sigma_x}{2}\sin 2\Theta + \frac{\tau_x}{2} - \frac{\tau_x}{2}\cos 2\Theta - \frac{\tau_x}{2} - \frac{\tau_x}{2}\cos 2\Theta$$

$$= \frac{\sigma_x}{2}\sin 2\Theta - \tau_x \cos 2\Theta \quad \text{------------------------ (2)}$$

The maximum and minimum normal stress may be obtained by equating the Shear stress to zero.

Maximum principal stress (σ_{p1}) = $\frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x}{2}^2 + \tau_x^2}$ ------------ **(3)**

Minimum principal stress (σ_{p2}) = $\frac{\sigma_x}{2} - \sqrt{(\frac{\sigma_x}{2})^2 + \tau_x^2}$ ------------ **(4)**

Case-V: **<u>Stresses on an oblique section subjected to normal stress along two mutually perpendicular directions and followed by shear stress</u>**

Normal stress (σ_n) = $\frac{\sigma_x + \sigma_y}{2}$ - $\frac{\sigma_x - \sigma_y}{2}$ **cos 2Θ** - τ_x sin 2Θ --------- (1)

Shear stress (τ) = $\frac{\sigma_x - \sigma_y}{2}$ **sin 2Θ** - τ_x cos 2Θ ----------------------------- (2)

Maximum principal stress (σ_{p1}) = $\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2}$ ------------- **(3)**

Minimum principal stress (σ_{p2}) = $\frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2}$ -------------- **(4)**

***** PROBLEM *****

Q-1) ***A rectangular block 10 cm × 5 cm in section is subjected to a tensile load of 500 kN. Determine the normal stress and shear stress on an oblique plane making an angle of 30^0 with the length of the block.***

Ans) <u>Data Given</u>

Tensile load (P) = 500 kN
angle made by the oblique plane (Θ) = 30^0
area of block (A) = 10 cm × 5 cm = 50 cm^2 = 5000 mm^2
Tensile stress (σ) = P/A = (500/5000) kN/mm^2 = 0.1 kN/mm^2 = 100 N/mm^2
Normal stress across the section (σ_n) = $\frac{\sigma}{2} - \frac{\sigma}{2}$ × (1- cos 2Θ)
= $\frac{100}{2} - \frac{100}{2}$ × (1- cos 2×30) = 25 N/mm^2
Shear stress across the section (τ) = $\frac{\sigma}{2}$ sin 2Θ = $\frac{100}{2}$ × sin 2×30 = 43.3 N/mm^2 (**ANS)**

Q-2) ***A rectangular block of 1000 mm^2 cross sectional area is subjected to a longitudinal compressive load of 1000 kN. Determine the normal stress across the cross section of the block. If the block is cut by an oblique plane making an angle of 40^0 with normal section of the block, determine i) normal and tangential stress on the oblique plane ii) resultant stress on the oblique plane.***

Ans) <u>Data Given</u>

Compressive load (P) = 1000 kN
area of cross section of block (A) = 1000 mm^2
angle made by the oblique plane (Θ) = 40^0
compressive stress (σ) = P/A = (1000/1000) kN/mm^2 = 1 kN/mm^2
Normal stress across the section (σ_n) = $\frac{\sigma}{2} - \frac{\sigma}{2}$ × (1- cos 2Θ)
= $\frac{1}{2} - \frac{1}{2}$ × (1- cos 2×40) = 0.58 kN/mm^2
Shear stress across the section (τ) = $\frac{\sigma}{2}$ sin 2Θ = $\frac{1}{2}$ × sin 2×40 = 0.49 N/mm^2
Resultant stress (σ_R) = $\sqrt{\sigma_n^2 + \tau^2}$ = $\sqrt{(0.58)^2 + (0.49)^2}$ = 0.76 kN/mm^2
Let α = angle made by the resultant stress with the oblique plane

then tan α = $\frac{\sigma_n}{\tau} = \frac{0.58}{0.49}$ = 1.19

∴ α = $\tan^{-1}$ (1.19) = 49.99^0 **(ANS)**

Q-3) ***An element in a strained body is subjected to two mutually perpendicular tensile stresses of 300 N/mm^2 and 200 N/mm^2. Determine on a plane inclined at 30^0 to the direction of smaller stress, i) normal stress ii) shear stress iii) resultant stress.***

Ans) Data Given

σ_x = tensile stress along X-X axis = 300 N/mm^2

σ_y = tensile stress along Y-Y axis = 200 N/mm^2

angle made by the oblique plane (Θ) = 30 ^{0}

Normal stress on the inclined plane (σ_n) = $\frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}$ cos 2Θ

= $\frac{300+200}{2} - \frac{300-200}{2}$ cos 2× 30^0

= 250 – 25 = 225 N/mm^2

Shear stress on the inclined plane (τ) = $\frac{\sigma_x - \sigma_y}{2}$ sin 2Θ

= $\frac{300-200}{2}$ sin 2× 30^0 = 43.3 N/mm^2

Resultant stress (σ_R) = $\sqrt{\sigma_n{}^2 + \tau^2}$ = $\sqrt{(225)^2 + (43.3)^2}$ = 303.10 N/mm^2

Let α = angle made by the resultant stress with the oblique plane

then tan α = $\frac{\sigma_n}{\tau} = \frac{225}{43.3}$ = 5.19 ∴

α = $\tan^{-1}$ (5.19) = 81.79^0 **(ANS)**

Q-4) ***The stresses at a point in a component are 100 MPa (tensile) ans 50 MPa (compressive). Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25^0 with tensile stress. Also determine the direction of the resultant stress and the magnitude of the maximum intensity of shear stress.***

Ans) Data Given

Tensile stress along X-X axis (σ_x) = 100 MPa

Compressive stress along Y-Y axis (σ_y) = 50 Mpa

Angle made by the plane with tensile stress (Θ) = 25^0

Normal stress on the inclined plane (σ_n) = $\frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}$ cos 2Θ

= $\frac{100+(-50)}{2} - \frac{100-(-50)}{2}$ cos 2× 25^0

= - 23.21 MPa

Shear stress on the inclined plane (τ) = $\frac{\sigma_x - \sigma_y}{2}$ sin 2Θ

= $\frac{100-(-50)}{2}$ sin 2× 25^0 = 57.45 MPa

Let α = angle made by the resultant stress with the oblique plane

then tan α = $\frac{\sigma_n}{\tau} = \frac{57.45}{-23.21}$ = - 2.4752

∴ α = $\tan^{-1}$ (- 2.4752) = - 68^0

Maximum shear stress (τ_{max}) = $\pm \frac{\sigma_x + \sigma_y}{2} = \pm \frac{100-(-50)}{2} = \pm$ 75 MPa **(ANS)**

Q –5) ***A plane element in a body is subjected to a tensile stress of 100 MPa accompanied by a shear stress of 25 MPa. Find i) the normal and shear stress on a plane inclined at an angle of 20^0 with the tensile stress and ii) the maximum shear stress on the plane.***

Ans) Data Given

Tensile stress along X-X axis (σ_x) = 100 MPa
Shear stress (τ_{xy}) = 25 MPa
angle made by the plane with tensile stress (Θ) = 20^0

Normal stress on the inclined plane (σ_n) = $\frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\Theta - \tau_{xy}\sin 2\Theta$

$= \frac{100}{2} - \frac{100}{2}\cos(2\times20^0) - 25\times\sin(2\times20^0)$

= - 4.37 MPa

Shear stress on the inclined plane (τ) $= \frac{\sigma_x}{2}\sin 2\Theta - \tau_{xy}\cos 2\Theta$

$= \frac{100}{2}\sin(2\times20^0) - 25\times\cos(2\times20^0)$

= 12.99 MPa

Maximum shear stress on the plane (τ_{max}) = $\sqrt{(\frac{\sigma_x}{2})^2 + (\tau_{xy})^2}$ = $\sqrt{(\frac{100}{2})^2 + (25)^2}$

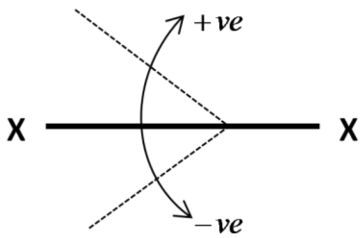
= 55.9 MPa **(ANS)**

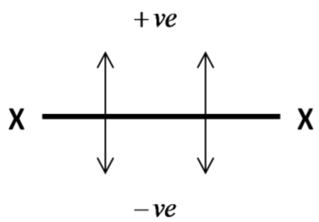
MOHR'S CIRCLE:

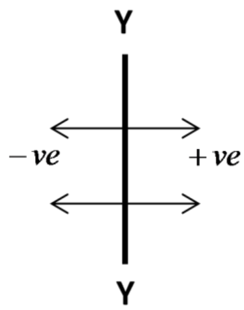
Mohr's circle is a graphical method which is used to determine the normal, shear and resultant stresses.

Sign Convention:

- The angle is taken with respect to X-X axis. All the angles shown in anticlockwise direction to X-X axis are negative and all the angles shown in clockwise direction to X-X axis are positive.







- The measurements above X-X axis are positive and below X-X axis are negative. The measurements to the right of Y-Y axis are positive and to the left of Y-Y axis are negative.

