UNIT-1

(UNITS & DIMENSIONS)

Physical quantities :

The quantities which can be measured and in terms of which the laws of Physics are expressed are called Physical quantities.

Fundamental Units :

The physical quantities like mass, length and time which can be defined independently are known as fundamental or base quantities.

There are seven fundamental quantities

i) Mass – M
ii) Length – L
iii) Time – T
iv) Temperature – K or θ
v) Electric current – A or I
vi) Luminous Intensity – Cd (candela)
vii) Amount of substance - Mole

Derived Quantities :

The physical quantities which can be derived by using the fundamental quantities are known as derived quantities.

Ex : velocity, Force, Momentum, work etc.

System of Units:

1. C.G.S System (Centimetre-Gram-Second) / French System : It is a system of units in which length, mass and time are taken as 1 centimetre, 1 gram and 1 second respectively.

2. M.K.S System (Metre-Kilogram-Second) / Metric System : It is a system of units in which length, mass and time are taken as 1 metre, 1 kilogram and 1 second respectively.

3. F.P.S System (Foot-Pound-Second) / British System : It is a system of units in which length, mass and time are taken as 1 foot, 1 pound and 1 second respectively.

4. S.I. System : It is a system of international standard in which there are seven fundamental and two supplementary units.

Fundamental Units:

- Mass – Kilogram
- Length- metre
- Time- second
- Electric current- Ampere
- Temperature- Kelvin
- Luminous Intensity—Candela
- Amount of substance- Mole

Supplementary Units:

- Angle – radian
- Solid angle - steradian

Dimensions :

The dimensions of a physical quantity are <u>the powers of the fundamental quantities</u> to express that physical quantity.

Dimensional Formula :

The dimensional formula of a physical quantity is the formula or expression in terms of the fundamental quantities to express that physical quantity.

Sl. No	Quantities	Dimensional formula	S.I. Units
1	Velocity	$\frac{Displacement}{Time} = \frac{L}{T} = [LT^{-1}]$ The dimensional formula of velocity is [LT^{-1}] The dimensions of velocity are (1, -1) of length and time respectively.	m/sec
2	Acceleration	$\frac{Velocity}{Time} = \frac{LT^{-1}}{T} = [LT^{-1}]$	m/sec^2
3	Force	Mass x Acceleration = M x LT^{-2} = [MLT^{-2}]	Newton
4	Momentum	Mass x velocity = M x LT^{-1} = [MLT^{-1}]	kg m/sec
5	Impulse	Force x Time = MLT^{-2} x T = [MLT^{-1}]	Newton.Sec
6	Pressure	$\frac{Force}{Area} = \frac{MLT^{-2}}{L^2} = [M^1L^{-1}T^{-2}]$	N/m^2
7	Density	$\frac{Mass}{Volume} = \frac{M}{L^3} = [ML^{-3}]$	Kg/m^3
8	Work	Force x Displacement = MLT^{-2} x L = [ML^2 T^{-2}]	Joule
9	Kinetic energy	$\frac{1}{2}mv^2$ = M x $[LT^{-1}]^2$ = [M^1 L^2 T^{-2}]	Joule
10	Potential energy	mgh = M x LT^{-2} x L = [M^1 L^2 T^{-2}]	Joule
11	Power	$\frac{Work}{Time} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$	Watt
12	Frequency	$\frac{1}{Time} = \frac{1}{Time} = [M^0L^0T^{-1}]$	$(Sec)^{-1}$ = Hertz
13	Gravitational Constant (G)	$\frac{Fr^2}{M_1M_2} = \frac{MLT^{-2}.L^2}{M^2} [M^{-1}L^3T^{-2}]$	$N.m^2/Kg^2$

14	Angle	$\frac{Arc}{Radius}=\frac{L}{L}=\left[M^0L^0T^0\right]$ (*Dimensionless quantity*)	radian
15	Angular Velocity	$\frac{Angular Displacement}{Time}=\frac{M^0L^0T^0}{T}=\left[M^0L^0T^{-1}\right]$	radian/sec
16	Angular acceleration	$\frac{Angular\ Velocity}{Time}=\frac{M^0L^0T^{-1}}{T}=\left[M^0L^0T^{-2}\right]$	radian/sec^2
17	Coefficient of friction	$\frac{Force\ of\ friction}{Normal\ reaction}=\frac{MLT^{-2}}{MLT^{-2}}=\left[M^0L^0T^0\right]$ (dimensionless)	No Unit
18	Relative Density	$\frac{Density\ of\ the\ Body}{Density\ of\ water}=\frac{ML^{-3}}{ML^{-3}}=\left[M^0L^0T^0\right]$	No Unit
19	Specific heat	$\frac{Amount\ of\ heat}{Mass\times Change\ in\ temp}=\frac{M^1L^2T^{-2}}{M\times K}=\left[M^0L^2T^{-2}K^{-1}\right]$	Kcal/Kg ^{0}C
20	Electric field	$\frac{Force}{Charge}=\frac{MLT^{-2}}{AT}=\left[MLT^{-3}A^{-1}\right]$	N / Coulomb

Principle of Homogenity:

The principal of homogeneity states that the dimensional formula of every term in both sides of a correct relation must be same.