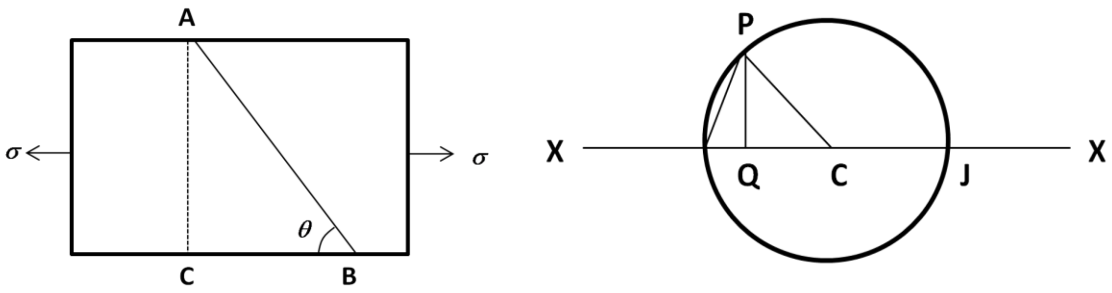
CONSTRUCTION OF MOHR'S CIRCLE

1. **Stresses on an oblique section of a body subjected to a direct stress in one plane:**

- Draw a horizontal line XOX
- Cut OJ equal to tensile stress (σ) to some suitable scale. Divide OJ into two equal parts as OC and CJ. The point O represents the stress system on plane BC and the plane J represents stress system on plane AC.
- Take C as the centre and OC as the radius and draw the circle. This is known as Mohr's circle for stresses.

- Through C draw a line CP at an angle of 2ϴ with CO in clockwise direction meeting the circle at P. The point P represents the section AB.
- Through P, draw PQ perpendicular to OX. Join OP.
- In figure OQ, QP and OP gives the value of normal stress, shear stress and resultant stress.
- The angle POJ is called the angle of obliquity (ϴ).
- When 2ϴ is equal to 90^0 or 270^0 then maximum shear stress will obtain. $\angle PCQ = 2\Theta$

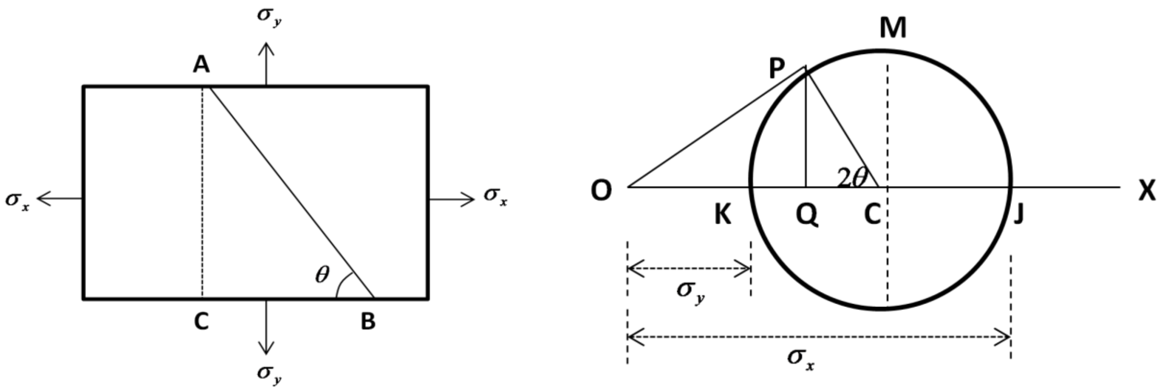
MOHR'S CIRCLE



2. **Stresses on an oblique section of a body subjected to direct stresses in two mutually perpendicular directions:**

- Draw a horizontal line OX. Cut off OJ and OK equal to the tensile stresses σx and σy to some suitable scale towards right of O.
- The point J represents the stress system on the plane AC and point K represents the stress system on the plane BC. Bisect JK at C.
- Take C as the centre and CJ as the radius and draw a circle. It is known as Mohr's circle of stresses.
- Draw a line CP through C making an angle 2ϴ with CK in clockwise direction. CP line meets the circle at P. The point P represents the stress systems on the section AB.
- Draw perpendicular PQ from point P to the axis OX. Join OP.
- In figure OQ, QP and OP gives the value of normal stress, shear stress and resultant stress.
- CM and CN give the maximum shear stress to the scale. The angle POC is called as the angle of obliquity.

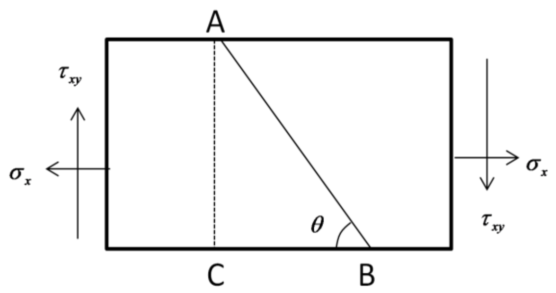
MOHR'S CIRCLE

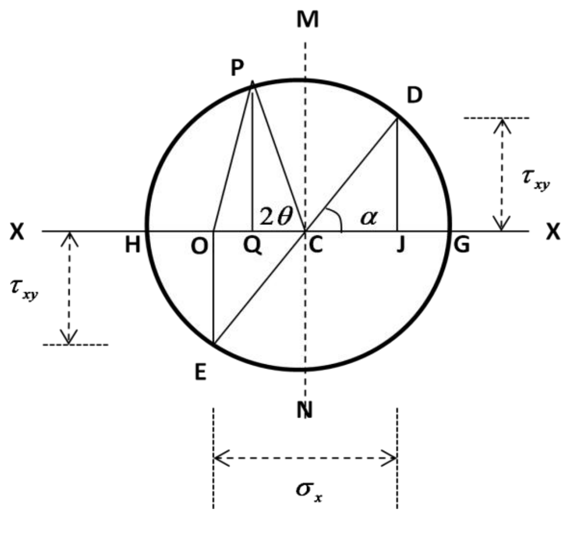


3. **Stresses on an oblique section of a body subjected to a direct stresses in one plane with a shear stress:**

- Draw a horizontal line XOX.
- Cut off OJ equal to the tensile stress σ_x to some suitable scale towards right.
- Draw a perpendicular from J vertically upward and cut JD equal to τ_{xy} to scale. The point D represents the stress system on plane AC.
- Draw a perpendicular from O vertically downward and cut OE equal to τ_{xy} to scale. The point E represents the stress system on plane BC.
- Join DE and bisect it at C.
- Take C as the centre and CD as the radius and draw a circle. It is known as the Mohr's circle of stresses.
- Draw a line CP through C making an angle 2Θ with CE in clockwise direction. The point P will lie on the circle which represents the stress system on plane AB.
- Draw a perpendicular PQ through P to meet the line XOX at Q. Join OP.
- In figure OQ, QP and OP give the normal, shear and resultant stresses to scale. The angle POC is called the angle of obliquity.

MOHR'S CIRCLE

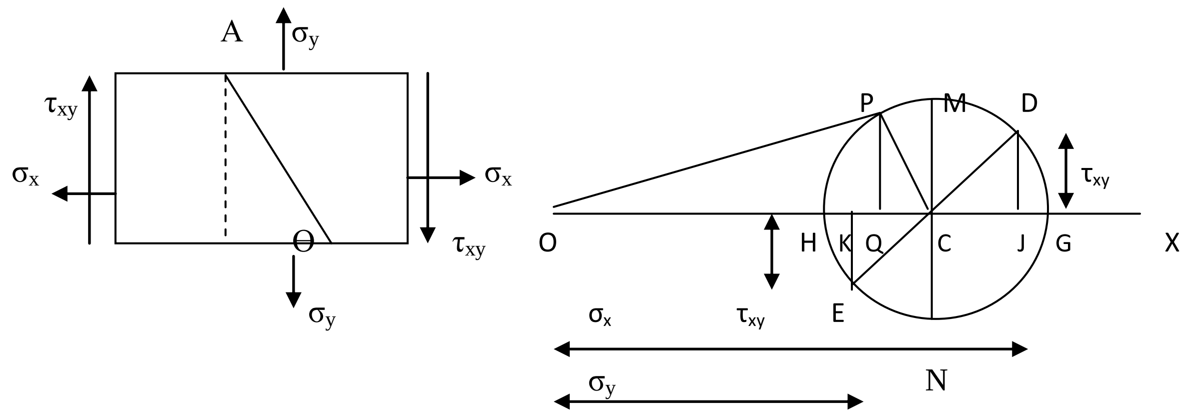




4. **Stresses on an oblique section of a body subjected to direct stresses in two mutually perpendicular directions with a shear stress:**

- Draw a horizontal line OX.
- Cut off OJ and OK equal to tensile stresses σ_x and σ_y respectively to some suitable scale towards right.
- Draw a perpendicular through J vertically upward and cut off JD equal to the shear stress τ_{xy} to scale. The point D represents the stress system on plane AC.
- Draw a perpendicular through K vertically downward and cut KE equal to the shear stress τ_{xy} to scale. The point E represents the stress system on plane BC.
- Join DE and bisect it at C.
- Take C as the centre and radius equal to CD and draw a circle. It is known as Mohr's circle of stresses.

- Draw a line CP through C making an angle 2Θ with CE in clockwise direction meeting the circle at P. The point P represents the stress system on section AB.
- Draw a perpendicular PQ to the line OX through P. Join OP.
- In figure OQ, QP and OP give the normal, shear and resultant stress respectively to scale.
- OG and OH give the maximum and minimum principal stresses to scale. The angle POC is called as the angle of obliquity.



MOHR'S CIRCLE

***** ASSIGNMENT – 04 *****

Q-1) The stresses at a point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55^0 with the axis of major tensile stress. Also find the magnitude of the maximum shear stresses in the component.

Q- 2) The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25^0 with tensile stresses. Also determine the direction of the resultant stresses and the magnitude of the maximum intensity of shear stress.

Q-3) A plane element in a rigid body is subjected to a tensile stress of 100 MPa accompanied by a clockwise shear stress of 25 MPa. Find (i) the normal and shear stress on a plane inclined at an angle of 20^0 with the tensile stress; and (ii) the maximum shear stress on the plane.

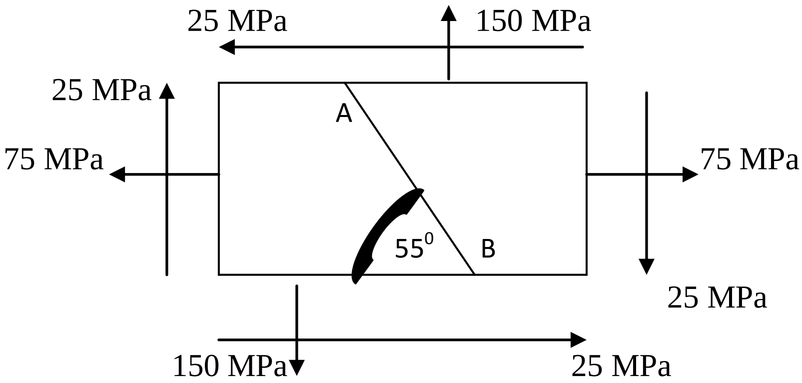
Q-4) An element in a strained body is subjected to a tensile stress of 150 MPa and a shear stress of 50 MPa tending to rotate the element in an anticlockwise direction. Find (i) the magnitude of the normal and shear stresses on a section inclined at 40^0 with the tensile stress; and (ii) the magnitude and direction of maximum shear stress that can exist on the element.

Q-5) An element in a strained body is subjected to a compressive stress of 200 MPa and a clockwise shear stress of 50 MPa on the same plane. Calculate the values of normal and shear stresses on a plane inclined at 35^0 with the compressive stress. Also calculate the value of maximum shear stress in the element.

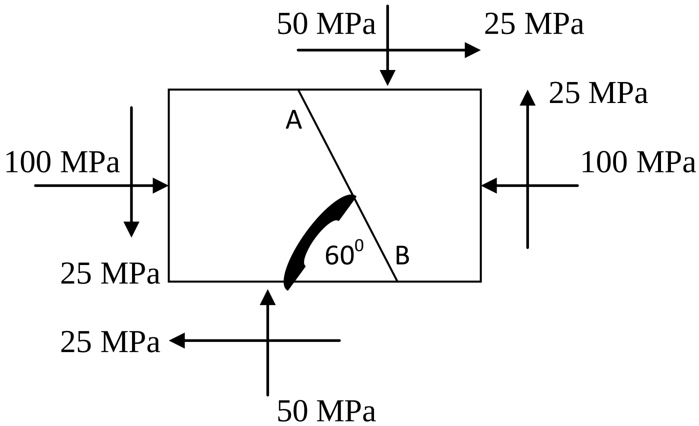
Q-6) A point is subjected to a tensile stress of 250 MPa in the horizontal direction and another tensile stress of 100 MPa in the vertical direction. The point is also subjected to a simple shear stress of 25 MPa, such that when it is associated with the major tensile stress, it tends to rotate the element in the clockwise direction. What is the magnitude of the normal and shear stresses inclined on a section at an angle of 20^0 with the major tensile stress?

Q-7) A plane element in a boiler is subjected to tensile stresses of 400 MPa on one plane and 150 MPa on the other at right angle to the former. Each of the above stresses is accompanied by shear stress of 100 MPa such that when associated with the major tensile stress tends to rotate the element in an anticlockwise direction. Find (i) principal stresses and their directions and (ii) maximum shearing stresses and directions of the plane on which they act.

Q-8) A point in a strained material is subjected to the stresses as shown in figure. Find the normal and shear stresses on the section AB.



Q-9) A machine component is subjected to the stresses as shown in figure. Find the normal and shearing stresses on the section AB inclined at an angle of 600 with X-X axis. Also find the resultant stress on the section.



✦✦✦✦

(Refer R.S Khurmi – Strength of Material book)

SHEAR FORCE & BENDING MOMENT

BEAM:

It is a member of a structure which can carry forces or couple acting on it.

TYPES OF BEAM:

- **Cantilever beam**: It is a beam whose one end is fixed and other end is free.
- **Simply supported beam**: It is a beam whose both ends are freely supported.
- **Overhanging beam**: It is a beam which is freely supported at any two points and having one or both ends projected beyond these two supports.
- **Continuous beam**: It is a beam which is supported at more than two supports.
- **Fixed beam**: It is a beam whose both ends are fixed or built into its supports.

STATICALLY DETERMINATE BEAMS:

If the reactions of the supports of the beams can be determine by the equations of static equilibrium, such beams are known as statically determinate beams. The value of the support reactions is independent of deformation.

STATICALLY INDETERMINATE BEAMS:

If the reactions of the supports of the beams can't be determine by the equations of static equilibrium, such beams are known as statically indeterminate beams. The value of the support reactions are based on deformation.

TYPES OF LOADING:

- **Concentrated or point load**: It is a load which acts at a point on the beam.
- **Uniformly distributed load**: It is a load which spreads over the entire length or part of the length of the beam at a uniform rate.
- **Non uniformly distributed or uniformly varying load**: It is a load which spreads over the entire length or part of the length of the beam at a non uniform rate.

SHEAR FORCE:

Shear force at any cross section of the beam is the algebraic sum of the vertical components of the forces acting on the beam.

BENDING MOMENT:

The bending moment at any point on a loaded beam is the algebraic sum of the moments of all the forces acting on one side of the point about the point.

SHEAR FORCE DIAGRAM:

It is the diagram which represents the variation/distribution of shear force along the length of the beam.

BENDING MOMENT DIAGRAM:

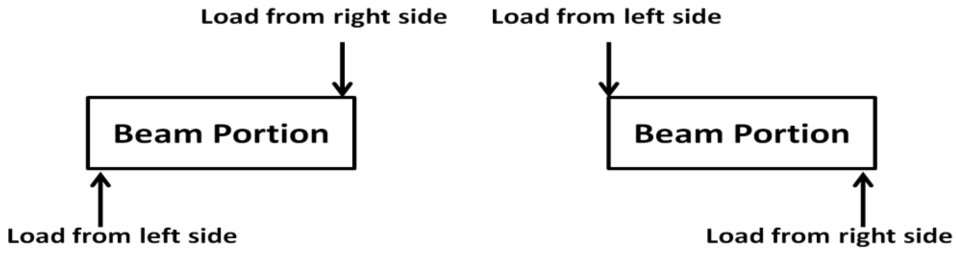
It is the diagram which represents the variation/distribution of bending moment along the length of the beam.

SIGN CONVENTION:

Sign convention is always selected according to the selection of the beam either from the left or right side of the beam.

Sign convention of shear force:

- ♣ Shear force is taken as positive, if the left hand portion of the beam tends to slide upwards and right hand side of the beam tends to slide downward. (figure - 1)
- ♣ Shear force is taken as negative, if the left hand portion tends to slide downwards and the right hand side tends to slide upwards. (figure - 2)



(Positive shear force) FIG-1

(Negative shear force) FIG-2

Sign convention of bending moment:

- ♣ Moment due to all downward forces is taken as negative (hogging moment) and the moment due to all upward forces (sagging moment) are taken as positive.
- ♣ BM is taken as positive, if it is acting in clockwise direction to the left or in anticlockwise direction to the right.
- ♣ BM is taken as negative, if it is acting in anticlockwise direction to the left or in clockwise direction to the right.

Sagging bending moment (positive)

Hogging bending moment (Negative)

SAGGING BENDING MOMENT:

It represents the positive bending moment. If the bending moment tends to bend the beam to produce concavity at the point of curvature, the bending moment is known as sagging bending moment.

HOGGING BENDING MOMENT:

It represents the negative bending moment. If the bending moment tends to bend the beam to produce convexity at the point of curvature, the bending moment is known as hogging bending moment.

RELATION BETWEEN LOADING, SHEAR FORCE AND BENDING MOMENT:

- ♣ If there is a point load at any section of a beam, then the SF suddenly changes but the BM does not change. The SF line is vertical.
- ♣ If there is no load between any two points of a beam section SF does not change but the BM changes linearly. The SF line is horizontal but BM line is an inclined straight line.
- ♣ If there is a U.D.L between two points, then the SF changes linearly and BM changes according to parabolic law. The SF line is an inclined straight line but the BM line is a parabola.

Determination of Shear force and Bending Moment and Shear force and Bending moment diagram for the following cases:

1. **Cantilever beam with point load (**concentrated loads**)**

   - ➢ *Determine the reaction force at the fixed end.*
   - ➢ *Determine the SF at the required points (including the free and fixed end).*
   - ➢ *Determine the BM at the required points (including the free and fixed end).*
   - ➢ *Draw the SFD*
   - ➢ *Draw the BMD*

2. **Cantilever beam with Uniformly distributed load (**U.D.L**)**

   - ➢ *Determine the reaction force at the fixed end.*
   - ➢ *Determine the SF at the required points (including the free and fixed end).*
   - ➢ *Determine the BM at the required points (including the free and fixed end).*
   - ➢ *Draw the SFD.*
   - ➢ *Draw the BMD.*

3. **Cantilever beam with U.D.L and point load**

   - ➢ *Determine the reaction force at the fixed end.*
   - ➢ *Determine the SF at the required points (including the free and fixed end).*
   - ➢ *Determine the BM at the required points (including the free and fixed end).*
   - ➢ *Draw the SFD.*
   - ➢ *Draw the BMD.*

4. **Simply supported beam with point load**

   - ➢ *Determine the reaction force at the simply supported ends.*

- *Determine the SF at the required points (including end points and the point where SF changes its sign).*
- *Determine the BM at the required points (including the end points where BM is zero and the point where there is maximum BM).*
- *Draw the SFD.*
- *Draw the BMD.*

5. Simply supported beam with U.D.L

- *Determine the reaction force at the simply supported ends.*
- *Determine the SF at the required points (including end points and the point where SF changes its sign).*
- *Determine the BM at the required points (including the end points where BM is zero and the point where there is maximum BM).*
- *Draw the SFD.*
- *Draw the BMD.*

6. Simply supported beam with U.D.L and point load

- *Determine the reaction force at the simply supported ends.*
- *Determine the SF at the required points (including end points and the point where SF changes its sign).*
- *Determine the BM at the required points (including the end points where BM is zero and the point where there is maximum BM).*
- *Draw the SFD.*
- *Draw the BMD.*

***** ASSIGNMENT – 05 *****

GROUP – A (2 mark questions)

1. Define cantilever beam.
2. Name the various types of beams.
3. Define Shear force and bending moment diagram.
4. Define point of contraflexure.
5. What is the maximum bending moment for a simply supported beam with UDL over the entire span?
6. What is the maximum bending moment for a simply supported beam carrying a point load at its middle?