Uses of Dimensions:

1. To convert the values of a physical quantity from one system to another.
2. To check the correctness of a given relation.
3. To derive a relation between various physical quantities.

Q. Check the correctness of the following equation by using dimensional method.

i) **v = u + at**
ii) **S = ut + ½ at^2**
iii) **v^2 – u^2 = 2aS**

Ans. **i)** **v = u + at**

Dimensional formula of V = [LT^{-1}]

Dimensional formula of u = [LT^{-1}]

Dimensional formula of at = [LT^{-2}] [T] = [LT^{-1}]

Since ,the dimensional formula of all the three terms are same, the equation is dimensionally correct.

ii) **S = ut + ½ at ^{2}**

Dimensional formula of S = [L]

Dimensional formula of ut = $[LT^{-1}][T] = [L]$

Dimensional formula of ½ at^2 = $[LT^{-2}][T^2] = [L]$

Since ,the dimensional formula of all the terms in the above equation are same, the equation is dimensionally correct.

iii) $v^2 - u^2 = 2aS$

Dimensional formula of $v^2 = [LT^{-1}]^2 = [L^2 T^{-2}]$

Dimensional formula of $u^2 = [LT^{-1}]^2 = [L^2 T^{-2}]$

Dimensional formula of $2aS = [LT^{-2}][L] = [L^2 T^{-2}]$

Since ,the dimensional formula of all the terms in the above equation are same, the equation is dimensionally correct.

Q. Using dimensional method check the correctness of the following equations.

$i) \quad g = \frac{GM}{R^2} \qquad ii) \quad T = 2\pi\sqrt{\frac{L}{g}} \qquad iii) \quad F = \frac{mv^2}{r}$

UNIT- 2

(SCALAR & VECTOR)

Scalar quantities:

The physical quantities having only magnitudes are known as scalar quantities.

Ex. Mass, Length, Volume, Temperature, Electrical Charge etc.

Vector quantities:

The physical quantities having both magnitudes and directions are known as vector quantities.

Ex. Displacement, Velocity, Acceleration, Force, Momentum etc.

A vector quantity is always represented by a line segment with an arrow head $(\vec{A})$

Types of Vectors:

- Equal Vector:

The vectors are said to be equal if they have the same magnitude and direction.

- Negative vector:

A vector is said to be negative of other vector if they are equal in magnitude and opposite in direction.

- Unit Vector :

A unit vector of a vector is that whose magnitude is 1 unit and its direction is along the original vector.

$$\vec{A} = \hat{A}\ |\vec{A}| \qquad \Rightarrow \qquad \boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

- Co-llinearear Vector:

i) Parallel vector: Two vectors acting along same direction are called parallel vectors angle between two vectors is 0^0.

ii) Anti parallel vector: Two vectors which are directed in opposite directions are called anti parallel vectors. Angle between two vectors is 180^0.

- Co-planar Vector :

Vectors situated in one plane irrespective of their directions are known as co-planar vector.

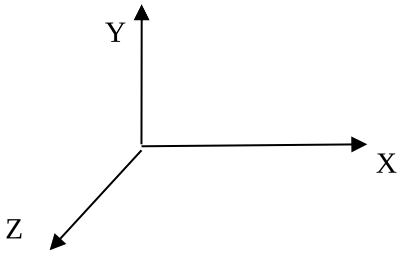
- Orthogonal Vector:

Two vectors are said to be orthogonal if they are perpendicular to each other. Angle between two vectors is 90^0.

- Orthogonal unit vector :

The vectors whose magnitudes are one unit each and orthogonal to each other are called orthogonal unit vectors.

$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along X – axis, Y-axis and Z-axis respectively.



<u>Traingle law of vector addition</u>

If two vectors are represented by the two sides of a triangle ,taken in the same order then the resultant vector is represented by the third side of the triangle in opposite order .

Magnitude of the Resultant Vector

R = $\sqrt{A^2 + B^2 + 2ABCOS\theta}$

The direction of the Resultant Vector

$\beta = \tan^{-1} \frac{B\sin\theta}{A+B\cos\theta}$

<u>Parallelogram law of vector addition</u>

The parallelogram law of vector addition states that if two vectors are represented by the adjacent sides of the parallelogram taken in the same order then the resultant vector is represented by the diagonal passing through the common point taken in opposite order.

Magnitude of the Resultant Vector

R = $\sqrt{A^2 + B^2 + 2ABCOS\theta}$

The direction of the Resultant Vector

$\beta = \tan^{-1} \frac{B\sin\theta}{A+B\cos\theta}$

Special Cases :

I) When the vectors acting along the same direction

i.e $\theta = 0°$, $\cos\theta = 1$, $\sin\theta = 0$

So , R = A+ B (Maximum) & $\beta = 0°$

II) When the vectors are perpendicular

i .e $\theta = 90°$, cos$\theta = 0$, sin$\theta = 1$

So , R = $\sqrt{A^2 + B^2}$ & β =tan^{-1} $(\frac{B}{A})$

III)When the vectors are acting in opposite direction

i.e $\theta = 180°$, cos$\theta = -1$, sin$\theta = 0$

So , R = A - B (Minimum) & β = 0°

Q .If the magnitude of resultant of two forces is equal to the magnitude of either of them. Find the angle between them .