GROUP – B (5 mark questions)

1. State the relationship between loading, shear force and bending moment.
2. Draw the shear force and bending moment diagram for a cantilever beam carrying UDL of w/unit metre over its whole span of *l*.
3. A cantilever beam of 8 m length is loaded with a point load of 50 kg at its free end and UDL of 10 kg/m over 4 m from its fixed end. Sketch the SF and BM diagram.

4. A cantilever beam AB, 2 m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw the SF and BM diagram for the beam.
5. A cantilever beam of length 2 m is subjected to load 500 N at its free end 800 N at a distance 0.5 m from its free end, Determine the SF and BM diagram for the beam.
6. Draw the SFD and BMD for a simply supported beam carrying a point load at its middle.
7. Draw the SFD and BMD for a simply supported beam carrying a UDL of w/unit meter length over its whole span.

GROUP – C (10 mark questions)

1. Draw the shear force and bending moment diagram for a cantilever beam of span 2 m subjected to load 500 N at the free end and loads 800 N, 300 N, 400 N at distance 0.5 m, 1 m, and 1.5 m from its free end.
2. A beam AB 10 m long is simply supported at its ends A and B. It carries a UDL of 20 kN/m for a distance of 5 m from the left end A and a concentrated load of 40 kN at a distance of 2 m from the right end B. Draw the S.F and B.M diagrams for the beam.
3. A beam AB 10 m long has supports at its ends A and B. It carries a point load of 5 kN at 3 m from A and a point load of 5 kN at 7 m from A and a UDL of 1 kN per metre between the point loads. Draw the S.F and B.M diagrams for the beam.

THEORY OF SIMPLE BENDING

SIMPLE BENDING:

When a straight beam is subjected to a couple at its ends, constant bending moment will produce at every cross section of the beam with no shear stress. This condition the beam is called as pure torsion or simple bending.

ASSUMPTION IN PURE BENDING:

- The material of the beam is same throughout (homogeneous) and possesses same elastic properties (isotropic) in all direction.
- The transverse section of the beam which is plane before bending remains same after bending.
- Young's modulus for the material is same in tension and compression.
- Each layer of the beam is free to expand or contract independently.
- The radius of curvature of the beam after bending is large as compared to its cross sectional dimensions.
- Loads act perpendicular to the axis of the beam.
- The beam is in equilibrium i.e. the resultant pull or thrust on the transverse section of the beam is zero.

VARIOUS TYPES OF SECTIONS:

Various types of engineering sections may be classified as symmetrical, unsymmetrical and built-up sections.

- Symmetrical sections: In this type of sections the centre of gravity lies at the geometrical centre of the sections. The neutral axis passes through the geometrical centre of the section. Ex – circular/ square/ rectangular sections.
- Unsymmetrical sections: In this type of sections the centre of gravity does not lies at the geometrical centre of the sections. Neutral axis passes through the centre of gravity.
  Ex – T, L, I sections.

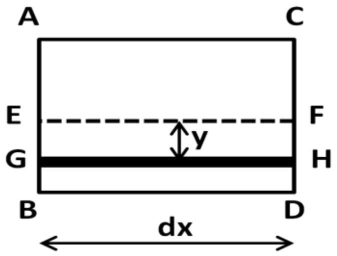
THEORY OF SIMPLE BENDING:

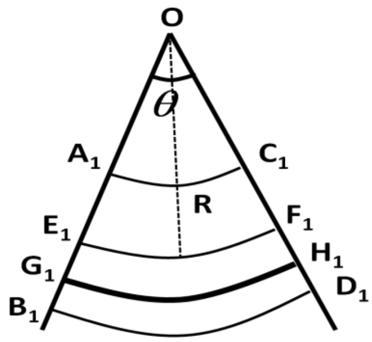
A small portion of a beam ABCD is shown in figure-1.
AB and CD are the two planes.
Let this portion is subjected to simple bending.
Let EF is the length of neutral layer.
Consider another layer GH at a distance 'y' from the neutral axis.
Consider all the layers bend as shown in figure – 2.
The layers AB, CD, EF and GH deformed to A_1B_1, C_1D_1, E_1F_1 and G_1H_1 respectively.
All the layers above EF get compressed and the layers below EF get elongated.

As EF is the neutral layer, its length will remain unchanged.
So $EF = E_1F_1$
Let all the layers bend into arcs with centre 'O' of radius of curvature 'R' and angle between the planes is 'θ'.
Before bending lengths of all layers is same i.e. $GH = EF = E_1F_1$
After deformation $G_1H_1 = (R+ y)\ \theta$
$EF = E_1F_1 = R\ \theta$

Change in length of the layer GH = $(R+y)\,\theta - R\,\theta = y\,\theta$

Strain in the layer GH = $e = \frac{change\ in\ length}{original\ length} = \frac{y\,\theta}{R\,\theta} = \frac{y}{R}$





Let, σ = intensity of stress of the layer

E = Young's modulus

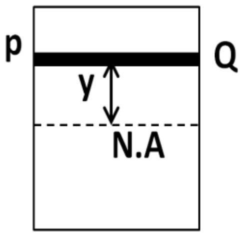
$\therefore\ e = \frac{y}{R} = \frac{\sigma}{E}$ or $\boldsymbol{\frac{\sigma}{y} = \frac{E}{R}}$ **---------- (1)**

or $\sigma = \frac{E}{R} \times y$

MOMENT OF RESISTANCE:

It is the algebraic sum of the moment about neutral axis of the internal forces developed in a beam due to bending.

Consider a beam section as shown in figure.
Consider a small layer PQ at a distance 'y' from the neutral axis.



Let, dA = elementary area of PQ

Intensity of shear stress in the layer PQ = $\sigma = \frac{E}{R} \times y$

Total stress/thrust in the layer PQ = $\frac{E}{R} \times y \times dA$

Moment of resistance offered by the elementary area = Moment of the-
thrust about the neutral axis = total stress × distance = $\frac{E}{R} \times y^2 \times dA$

Total moment of resistance offered by the beam section = $\frac{E}{R} \sum y^2\ dA$

$\sum y^2\ dA$= moment of inertia of the beam section about the neutral axis

Thus $M = \frac{E}{R} \times I$

$\Rightarrow \boldsymbol{\frac{M}{I} = \frac{E}{R}}$ --------------- (2)

We know that, $\boldsymbol{\frac{E}{R} = \frac{\sigma}{y}}$

So we may write $\boldsymbol{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$ __________ **(3)**

This is known as Bending equation.

SECTION MODULUS:

It is the ratio between moment of inertia of the section of the beam and the maximum distance of layer from the neutral axis.

Mathematically, *section modulus* (Z) = $\frac{I}{Y}$

So we may write, $\mathbf{M = \sigma \times Z}$ ________ **(4)**

POLAR MODULUS:

It is the measure of the strength of the shaft in torsion. It is the ratio between the polar moment of inertia of the beam section and the radius of curvature. It is denoted by Z_P. Mathematically, $\mathbf{Z_P = \frac{I_p}{R}}$

***** PROBLEM *****

Q – 1) *A steel plate is bent into a circular arc of radius 10 m. If the plate section be 120 mm wide and 20 mm thick, find the maximum stress induced and the bending moment which can produce this stress. Take E = 2 × 10^5 N/mm^2.*

Ans) Data Given

Radius of curvature (R) = 10 m   width = 120 mm

thickness = 20 mm   E = 2 × 10^5 N/mm^2

Moment of inertia of the section about the neutral axis = I = $\frac{120 \times 20^3}{12}$ = **80000 mm^4**

we know that, $\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$ ⟹ $\sigma = \frac{E}{R} \times y$

∴ $\sigma_{max} = \frac{2 \times 10^5}{10 \times 10^3} \times \left(\frac{20}{2}\right) = \mathbf{200\ N/mm^2}$

Bending moment (M) = $\frac{E}{R} \times l = \frac{2 \times 10^5}{10 \times 10^3} \times 8 \times 10^4 = 16 \times 10^5$ N-mm **(ANS)**

Q – 2) *A cast iron test beam 20 mm × 20 mm in section and 1 m long and supported at the ends fails when a central load of 640 N is applied. What uniformly distributed load will break a cantilever of same material 50 mm wide, 100 mm deep and 2 m long?*

Ans) Data Given

Cast iron test beam : Cross section of = 20 mm × 20 mm
length = 1 m = 1000 mm
load (W) = 640 N

Cantilever : width = 50 mm   depth = 100 mm   length = 2 m = 2000 mm

for the test beam, maximum bending moment = M = $\frac{WL}{4} = \frac{640 \times 1000}{4} = 16 \times 10^4$ N-mm

moment of resistance = $\frac{1}{6}\sigma b d^2 = \frac{1}{6}\sigma \times 20 \times 20^2 = \frac{4000}{3} \times \sigma$ N-mm

Equating these two we get, $\frac{4000}{3} \times \sigma = 16 \times 10^4$

∴ $\sigma = 120$ N/mm^2

for the cantilever consider the UDL required is w/metre run

so, maximum bending moment = M = $\frac{Wl^2}{2} = \frac{W \times 2000^2}{2}$ = 2000 w N-mm

moment of resistance of the section = $\frac{1}{6}\sigma b d^2 = \frac{1}{6} \times 120 \times 50 \times 100^2 = 10 \times 10^6$ N-mm

Equating these two we get, 2000 w = 10 × 10^6

∴ w = 5000 N/m **(ANS)**

***** ASSIGNMENT – 7 *****

GROUP - A (2 mark questions)

4. Define section modulus of a beam.
5. What do you mean by symmetrical section? Give some example.
6. State bending equation for a beam section.
7. Define flexural rigidity.
8. Write the relation between bending moment and bending stress.
9. Write the expression of section modulus of a rectangular section of dimension (b × d).
10. Write the expression of section modulus of a circular section of diameter d.