Ans. Given that R = A = B

So R = $\sqrt{A^2 + B^2 + 2ABCOS\theta}$

$A^2 = A^2 + A^2 + 2A^2\cos\theta$

$A^2 = 2A^2 + 2A^2\cos\theta$

$A^2 = 2A^2 (1 + \cos\theta)$ ⇒ $(1 + \cos\theta) = 1/2$ ⇒ $\text{Cos}\theta = -\frac{1}{2}$ ⇒ $\theta = 120°$

Q .The magnitude of resultant of two forces (A + B) and (A – B) is $\sqrt{(3A^2 + B^2)}$. Find the angle between them .

Ans : Given that P = (A + B) , Q = (A – B) , R =$\sqrt{(3A^2 + B^2)}$

Since R = $\sqrt{P^2 + Q^2 + 2PQCOS\theta}$

$\sqrt{(3A^2 + B^2)} = \sqrt{(A + B)^2 + A - B)^2 + 2(A + B)(A - B)cos\theta}$

$3A^2 + B^2 = 2A^2 + 2B^2 + 2(A^2 - B^2)\cos\theta$

$A^2 - B^2 = 2(A^2 - B^2)\cos\theta$ ⇒ $\cos\theta = \frac{1}{2}$ ⇒ $\theta = 60°$

<u>Resolution of a vector.</u>

It means splitting of a vector into it's components which when added gives the original vector.

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

here,

$$\cos\theta = \frac{A_x}{A}, \qquad \sin\theta = \frac{A_y}{A}$$

$A_x = A\cos\theta \qquad A_y = A\sin\theta$

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z k$$
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Q. One component of a force making an angle 60^0 with horizontal is 20 N . Find the other component.

Ans. One component

$F_x = F\cos\theta \quad \Rightarrow \quad 20 = F\cos\theta$

$\Rightarrow \quad 20 = \dfrac{F}{2} \quad \Rightarrow \quad F = 40$ N

So other component

$F_y = F\sin\theta = 40\sin 60^0 = 40 \times \dfrac{\sqrt{3}}{2}$

$F_y = 20\sqrt{3}$ N

Q. One component of a force 65 N is 25 N ,then find the other component.

Ans Since, $F = \sqrt{F_x^2 + F_y^2} \quad \Rightarrow (65)^2 = (25)^2 + F_y^2$

$\Rightarrow \quad F_y^2 = 4225 - 625 = 3600 \quad \Rightarrow \quad F_y = 60$ N

DOT PRODUCT OF THE VECTORS (SCALAR PRODUCT)

- The dot product of two vectors is defined as the product of their magnitude and the cosine of the angle between them .i.e $\vec{A}\bullet\vec{B} = ABCOS\theta$
- The dot product of two vectors is always a scalar quantity.
- Example : W = $\vec{F}\bullet\vec{S} = FSCOS\theta$
- $If\ \theta = 0°$, $\vec{A}\bullet\vec{B} = AB$
- $If\ \theta = 90°$, $\vec{A}\bullet\vec{B} = 0$
- $If\ \theta = 180°$, $\vec{A}\bullet\vec{B} = -AB$

- The dot product is commutative in nature i.e $\vec{A}\bullet\vec{B}=\vec{B}\bullet\vec{A}$
- The dot product is distributive in nature i.e $\vec{A}\bullet(\vec{B}+\vec{C})=\vec{A}\bullet\vec{B}+\vec{A}\bullet\vec{C}$
- Unit vectors of dot product

  $\hat{i}\bullet\hat{i}=\hat{j}\bullet\hat{j}=\hat{k}\bullet\hat{k}=1$

  $\hat{i}\bullet\hat{j}=\hat{j}\bullet\hat{k}=\hat{k}\bullet\hat{i}=0$, $\hat{j}\bullet\hat{i}=\hat{k}\bullet\hat{j}=\hat{i}\bullet\hat{k}=0$
- Dot product in terms of rectangular components

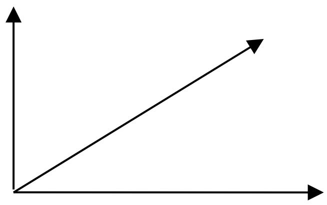
  $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_Z\hat{k}$ & $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_Z\hat{k}$

  $\vec{A}\bullet\vec{B}=\left(A_x\hat{i}+A_y\hat{j}+A_Z\hat{k}\right)\bullet\left(B_x\hat{i}+B_y\hat{j}+B_Z\hat{k}\right)$

  $\vec{A}\bullet\vec{B}=A_XB_X+A_YB_Y+A_ZB_Z$
  - $\vec{A}\bullet\vec{B}=ABCOS\theta$, $COS\theta=\dfrac{\vec{A}\bullet\vec{B}}{AB}$