GROUP – B (6 mark questions)

1. Explain the theory of simple bending.
2. State the assumption of pure bending.
3. A cast iron cantilever at length 1.5 m fails when a load of 1920 N is applied at the free end. Determine the stress at failure if the section of the cantilever is 40 mm × 60 mm.
4. A cast iron pipe of external diameter 60 mm, 10 mm thickness and 5 m long is supported at its ends. The pipe carries a point load of 100 N at its centre. Calculate the maximum flexural stress induced due to point load.
5. Find the maximum UDL that a beam can support over a span of 10 m if the maximum stress is limited to 500 kg/cm^2 and section modulus of the beam is 128 cm^3.

COMBINED DIRECT & BENDING STRESS

Column: It is a vertical bar or member of a structure, which always subjected to axial compressive forces.

Short column: It is a column whose length is less than 8 times the diameter and slenderness ratio less than 32. This type of column may fail due to direct stress.

Medium column: It is a column whose length varies from 8 to 30 times their diameter and slenderness ratio varies from 32 to 120.

Long column: It is a column whose length is more than 30 times the diameter and slenderness ratio more than 120. This type of column may fail due to bending stress.

Slenderness ratio: It is the ratio of the length of the column to the minimum radius of gyration of the cross sectional area of the column.

Buckling Load: It is the minimum limiting load acting axially on a column due to which the lateral displacement or buckling may appear on a column. It is also known as crippling or critical load.

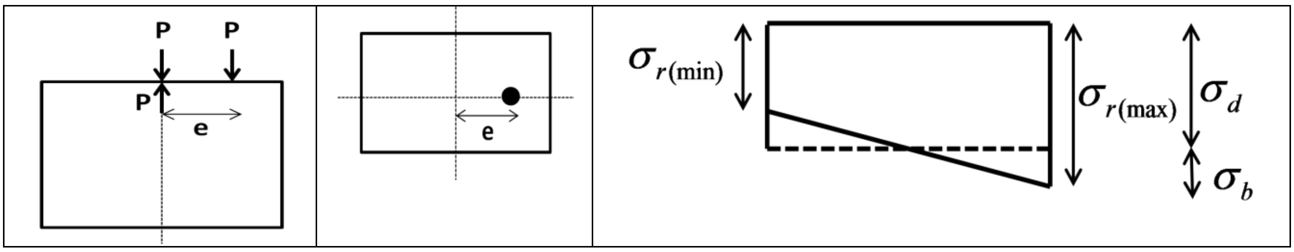
Eccentric load: If the line of action of a load does not coincide with the line passing through the body subjected to load, such a load is called eccentric load.

Eccentricity: It is the distance between the line of action of eccentric load from the centroid of the body.

Stress in short column under eccentric loading:

1. Load acting eccentrically to one axis:

Consider a short column subjected to an eccentric load ‘P’ at a distance ‘e’ from its axis as shown in figure. From figure; at point ‘O’, there is two equal and opposite loads ‘P’.
The downward load ‘P’ acting at ‘O’ causes a direct stress.
The upward force acting at ‘O’ and the eccentric load causes a clockwise couple.



Let A = area of cross section of the column
σ_d = direct stress due to load ‘P’ applied axially
σ_b = bending stress at a distance ‘y’ from the neutral axis.
σ_r = resultant of direct and bending stress

Thus Direct stress $= \boldsymbol{\sigma_d} = \frac{\mathbf{P}}{\mathbf{A}}$ ---------------------- (1)

Bending stress $= \boldsymbol{\sigma_b} = \frac{\mathbf{M.y}}{\boldsymbol{l}} = \frac{\mathbf{P.e.y}}{\boldsymbol{l}}$ --------------------- (2)

Resultant stress $= \boldsymbol{\sigma_r} = \boldsymbol{\sigma_d} \pm \boldsymbol{\sigma_b} = \frac{\mathbf{P}}{\mathbf{A}} \pm \frac{\mathbf{P.e.y}}{\boldsymbol{l}}$ -------- (3)

Maximum resultant stress $= \boldsymbol{\sigma_d} + \boldsymbol{\sigma_b}$, when σ_b is tensile
Minimum resultant stress $= \boldsymbol{\sigma_d} - \boldsymbol{\sigma_b}$, when σ_b is compressive

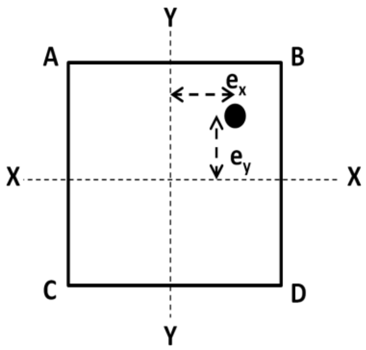
Conditions:

- When $\boldsymbol{\sigma_d} > \boldsymbol{\sigma_b}$; $\sigma_{r\,(max)}$ and $\sigma_{r\,(min)}$ both are positive. Compressive stresses act anywhere of the section.
- When $\boldsymbol{\sigma_d} = \boldsymbol{\sigma_b}$; $\sigma_{r\,(max)} = 2\,\sigma_d = 2\,\sigma_b$ and $\sigma_{r\,(min)} = 0$. Compressive stresses act anywhere of the section and varies from zero at one edge to maximum at other.
- When $\boldsymbol{\sigma_d} < \boldsymbol{\sigma_b}$; $\sigma_{r\,(max)} = \sigma_d + \sigma_d$, which is positive and $\sigma_{r\,(min)} = \sigma_d - \sigma_b$, which is negative. $\sigma_{r\,(max)}$ is compressive and $\sigma_{r\,(min)}$ is tensile.

2. Load acting eccentrically to both axis:

Consider a short column subjected to an eccentric load 'P' with eccentricity about both axes as shown in figure.

Let A = area of cross section of the column
P = load acting on the column
e_x = eccentricity of the load about Y-Y axis
e_y = eccentricity of the load about X-X axis



Moment of the load about X-X axis $= M_x = P \times e_y$
Moment of the load about Y-Y axis $= M_y = P \times e_x$
Direct stress due to load P $= \sigma_d = \frac{P}{A}$
Bending stress due to eccentricity $e_x = \sigma_{by} = \frac{P.e_x.x}{I_{yy}} = \frac{M_y.x}{I_{yy}}$
Bending stress due to eccentricity $e_y = \sigma_{bx} = \frac{P.e_y.y}{I_{xx}} = \frac{M_x.y}{I_{xx}}$

Resultant stress $= \boldsymbol{\sigma_r} = \frac{\mathbf{P}}{\mathbf{A}} \pm \frac{\mathbf{M_x.y}}{\mathbf{I_{xx}}} \pm \frac{\mathbf{M_y.x}}{\mathbf{I_{yy}}}$

Maximum stress at B $= \sigma_{B\,(max)} = \frac{P}{A} + \frac{M_x.y}{I_{xx}} + \frac{M_y.x}{I_{yy}}$ -------------- (1)

Minimum stress at C $= \sigma_{C\,(min)} = \frac{P}{A} - \frac{M_x.y}{I_{xx}} - \frac{M_y.x}{I_{yy}}$ -------------- (2)

Stress at A $= \sigma_A = \frac{P}{A} + \frac{M_x.y}{I_{xx}} - \frac{M_y.x}{I_{yy}}$ ------------------------ (3)

Stress at D $= \sigma_D = \frac{P}{A} - \frac{M_x.y}{I_{xx}} + \frac{M_y.x}{I_{yy}}$ ----------------------- (4)

Limit of eccentricity:

Consider the stresses are compressive and no tensile stress in a column.

For this case $\sigma_b \le \sigma_d \quad \Rightarrow \frac{P.e}{Z} \le \frac{P}{A}$

$\therefore \quad \mathbf{e} \le \frac{\mathbf{Z}}{\mathbf{A}}$ --------------------------- (5)

For rectangular section, $\mathbf{e} \le \frac{\mathbf{b}}{\mathbf{6}}$

For circular section, $e \le \frac{\mathbf{d}}{\mathbf{8}}$

Euler's formula for various end conditions of the column:

Conditions	Formula for Cripping load	
When both ends hinged	$P = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 E I}{l_e^{\,2}}$	P = least buckling load
When both ends fixed	$P = \frac{4\pi^2 E I}{L^2} = \frac{4\pi^2 E I}{(2 l_e)^2}$	E = Young's modulus
When one end fixed and other end hinged	$P = \frac{2\pi^2 E I}{L^2} = \frac{2\pi^2 E I}{(\sqrt{2}\, l_e)^2}$	I = moment of inertia
When one end fixed and other end free	$P = \frac{\pi^2 E I}{4L^2} = \frac{4\pi^2 E I}{4\left(\frac{l_e}{2}\right)^2}$	L = total length of column l_e = equivalent length

***** PROBLEM *****

Q-1) A rectangular strut is 150 mm wide and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.