<u>CROSS PRODUCT OF THE VECTORS (VECTOR PRODUCT)</u>

- The cross product of two vectors is defined as the product of their magnitude , the sine of the angle between them and the direction is perpendicular to the plane containing the vectors .i.e $\vec{A}\times\vec{B}=AB\sin\theta\hat{n}$



- The dot product of two vectors is always a vector quantity.
- Example : $\vec{L}=\vec{r}\times\vec{p}$
- $If\ \theta=0°$, $\vec{A}\times\vec{B}=0$
- $If\ \theta=90°$, $\vec{A}\times\vec{B}=1$
- $If\ \theta=180°$, $\vec{A}\times\vec{B}=0$
- The cross product is not commutative in nature i.e $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ &
- The cross product is distributive in nature i.e $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$
- Unit vectors of cross product

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{i} = \hat{j}$$

$$\hat{j}\times\hat{i} = -\hat{k} \qquad \hat{k}\times\hat{j} = -\hat{i} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

Cross product in terms of rectangular components

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_Z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_Z\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_Z \\ B_x & B_y & B_z \end{vmatrix} = i\,(A_yB_z - A_zBy) - j\,(A_xB_z - A_zBx) + k\,(A_xB_y - A_yBx)$$

Q If $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, and $\vec{B}=5\hat{i}+2\hat{j}-3k$ then find $\vec{A}\bullet\vec{B}$?

Q If $\vec{A}=5\hat{i}-2\hat{j}+3\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=3\hat{i}+2\hat{k}$ then find $\vec{A}\bullet(\vec{B}\times\vec{C})$

Q If $\vec{A}\bullet\vec{B}=\sqrt{3}\,\left|\vec{A}\times\vec{B}\right|$ find the angle between two vectors.

Q If $\vec{A}\bullet\vec{B}=\left|\vec{A}\times\vec{B}\right|$ find the angle between two vectors.

Q $\vec{A}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{B}=\hat{i}+5\hat{j}-2\hat{k}$ find $\vec{A}\bullet\vec{B}$ and $\vec{A}\times\vec{B}$

Q Find the angle between two vectors $(\hat{i}-2\hat{j}-5\hat{k})$ and $(2\hat{i}+\hat{j}-4\hat{k})$.

Q $\vec{F}=5\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{S}=4\hat{i}+7\hat{j}+\hat{k}$ find the work done.

Q If $\vec{A}=6\hat{i}-n\hat{j}+3\hat{k}$, $\vec{B}=5\hat{i}+2\hat{j}-n\hat{k}$, are orthogonal to each other then find the value of n.

Q $\vec{A}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{B}=5\hat{i}+5\hat{j}$ Find $\vec{A}\times\vec{B}$

UNIT - 3

KINEMATICS

Equations of motion

Let, u - Initial velocity

v – Final velocity

a- acceleration

S- displacement

t - Timeperiod

1) Velocity after t- second, v = u + at

2) Displacement after t second, S= ut+1/2 at^2

3) Velocity after a displacement S, $v^2 - u^2 = 2aS$

4) Displacement during nth second, S_{nth} = u+a/2(2n-1)

* When a body starts from rest, u=0

* When a body comes to rest v=0

* In vertically downward direction, a=g, u= 0

* In vertically upward direction, a= -g , at the highest point v =0

Q. The velocity of a body increases, at a constant rate, from 10 ms^{-1} to 25ms^{-1} in 6 minute. Find the acceleration and the distance travelled.

Ans: Given, u = 10 ms^{-1} , v = 25ms^{-1} , t = 6 minute = 360 Sec

Using the equation of motion, v = u + at

$$a = \frac{v-u}{t} = \frac{25-10}{360} = \frac{15}{360} = \frac{1}{24} ms^{-2}$$

Using, $v^2 - u^2 = 2aS$

$$S = \frac{v^2-u^2}{2a} = \frac{625-100}{2\times 1/24} = 6300\,m = 63\,Km$$

Q. A car moving with a velocity of 30 ms^{-1} is stopped by the application of brakes which impart a retardation of 6 ms^{-2} to the car. how long does it take for the car to come to a stop ? How far does the car travel during the time brakes are applied?

Ans: Given, u = 30 ms^{-1} , v = 0 , a = - 6 ms^{-2}

Using the equation of motion, v = u + at

0 = 30 – 6t

6t = 30 , so t = 5 Sec.

Using, Using, $v^2 - u^2 = 2aS$

$$S = \frac{v^2-u^2}{2a} = \frac{0-900}{2\times(-6)} = 75\,m$$

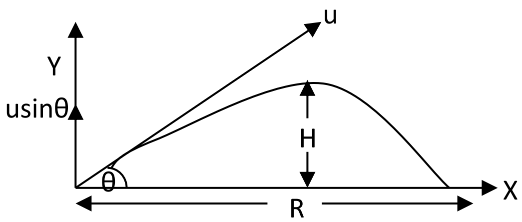
PROJECTILE MOTION:

A projectile is a body which is thrown with some initial velocity and as it moves no other energy is given to it. It moves under the action of gravity. The motion of the body is known as projectile motion.