Ans) Data Given

Width (b) = 150 mm thickness (t) = 120 mm

load (P) = 180 kN = 180×10^3 N eccentricity (e) = 10 mm

area of the strut = $150 \times 120 = 18\,000$ mm²

maximum intensities of stress in the section (σ_{max}) = $\frac{P}{A}\left(1 + \frac{6\,e}{b}\right)$

$= \frac{180 \times 10^3}{18000}\left(1 + \frac{6 \times 10}{150}\right) = 10\,(1 + 0.4) = 14 \text{ N/mm}^2 = 14 \text{ MPa}$

minimum intensities of stress in the section (σ_{min}) = $\frac{P}{A}\left(1 - \frac{6\,e}{b}\right)$

$= \frac{180 \times 10^3}{18000}\left(1 - \frac{6 \times 10}{150}\right) = 10\,(1 - 0.4) = 6 \text{ N/mm}^2 = 6 \text{ MPa}$

Q-2) A hollow circular column having external and internal diameters of 300 mm and 250 mm respectively carries a vertical load of 100 kN at the outer edge of the column. Calculate the maximum and minimum intensities of stress in the section.

Ans) Data Given

External diameter (D) = 300 mm internal diameter (d) = 250 mm

load (P) = 100 kN = 100×10^3 N

area of the circular column (A) = $\frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(300)^2 - (250)^2] = 21.6 \times 10^3 \text{ mm}^2$

section modulus (z) = $\frac{\pi}{32} \times [\frac{D^4 - d^4}{D}] = \frac{\pi}{32} \times [\frac{300^4 - 250^4}{300}] = 1372 \times 10^3 \text{ mm}^3$

as the column carries the vertical load at its outer edge, so we can write e = 150 mm

Moment due to eccentricity of load = M = P . e = $(100 \times 10^3) \times 150 = 15 \times 10^6$ N-mm

Maximum intensities of stress in the section (σ_{max}) = $\frac{P}{A} + \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} + \frac{15 \times 10^6}{1372 \times 10^3}$

= $4.63 + 10.93 = 15.56 \text{ N/mm}^2 = 15.5$ MPa

Minimum intensities of stress in the section (σ_{min}) = $\frac{P}{A} - \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} - \frac{15 \times 10^6}{1372 \times 10^3}$ N/mm^2

= $4.63 - 10.93 = -6.3 \text{ N/mm}^2 = 6.3$ MPa (tension)

***** ASSIGNMENT – 6 *****

GROUP – A (2 mark questions)

1. What is a column?
2. Differentiate between long and short column.
3. Define crippling load.
4. Write the expressions for crippling load for columns for various end conditions.
5. What is eccentric loading?

GROUP – B (5 mark questions)

1. Write short notes on crippling load.
2. Define a column. State the expression for crippling load under various end conditions.
3. A rectangular column 200 mm wide and 150 mm thick is carrying a vertical load of 120 kN at an eccentricity of 50 mm in a plane bisecting the thickness. Determine the maximum and minimum intensities of stress in the section.

GROUP – C (10 mark questions)

1. A rectangular column of 240 mm × 150 mm is subjected to a vertical load of 10 kN placed at an eccentricity of 60 mm in a plane bisecting 150 mm side. Determine stress intensities in the section. **(*Ans*: 695 kN/m^2, -139 kN/m^2)**
2. The line thrust in a compression testing specimen of 2 cm diameter is parallel to the axis of the specimen but is displaced from it. Calculate the distance of the line of thrust from the axis when the maximum stress is 20% greater than the mean stress on a normal section. **(*Ans*: 0.05cm)**
3. A hollow rectangular masonry pier is 1.2 m × 0.8 m wide and 150 mm thickness. A vertical load of 2 MN is transmitted in the vertical plane bisecting 1.2 m side and at an eccentricity of 100 mm from the geometric axis of the section. Calculate the maximum and minimum stress intensities in the section.

TORSION

TORSION:

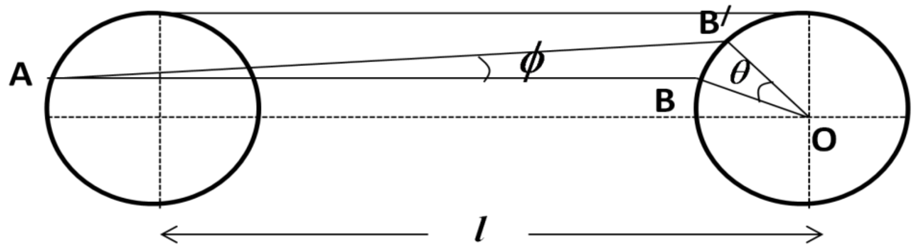
When two equal and opposite torques acts at the two ends of the shaft to produce the possibility of shearing of shaft perpendicular to its longitudinal axis, the shaft is said to be in torsion. Due to torsion or twisting moment every section of shaft subjected to some shear stress.

ASSUMPTION OF PURE TORSION:

- ♣ The material of the shaft is uniform throughout.
- ♣ The twist along the length of the shaft is uniform throughout.
- ♣ The shaft circular in section remains circular after twisting.
- ♣ All diameters which are straight before twist remains straight after twist.
- ♣ The plane of the shaft normal to its axis before twist remains plane after twist.
- ♣ Maximum shear stress induced in the shaft does not exceed its elastic limit.

THEORY OF PURE TORSION:

Consider a circular shaft of radius R and length *l* fixed at one end and its other end is subjected to twisting moment as shown in figure.



Let, the line AB on the surface of the shaft is deformed to $AB^/$ and OB to $OB^/$.

Let, $\angle BAB^/ = ø$ = shear strain and $\angle BOB^/ = \Theta$ = angle of twist

∴ shear strain = $ø = \frac{BB^/}{L} = \frac{R\Theta}{L}$

Let, τ = shear stress on the surface of the shaft, C = modulus of rigidity

we know that, $\frac{\tau}{ø} = C$ or $\frac{\tau}{C} = ø = \frac{R\Theta}{L}$ ⇒ $\frac{\tau}{R} = \frac{C\,\Theta}{L}$ **= constant** ------------- (1)

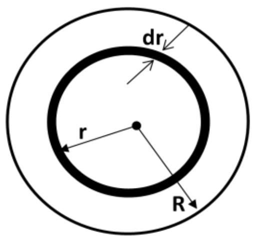
For a shaft the value of C,Θ and L are constants.

Shear stress is maximum at the outer surface and zero at the centre.

TORSIONAL MOMENT OF RESISTANCE:

Consider an elementary ring of the shaft of thickness ‘dr’ at a radius ‘r’ from the centre of the shaft as shown in figure.

Let, τ_1 = shear stress at this radius dA = elementary area of the of the ring



Total force acting on the ring = area of the ring × stress on the ring

$= 2\,\pi\, r \times dr \times \tau_1$

Moment of the force about axis of the shaft $= 2\,\pi\, r \times dr \times \tau_1 \times r$

$= 2\,\pi\, r^2 \times dr \times \tau_1$

$= 2\,\pi\, r^2 \times dr \times \frac{\tau_1}{R} \times r$

$= 2\,\pi\, r^3 \times dr \times \frac{\tau}{R}$

As $\frac{\tau}{R} = \frac{\tau_1}{r} = \frac{C\,\theta}{L}$ = constant, we may write $\boldsymbol{\tau_1 = \frac{\tau}{R} \times r}$

Total moment of resistance of the shaft section $= \int_0^R 2\,\pi\, r^3 \times dr \times \frac{\tau}{R}$

Total resisting moment = Applied torque

So, we may write

$$T = \int_0^R 2\,\pi\, r^3 \times dr \times \frac{\tau}{R} = 2\,\pi \frac{\tau}{R} \int_0^R r^3 \times dr$$

$$= 2\,\pi \times \frac{\tau}{R} \times \left[\frac{R^4}{4}\right]_0^R = 2\,\pi \times \frac{\tau}{R} \times \frac{R^3}{4}$$

$$= \frac{2\,\pi\,\tau\,(D/2)^2}{4} = \frac{\pi}{16} \times D^3 \times \tau$$

$$\therefore \quad \mathbf{T = \frac{\pi}{16} \times \tau \times D^3}$$

Relation:

$$T = \frac{\pi}{16} \times \tau \times D^3 \times \frac{D}{2} \times \frac{2}{D} = \frac{\pi}{32} \times \tau \times D^4 \times \frac{2}{2R}$$

$$= I_P \times \frac{\tau}{R}$$

Where $I_p = \frac{\pi}{32} \times D^4$ = **polar moment of inertia** of the section of the shaft.

$\Rightarrow \quad \frac{T}{I_P} = \frac{\tau}{R}$ We know that, $\frac{\tau}{R} = \frac{C\,\theta}{L}$

Comparing the two equation we get

$$\frac{T}{I_P} = \frac{\tau}{R} = \frac{C\,\theta}{L} \quad \text{-------------------- (2)}$$

This is known as the **torsion equation.**

POLAR MOMENT OF INERTIA:

The moment of inertia of a plane area about an axis perpendicular to the plane of the area is called polar moment of inertia of the area with respect to the point at which the axis intersects the plane.

For solid shafts it is given by $I_p = \frac{\pi}{32} \times D^4$

For hollow shafts it is given by $I_p = \frac{\pi}{32} \times (D^4 - d^4)$

POLAR MODULUS:

It is the ratio of polar moment of inertia to the radius of the shaft.