* The path followed by the projectiles is known as trajectory.

Ex. 1) A bomb dropped from an aeroplane.
2) A bullet fired from a gun
3) A ball thrown into space.

PROJECTILE FIRED MAKING AN ANGLE θ WITH HORIZONTAL :



Consider a projectile fired making an angle θ with horizontal, with a velocity u.

Resolving the velocity into 2 components.

i) ucosθ, along horizontal direction

ii) usinθ, along vertical direction

it is non-uniform as the body moves. It gradually decreases and becomes zero at the height point P.

Equation of trajectory (path of the projectile) :

It is a relation between the horizontal and vertical equation of motion of the projectile.

Horizontal equation of motion :

x = (ucosθ) t

$\Rightarrow$ $t = \dfrac{x}{u\cos\theta}$

Vertical equation of motion

$y = (u\sin\theta)t - \frac{1}{2} gt^2$

$$\Rightarrow \quad y = (usion\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \quad y = \frac{usion\theta x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$\Rightarrow \quad y = \tan\theta x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

This equation represents a parabola. Thus the path followed by a projected is parabolic in nature.

Maximum Height

It is the maximum distance covered by the projectile in vertical direction.

Consider the motion in vertically upward direction.

$(u\sin\theta)^2 = 2(-g)\,H$

$\Rightarrow \quad -u^2\sin^2\theta = -2gH$

$$\Rightarrow \quad H = \frac{u^2 \sin^2\theta}{2g}$$

Time of ascent:

It is the time taken by the projectile to reach at the highest point from the point of projection.

Consider the motion in vertically upward direction.

Applying, $\quad v = u + at$

$\Rightarrow \quad 0 = u\sin\theta - gt \quad \Rightarrow \quad t = \dfrac{u\sin\theta}{g}$

Time of Flight:

It is the total time taken by the projectile during motion.

Since the time of ascent is equal to the time of descent so that total time taken T = 2t

$$\Rightarrow \quad \boxed{T = \frac{2u\sin\theta}{g}}$$

Horizontal Range:

It is the distance travelled by the projectile in horizontal direction.

$$R = (u\cos\theta)T \qquad \Rightarrow \quad R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$$

$$\Rightarrow \quad R = \frac{u^2\, 2\sin\theta.\cos\theta}{g}$$

$$\Rightarrow \quad \boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

* The range will be maximum if, $\sin 2\theta = 1$

$\Rightarrow \quad 2\theta = 90^0 \qquad \Rightarrow \quad \theta = 45^0$

So, maximum horizontal Range , $R_{\max} = \dfrac{u^2}{g}$

Q. Find the angel of projection for which the horizontal range is equal to the maximum height.

A: R = H

$$\Rightarrow \quad \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\Rightarrow \quad 2\sin\theta.\cos\theta = \frac{\sin^2\theta}{2} \qquad \Rightarrow \ 2\cos\theta = \frac{\sin\theta}{2}$$

$$\Rightarrow \quad 4\cos\theta = \sin\theta \qquad \Rightarrow \quad 4 = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \quad 4 = \tan\theta$$

$\Rightarrow$ $\theta = \tan^{-1}(4)$ or $\cot^{-1}(1/4)$.

Q. For what angle the horizontal range should be maximum?

A $$R = \frac{u^2 \sin_2 \theta}{g}$$

If $\sin 2\theta = 1 \Rightarrow 2\theta = 90 \quad \Rightarrow$ **$\theta = 45°$**

Angular Displacement (θ) :

When a particle moves in a circular path. The angular displacement is the angle subtended by the radius vector at the cenre. Here θ is the angular displacement.

Dimension = [$M^0L^0T^0$]

$\Rightarrow$ When a body completes a circular path its angular displacement is 2π

Angular velocity ($\vec{\omega}$)

It is the rate of change of angular displacement of the body.

$$\vec{\omega} = \frac{\vec{\theta}}{t}$$

Unit: radian / sec

Dimension: [$M^0L^0T^{-1}$]

$\Rightarrow$ Average angular velocity = $\dfrac{Change\ in\ angular\ displacement}{Change\ in\ time}$

$$\vec{\omega}_{av} = \frac{\vec{\theta}_2 - \vec{\theta}_1}{t_2 - t_1}$$

$\Rightarrow$ Instantaneous angular velocity

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

$\Rightarrow$ If V is the linear velocity

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt}$$

⇒ V = rω

⇒ If a body completes a circular path in a time period T, the angular velocity.

$$\omega = \frac{2\pi}{T}$$

⇒ If η or *f* is the frequency of revolution,

$$f = \frac{1}{T}$$

So, $\omega = \frac{2\pi}{T} = 2\pi f$

Angular Acceleration (α) :

It is defined as the rate of change of angular velocity , $\alpha = \frac{\omega}{t}$

Unit : radian /sec^2

Dimension : [$M^0 L^0 T^{-2}$]

$$\vec{\alpha}\,Avg = \frac{change\,in\,angular\,velocity}{change\,in\,time} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$$

⇒ Instantaneous angular acceleration

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

⇒ If ‘a’ is the linear acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}(a\omega) = r\frac{d\omega}{dt} = r\alpha$$

Q. Find the angular velocity of the second hand of a watch.

A $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}\,rad/\sec.$

* For minute hand $\omega = \frac{2\pi}{60\times 60} = \frac{\pi}{1800}\,rad/\sec.$

* For hour hand $\omega = \frac{2\pi}{12\times 3600} = \frac{\pi}{21600}\,rad/\sec.$

Q. The length of the second hand of a clock is sem find it, linear velocity

A. r = 5cm

Angular velocity of second hand

$$\omega = \frac{2\pi}{60} = \frac{\pi}{30} rad / \sec.$$

v = rw

$$\Rightarrow \qquad v = 5 \times \frac{\pi}{30} = \frac{\pi}{6} = \frac{3.14}{6} cm / \sec = 0.52 cm / \sec.$$

UNIT - 4

WORK AND FRICTION

WORK

- Work is said to be done when the force acting on a body and the body displaces through certain displacement and force has a component along the displacement.
- $W = \vec{F} \bullet \vec{S} = FSCOS\,\theta$
- Case I (If $\theta = 0°$, $\cos\theta = 1$ $\Rightarrow$ W $= FS$ (Positive work)

  i.e The work done is positive if the force and displacement are in same direction .
- Case II (If $\theta = 180°$, $\cos\theta = -1$ $\Rightarrow$ W $= -FS$ (Negative work)

  i.e The work done is negative if the force and displacement are in opposite direction .
- Case III (If $\theta = 90°$, $\cos\theta = 0$ $\Rightarrow$ W $= 0$ (Zero work)

  i.e No work is done when the force and displacement are perpendicular to eachother .
- Dimensional formula of work =[$M^1L^2T^{-2}$]
- SI unit of work Joule

  1 Joule = 1 N × 1m
- CGS unit of work erg

  1 erg = 1 dyne × 1 cm
- 1 Joule = 10^7 erg

FRICTION

- When a body moves over a rough surface the force which opposes the motion of the body is called force of friction .
- The foece of friction is always opposite to the direction of motion .
- It acts parallel to the surface .
- The force of friction depends upon
  1) Nature of two surfaces in contact.
  2) The normal reaction with which the body and the surface being pressed together .

STATIC FRICTION

The force of friction between the body and the surface so long as there is no relative motion is called static friction .

LIMITING FRICTION

The maximum value of the static friction is known as limiting friction .

DYNAMIC FRICTION

The force of friction between the body and the surface so long as there is a relative motion between them is called dynamic friction .

LAWS OF LIMITING FRICTION

1. The force of friction depends upon the nature of two surfaces and it is opposite to direction of motion of the body.
2. The force of friction acts parallel to the surface .
3. The force of friction directly proportional to normal reaction .

   i..e $F \propto R \Rightarrow F = \mu R$ (μ is the coefficient of friction)

4. The force of limiting friction is independent of the area of contact till the normal reaction remains constant

.

COEFFICIENT OF FRICTION

The coefficient of friction between two surfaces is defined as the ratio between the force of friction and the normal reaction .

$$F = \mu R \Rightarrow \mu = F / R$$

μ has no unit and dimension .

METHODS OF REDUCING FRICTIONAL FORCE

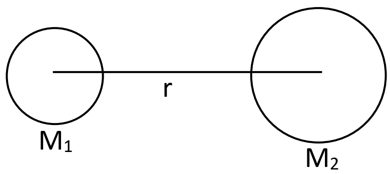
1. By rubbing and polishing the surfaces .
2. By using the lubricants.
3. By decreasing the area of contact between the body and surface .
4. By streamlining .

Gravitation

Newton's law of Gravitation:

The gravitational force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Consider two bodies of masses M_1 and M_2 separated by a distance r.



$$F \, \alpha \, M_1 M_2$$

$$F \, \alpha \, 1/r^2$$

Combining these two equations

$$\Rightarrow \qquad F = G\frac{M_1 M_2}{r^2}$$

Where G is known as universal gravitational constant.

$$G = \frac{Fr^2}{M_1 M_2}$$

If $M_1 = M_2 = 1$ unit, r = 1 unit.

then , G = F

so, the gravitational constant is defined as the gravitational force of attraction between two bodies each of mass 1 unit separated by a distance of 1 unit.

- The value of G is same everywhere in the universe.

  $G = 6.67 \times 10^{-11}$ Nm^2 / Kg^2 (in M.K.S system)

  $G = 6.67 \times 10^{-8}$ dyne cm^2 / g^2 (in C.G.S. system)

- Dimension : [$M^{-1}L^3T^{-2}$]

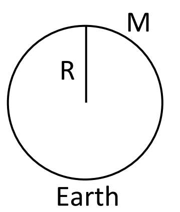
Relation between g and G

The force with which a body is attracted towards the center of earth is called gravity.