We know that, $\frac{T}{I_P} = \frac{\tau}{R}$

$\Rightarrow \quad \mathbf{T = \tau \times \frac{I_P}{R} = \tau \times Z_P}$ where, Z_P = polar modulus of the shaft section

STRENGTH OF A SOLID SHAFT:

Consider an elementary ring of the shaft of thickness 'dr' at a radius 'r' from the centre of the shaft as shown in figure.

Let, τ_1 = shear stress at this radius

dA = elementary area of the of the ring

Total force acting on the ring = area of the ring × stress on the ring
$= 2\pi r \times dr \times \tau_1$

Moment of the force about axis of the shaft $= 2\pi r \times dr \times \tau_1 \times r$
$= 2\pi r^2 \times dr \times \tau_1$
$= 2\pi r^2 \times dr \times \frac{\tau_1}{R} \times r$
$= 2\pi r^3 \times dr \times \frac{\tau}{R}$

As $\frac{\tau}{R} = \frac{\tau_1}{r} = \frac{C\,\theta}{L}$ = constant, we may write $\boldsymbol{\tau_1 = \frac{\tau}{R} \times r}$

Total moment of resistance of the shaft section = $\int_0^R 2\pi r^3 \times dr \times \frac{\tau}{R}$

Total resisting moment = Applied torque

So, we may write

$T = \int_0^R 2\pi r^3 \times dr \times \frac{\tau}{R} = 2\pi \frac{\tau}{R} \int_0^R r^3 \times dr$

$= 2\pi \times \frac{\tau}{R} \times [\frac{R^4}{4}]_0^R = 2\pi \times \frac{\tau}{R} \times \frac{R^3}{4}$

$= \frac{2\pi\tau\,(D/2)^2}{4} = \frac{\pi}{16} \times D^3 \times \tau$

$\therefore$ $\mathbf{T = \frac{\pi}{16} \times \tau \times D^3}$

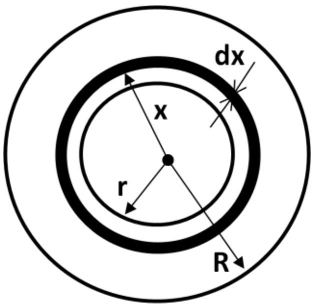
STRENGTH OF A HOLLOW SHAFT:

Consider a hollow shaft subjected to torque T.

Let, R = outer radius of shaft; r = inner radius of the hollow shaft

D = outer diameter of the hollow shaft; d = inner diameter of the hollow shaft

τ = maximum shear stress



Consider a small circular ring of thickness 'dx' at a radius x form the centre of the shaft.

τ_1 = shear stress at this radius

Total force acting on the ring = area of the ring × stress on the ring
$= 2\pi x \times dx \times \tau_1$

Moment of resistance of force on the ring $= 2\pi x \times dx \times \tau_1 \times x$
$= 2\pi x^2 \times dr \times \tau_1$
$= 2\pi x^2 \times dx \times \frac{\tau_1}{R} \times x$
$= 2\pi x^3 \times dx \times \frac{\tau}{R}$

As $\frac{\tau}{R} = \frac{\tau_1}{x} = \frac{C\,\theta}{L}$ = constant, we may write $\boldsymbol{\tau_1 = \frac{\tau}{R} \times x}$

Total moment of resistance of the shaft section = $\int_r^R 2\pi x^3 \times dx \times \frac{\tau}{R}$

Total resisting moment = Applied torque

So, we may write $T = \int_r^R 2\pi x^3 \times dx \times \frac{\tau}{R} = 2\pi \frac{\tau}{R} \int_0^R x^3 \times dx$

$= 2\pi \times \frac{\tau}{R} \times [\frac{x^4}{4}]_r^R = 2\pi \times \frac{\tau}{R} \times [\frac{R^4}{4} - \frac{r^4}{4}]$

$$= \quad 2\pi \times \frac{\tau}{4R} \times (R^4 - r^4) = \frac{\tau}{R} \times \frac{\pi}{2} \times (R^4 - r^4)$$

$$= \quad \frac{\tau}{\frac{D}{2}} \times \frac{\pi}{32} \times (D^4 - d^4) = \frac{\pi}{16}\ \tau \times \left[\frac{D^4 - d^4}{D}\right]$$

This is the required equation of the hollow shaft.

POWER TRANSMITTED BY A SHAFT:

Work done by the shaft per minute = Torque × angle turned in one minute

$= T_{mean} \times 2\pi N$ N-m

Power transmitted by the shaft (P) $= \frac{\text{Work done per minute}}{60000}$ KW

$$\Rightarrow \quad \mathbf{P} = \frac{\mathbf{Tmean \times 2\pi N}}{\mathbf{60000}}\ \text{kW}$$

TORSIONAL RIGIDITY:

It is the torque which produces the twist of one radian in a shaft of unit length. It is denoted by **$C.I_P$**. Mathematically, $\frac{T}{I_P} = \frac{C\theta}{L}$ or $C\theta = \frac{\mathbf{T}}{\mathbf{I_P}} \times \mathbf{L}$

COMPARISON BETWEEN STRENGTH OF SOLID AND HOLLOW SHAFT:

We know that, Strength of solid shaft = $T_s = \frac{\pi}{16} \times \tau \times D_s^3$

Strength of hollow shaft = $T_H = \frac{\pi}{16}\ \tau \times \left[\frac{D^4 - d^4}{D}\right]$

$$\therefore \quad \frac{T_H}{T_S} = \frac{\frac{\pi}{16}\ \tau \times \left[\frac{D^4 - d^4}{D}\right]}{\frac{\pi}{16} \times \tau \times (D_S)^3} = \frac{D^4 - d^4}{D\,(D_S)^3} \quad \text{----------- (1)}$$

Let, $\frac{D}{d} = n$ or $D = nd$, substituting this value in equation-1 we get,

$$\frac{T_H}{T_S} = \frac{n^4 d^4 - d^4}{n\,d\,(D_S)^3} = \frac{(n^4 - 1)d^4}{n\,d\,(D_S)^3} = \frac{(n^4 - 1)d^3}{n\,(D_S)^3} \quad \text{----------------- (2)}$$

(T_H/T_S) ratio value is 1.44.

So we consider that, torque transmitted to the hollow shaft is greater than the solid shaft. Thus the hollow shaft is stronger than solid shaft.

COMBINED BENDING AND TORSION:

In general shafts are subjected to both bending and torsion.

From torsion equation, $\frac{\mathbf{T}}{\mathbf{I_P}} = \frac{\boldsymbol{\tau}}{\boldsymbol{R}}$

we know that, $T = \frac{\pi}{16} \times \tau \times D^3$

From bending equation, $\frac{\mathbf{M}}{\mathbf{I}} = = \frac{\boldsymbol{\sigma_b}}{\boldsymbol{y}}$

we know that, $M = \frac{\boldsymbol{\sigma_b} .\pi D^3}{32}$

If a material is loaded and direct stress and shear stress induce in it, then according to principal stress and maximum shear stress theory we may write –

$$\sigma_{max} = \frac{\boldsymbol{\sigma_d}}{2} + \sqrt{\left(\frac{\boldsymbol{\sigma_d}}{2}\right)^2 + \tau^3} \quad \text{----------------- (1)}$$

$$\tau_{max} = \sqrt{\left(\frac{\boldsymbol{\sigma_d}}{2}\right)^2 + \tau^3} \quad \text{----------------- (2)}$$

Multiplying both side of equation-1 with $\frac{\pi D^3}{32}$, we get

$$\sigma_{max} \times \frac{\pi D^3}{32} = \frac{\boldsymbol{\sigma_d}}{2} \times \frac{\pi D^3}{32} + \sqrt{\left(\frac{\boldsymbol{\sigma_d}}{2} \times \frac{\pi D^3}{32}\right)^2 + \left(\tau \times \frac{\pi D^3}{32}\right)^2}$$

$$\Rightarrow \quad M_e = \frac{\mathbf{M}}{\mathbf{2}} + \sqrt{(\frac{\mathbf{M}}{\mathbf{2}})^2 + (\frac{\mathbf{T}}{\mathbf{2}})^2}$$

$$\therefore \quad \mathbf{Me} = \frac{\mathbf{M} + \sqrt{\boldsymbol{M}^2 + \boldsymbol{T}^2}}{\mathbf{2}} \quad \text{------------------- (3)}$$

where $\mathbf{M_e}$ is known as equivalent bending moment.

Multiplying both side of equation-2 with $\frac{\pi D^3}{16}$, we get

$$\tau_{max} \times \frac{\pi D^3}{16} = \sqrt{(\frac{\sigma_d}{2} \times \frac{\pi D^3}{16})^2 + (\tau \times \frac{\pi D^3}{16})^2}$$

$$\Rightarrow \quad \mathbf{T_e} = \sqrt{(\mathbf{M})^2 + (\mathbf{T})^2} \quad \text{------------- (4)}$$

where $\mathbf{T_e}$ is known as equivalent twisting moment.

NOTES:

Shear stress is maximum at the outer surface of shaft and zero at its axis.
The shaft which can resist greatest twisting moment possesses greatest polar modulus.
Strength of hollow shaft is higher than solid shaft.
When a solid shaft is replaced by hollow shaft or hollow shaft by solid shaft, the torque transmitted remains same for both shafts.