Consider a body of mass 'm' placed on the surface of earth

The weight of the body = mg

The force of attraction between the earth and the body



$$F = G\frac{Mm}{R^2}$$

Where, M $\rightarrow$ Mass of earth

R $\rightarrow$ Radius of earth

So, $\quad mg = G\frac{Mm}{R^2}$

$$\Rightarrow \boxed{g = G\frac{M}{R^2}}$$ (This is the relation between g and G)

- The value of ‘g’ on the surface of earth.

  g = 9.8 m/sec^2 = 980 cm / sec^2 = 32 ft / sec^2
- The value of ‘g’ is independent of the mass of the body.
- It depends upon mass and the radius of the planet
- Since the value of g is different for different places, the weight of a body is different at different places.

Difference between Mass and Weight

MASS	WEIGHT
• The mass of a body is the amount of substance contained in the body.	• The weight of a body is the force with which the body is attracted towards the center of earth
• Mass of body is same at all points.	• Weight of a body is different at different places due to variation of g.
• Mass is a fundamental quantity.	• Weight is a derived quantity.
• Mass is a scalar quantity.	• Weight is a vector quantity.
• Unit : Kilogram gram	• Unit : Newton , Kgf Dyne , gmf

Variation of g with Altitude / Height:

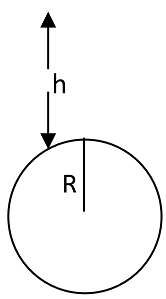
The acceleration due to gravity at any point on the surface of earth.

$$g = G\frac{M}{R^2} \qquad - (1)$$

where, M = Mass of earth.

R = Radius of earth.

The acceleration due to gravity at a height ‘h’ from the surface of earth



$$g1 = G\frac{M}{(R+h)^2} \qquad - (2)$$

$$\Rightarrow \quad \frac{g^1}{g} = 1 - \frac{2h}{R}$$

$$\Rightarrow \quad g^1 = g\left(1 - \frac{2h}{R}\right)$$

Or $$g^1 = g - \frac{2gh}{R}$$

$$\Rightarrow \quad g - g^1 \,\alpha\, h$$

i.e with increase in height from the surfaces of earth the acceleration due to gravity decreases.

- Loss in weight of a body.

$$mg - mg^1 = \frac{2mgh}{R}$$

Variation of g with Depth :

The acceleration due to gravity at any point on the surface of earth.

$$g = G\frac{M}{R^2}$$

Where, M $\rightarrow$ mass of earth, R $\rightarrow$ Radius of earth

At a depth‘d’ from the surface of earth

$g^1 = g\,(1\text{-}d/R)$ $\Rightarrow$ $g^1 = g - \frac{gd}{R}$

$\Rightarrow$ $g - g^1 = \frac{gd}{R}$ $\Rightarrow$ $\left(g - g^1\right) \alpha\ d$

With the increase in depth from the surface of earth the acceleration due to gravity decreases.

At the centre of earth.

d = R

So, g = g (1- R/R) = g (1-1) = 0

$\Rightarrow$ $g_{centre} = 0$

i.e The acceleration due to gravity at the centre of earth is zero. So the weight of a body at the centre of earth is zero.

Q. The mass of moon is $1/80$ times and its radius is $1/4$ that of earth.

Find the acceleration due to gravity on the surface of moon.

If a body weights 5o kg on the earth . What will be its weight on the surface of moon.

A: i) $M_m = \frac{1}{80}M$, $R_m = \frac{1}{4}R$

On earth, $g = G\frac{M}{R^2}$

On moon, $g_m = G\frac{M_m}{{R_m}^2}$

Dividing, $\frac{gm}{g} = \frac{M_m}{M} \times \frac{R^2}{R_m^2}$

$$\Rightarrow \quad \frac{g_m}{g} = \frac{1}{80} \times (4)^2 = \frac{16}{80} = \frac{1}{5}$$

$$\Rightarrow \quad g_m = \frac{g}{5} = \frac{9.8}{5} = 1.96\, m/s^2$$

ii) $$\frac{wt.of\ a\,body\,on\,moon}{wt.of\ the\,body\,on\,earth} = \frac{g_m}{g} = \frac{1}{5}$$

$\Rightarrow$ wt. of the body on moon $= \frac{1}{5}\times wt.$ of the body on earth.

Q. A body weights 36 kg wt on the surface of earth. How much will it weight on the surface of a planet. Whose mass is $\frac{1}{9}$ and radius $\frac{1}{2}$ that of earth.