Terms	**Symbol**	**Formula**	**Units**		
Torsion equation	--	$\frac{\mathbf{T}}{\mathbf{I_P}} = \frac{\boldsymbol{\tau}}{\boldsymbol{R}} = \frac{\mathbf{C\,\Theta}}{\mathbf{L}}$	------		
Polar moment of inertia	Ip	$I_p = \frac{\pi}{32} \times D^4$ for solid shaft and	m^4 mm^4 cm^4		
		$I_p = \frac{\pi}{32} \times (D^4 - d^4)$ for hollow shaft			
Torque transmitted	T	$\frac{\pi}{16} \times \tau \times D^3$ for solid shaft	N-m	KN-m	-----
		$\frac{\pi}{16}\ \tau \times [\frac{D^4 - d^4}{D}]$ for hollow shaft	N-mm	KN-mm	-----
Power transmitted	P	$\frac{\text{Tmean} \times 2\pi N}{60000}$	KW		
Mean or Average torque	T or T_{mean} or $T_{average}$	$\frac{60\,P}{2\pi N}$	Watt		
Polar modulus	Z_p	$\frac{I_P}{R}$	m^3 mm^3 cm^3		
Torsional rigidity	k or μ	CI_p per unit length	$N\text{-}m^2$ $N\text{-}mm^2$		
Equivalent twisting moment	T_e	$\sqrt{(\mathbf{M})^2 + (\mathbf{T})^2}$	Same as twisting moment		
Equivalent bending moment	M_e	$\frac{\mathbf{M} + \sqrt{\boldsymbol{M}^2 + \boldsymbol{T}^2}}{\mathbf{2}}$	Same as bending moment		

***** PROBLEM *****

Q -1) *The average torque transmitted by a shaft is 2255 Nm. The maximum torque is 40% of the average torque. If the allowable shear stress in the shaft material is 45 N/mm^2, determine available diameter of the shaft.*

Ans) Data Given

Average torque **(**T_{av}**) =** 2255 Nm = 2255 × 10^3 N-mm
allowable shear stress (τ) = 45 N/mm^2
Maximum torque (T) = 1.40 T_{av} = 1.40 × 2255 × 10^3 N-mm = 3157 × 10^3 N-mm
we know that $T = \frac{\pi}{16} \times \tau \times D^3$

$$\Rightarrow \quad 3157 \times 10^3 = \frac{\pi}{16} \times 45 \times D^3$$

$$\Rightarrow \quad D^3 = \frac{16 \times 3157 \times 10^3}{\pi \times 45} = 357299.31$$

$$\therefore \quad D = 70.96 \text{ mm}$$

Q-2) *A circular shaft of 60 mm, diameter is running at 150 r.p.m. If the shear stress is not to exceed 50 MPa, find the power which can be transmitted by the shaft.*

Ans) Data Given

Diameter (D) = 60 mm speed (N) = 150 r.p.m
maximum shear stress (τ) = 50 MPa = 50 N/mm^2
Torque transmitted by the shaft (T) = $\frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 50 \times 60^3 = 2.12 \times 10^6$ N-mm = 2.12 kN-m
power transmitted by the shaft (P) = $\frac{2\,\pi\,N\,T}{60} = \frac{2\,\pi \times 150 \times 2.12}{60}$ = 33.3 kW

Q-3) *A hollow shaft is to transmit 200 kW at 80 r.p.m. If the shear stress is not to exceed 60 MPa and internal diameter is 0.6 of the external diameter, find the diameters of the shaft.*

Ans) Data Given

Power (P) = 200 kW Speed (N) = 80 r.p.m
Maximum shear stress (τ) = 60 MPa = 60 N/mm^2
internal diameter of the shaft (d) = 0.6 × external diameter = 0.6 × D
Torque transmitted by shaft (T) $= \frac{\pi}{16}\, \tau \times \left[\frac{D^4 - d^4}{D} \right] = \frac{\pi}{16} \times 60 \times \left[\frac{D^4 - (0.6\,D)^4}{D} \right]$
$= 10.3\, D^3$ N-mm
Power transmitted by the shaft (P) = $\frac{2\,\pi\,N\,T}{60}$

$$\Rightarrow \quad 200 \times 10^3 = \frac{2\,\pi \times 80 \times (10.3 \times D^3)}{60} = 86.3 \times 10^{-3}\, D^3$$

$$\Rightarrow \quad D^3 = \frac{200 \times 10^3}{86.3 \times 10^{-3}} = 2.32 \times 10^6 \text{ mm}^3$$

$$\therefore \quad D = 1.32 \times 10^2 = 132 \text{ mm}$$

and $d = 0.6 \times 132 = 79.2$ mm

Q-4) *A solid shaft of 120 mm diameter is required to transmit 200 kW at 100 r.p.m. If the angle of twist not to exceed 2^0, find the length of the shaft. Take modulus of rigidity for the shaft material as 90 GPa.*

Ans) Data Given

Diameter (D) = 120 mm   Power (P) = 200 kW   speed (N) = 100 r.p.m

angle of twist (Θ) = $2^0 = \frac{2\pi}{180}$ = rad   modulus of rigidity (C) = 90 GPa = 90×10^3

power transmitted by shaft (P) = $\frac{2\pi N T}{60}$

$$\Rightarrow 200 = \frac{2\pi \times 100 \times T}{60} = 10.5\ T$$

$$\Rightarrow T = 200/10.5 = 19\ \text{kN-m} = 19 \times 10^6\ \text{kN-mm}$$

polar moment of inertia of a solid shaft (I_p) = $\frac{\pi}{32} \times D^4 = \frac{\pi}{32} \times (120)^4 = 0.4 \times 10^6\ \text{mm}^4$

we know that, $\frac{\mathbf{T}}{\mathbf{I_P}} = \frac{\mathbf{C\,\Theta}}{\mathbf{L}}$

$$\Rightarrow \frac{19 \times 10^6}{20.4 \times 10^5} = \frac{90 \times 10^3 \times (2\pi/180)}{l}$$

$$\Rightarrow 0.931 = \frac{3.14 \times 10^3}{l}$$

$$\Rightarrow l = \frac{3.14 \times 10^3}{0.931} = 3.37 \times 10^3 = 3.37\ \text{m}$$

Q – 5) *Prove that a hollow shaft is always stronger than a solid shaft of the same material, weight and length subjected to same torque.*

Ans) Data Given

Let d = internal diameter of hollow shaft

nd = external diameter of hollow shaft ; n greater than 1

D = diameter of solid shaft

As the two shafts having equal length and weight, their area of cross section will be same.

i.e. $\frac{\pi}{4} \times D^2 = \frac{\pi}{4} (n^2 d^2 - d^2)$

$\Rightarrow D^2 = d^2 (n^2 - 1) \quad \Rightarrow \quad D = d\sqrt{(n^2 - 1)}$

Torque transmitted by hollow shaft $= \frac{\pi}{16}\ \tau \times \left[\frac{n^4 d^4 - d^4}{nd}\right] = \frac{\pi}{16}\ \tau \times \left[\frac{n^4 - 1}{n}\right] \times d^3$ ------- (1)

Torque transmitted by solid shaft = $\frac{\pi}{16}\ \tau \times D^3 = \frac{\pi}{16}\ \tau \times (n^2 - 1)^{3/2}$ ----------------- (2)

If the expression 1 will be greater than 2, then hollow shaft will be stronger than solid shaft.

$$\Rightarrow \frac{n^4 - 1}{n} > (n^2 - 1)^{3/2}$$

$$\Rightarrow \frac{n^2 + 1}{n} > (n^2 - 1)^{1/2}$$

$$\Rightarrow (n^2 + 1)^2 > n^2 (n^2 - 1)$$

$$\Rightarrow n^4 + 2n^2 + 1 > n^4 - n^2$$

$\Rightarrow 3n^2 + 1 > 0$, which is true

$\therefore$ Hollow shaft is stronger than solid shaft.

***** ASSIGNMENT – 8 *****

GROUP – A (2 mark questions)

1. Write the equation of torsion.
2. What do you mean by torsion?
3. What is polar moment of inertia?
4. Define torsional rigidity.
5. Write the equation for equivalent bending and twisting moment.
6. State the assumptions of pure torsion.

GROUP – B (6 mark questions)

1. Derive the torsion equation.
2. State the assumptions for pure torsion.
3. A solid shaft of 100 mm diameter is transmitting 120 kW at 150 r.p.m. Find the intensity of shear stress in the shaft.
4. Calculate the maximum torque that a shaft of 125 mm diameter can transmit, if the maximum angle of twist is 1^0 in a length of 1.5 m. Take C = 70 GPa.
5. Find the angle of twist per metre length of a hollow shaft of 100 mm external and 60 mm internal diameter, if the shear stress is not to exceed 35 MPa. Take C = 85 GPa.
6. A solid shaft is subjected to a torque of 1.6 kN-m. Find the necessary diameter of the shaft, if the allowable shear stress is 60 MPa. The allowable twist is is 1^0 for every 20 diameters of the shaft. Take C = 80 GPa.

GROUP – C (8 mark questions)

1. A solid shaft of 200 mm diameter has the same cross-sectional area as that of a hollow shaft of the same material with inside diameter 150 mm. Find the ratio of power transmitted by the two shafts at the same speed.
2. A solid shaft transmits 75 kW at 200 r.p.m. Calculate the shaft diameter if the twist in the shaft is not to exceed 1^0 in 2 m of shaft and the shearing stress is limited to 50 N/mm^2. Take $C = 1 \times 10^5$ N/mm^2.
3. A solid steel shaft has to transmit 100 kW at 160 r.p.m. Taking allowable shear stress as 70 MPa, find the suitable diameter of the shaft. The maximum torque transmitted in each revolution exceeds the mean by 20%.
4. Determine the diameter of a solid shaft which will transmit 90 KW at 160 r.p.m, if the shear stress in the shaft is limited to 60 N/mm^2. Find also the length of the shaft, if the twist must not exceed 1^0 over the entire length. Take $C = 8 \times 10^4$ N/mm^2.

The End