A : $$M_p = \frac{M_e}{9},\ R_p = \frac{R_e}{2}$$

$$\frac{g_p}{g_e} = \frac{M_p}{M_e}\times\left(\frac{R_e}{R_p}\right)^2 = \frac{1}{9}\times(2)^2 = \frac{4}{9}$$

$$\Rightarrow \frac{w_p}{w_e} = \frac{mg_p}{mg_e} = \frac{g_p}{g_e} = \frac{4}{9}$$

$$\Rightarrow w_p = \frac{4}{9}\times w_e = \frac{4}{9}\times 36 = 16kgwt$$

$$= \frac{1}{5}\times 50 = 10\,kgf$$

Q. **Find the acceleration due to gravity at a depth equals to half of the radius of earth.**

A: d= R/2

At a depth, $$g^1 = g\left(1-\frac{d}{R}\right)$$

$$\Rightarrow g^1 = g\left(1-\frac{R/2}{R}\right) = g\left(1-\frac{1}{2}\right)$$

$$\Rightarrow g^1 = \frac{g}{2} = \frac{9.8}{2} = 4.9\,m/\sec^2$$

<u>Kepler's laws of planetary motion</u>:

1st law (law of orbit)

All the planets revolve around the sun in elliptical orbits and the sun is situated at one of its foci.

2nd law (law of area) :

When a planet revolves around the sun it sweeps out equal area in equal interval of time

i.e. during planetary motion the areal velocity remains constant.

$$\frac{dA}{dt} = \text{Constant.}$$

3rd law (Law of time period <u>or</u> Harmonic law) :

When a planet revolves around the sun the square of the time period is proportional to the cube of the semi major axis.

$$T^2 \alpha\ R^3 \qquad \Rightarrow \qquad \frac{T^2}{R^3} = \text{constant.}$$

For two planets, $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

Oscillations and Waves

Simple Harmonic Motton (SHM)

The motion of a particle is said to be simple harmonic if

i) Its acceleration is directly proportional to displacement .

ii) The acceleration is directed towards the mean position.

i.e Acceleration and displacement are in opposite direction.

a α y

a = - ω^2y, -ve sign is due to the acceleration and the displacement are opposite to each other.

DISPLACEMENT;

The displacement of a particle executing SHM at any time is the distance from the mean position.

In Δ OMP

$$\sin \omega t = \frac{OM}{OP} = \frac{y}{r}$$

$$\boxed{y = r \sin \omega t}$$

At mean position, y = 0

At extreme position, y= r (Maximum)

- **The maximum displacement of a particle executing SHM is known as the amplitude.**

VELOCITY:

The rate of change of displacement of the particle is called velocity.

$$V = \frac{dy}{dt}$$

$$V = \frac{d}{dt}(r \sin \omega t)$$

$$\boxed{V = r\omega \cos \omega t}$$

$$Cos\,\omega t = \frac{PM}{OP} = \frac{\sqrt{r^2 - y^2}}{r} \qquad \text{So, } \boxed{V = \omega\sqrt{r^2 - y^2}}$$

At mean position, y= 0, V= rω (Maximum) ,

At extreme position, y=r, V=0 (Minimum)

ACCELERATION:

It is the rate of change of velocity of the particle.

$$a = \frac{dv}{dt} \quad \Rightarrow \quad a = \frac{d}{dt}(r\omega \cos \omega t)$$

$$\boxed{a = -\omega^2 y}$$

At mean position, a=0 (minimum)

At extreme position, y=r, a= ω^2r (Maximum)

Q. A particle exciting S.H.M has maximum velocity of 1.00 ms^{-1} and a maximum acceleration of 1.57 ms^{-2} calculate its time period.

A. Dividing $\frac{\omega^2 r}{\omega r} = \frac{1.57}{1.00}$

ω = 1.57 rad 5^1

$\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.57} = 4\,\text{sec}$

Q. A particle vibrating S.H.M along the line of length 20 cm. If its period is 3.14 sec. Find its velocity at mean position.

A Here, 2r = 20

$\Rightarrow$ r= 10 cm

Since, Velocity at mean position $V = \omega r$

$= \frac{2\pi}{T} \times 10 = \frac{2 \times 3.14}{3.14} \times 10$ =20 cms^{-1}

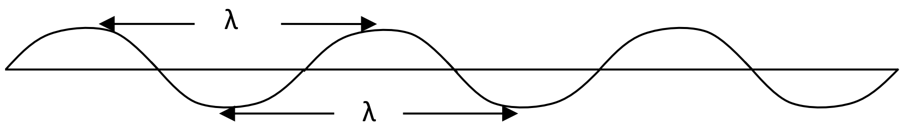
Transverse Wave	Longitudinal Wave
• It is that type of wave motion in which the particles of the medium vibrate in a direction perpendicular to the direction of motion of the wave.	• It is that type of wave motion in which the particles of the medium vibrate in a direction same as the direction of motion of the wave.
• It forms crests and troughs.	• It forms compressions and rarefactions.
• **Ex -** Light wave Electromagnetic wave Vibration in a stretched string.	• **Ex-** Sound wave Vibration in air column (organ pipe)

<u>Wave Length (λ):</u>

It is the distance covered by the wave in a complete time period.

Or

It is the distance between two consecutive crests or between two consecutive troughs.



<u>Relation Between Velocity, Wagelength & Frequeny of a wave</u> :

Velocity = $\frac{Distance}{Time}$

Since, λ is the distance covered by the wave in a time period T,

$v = \frac{\lambda}{T} \Rightarrow v = \lambda f$, Where $f = \frac{1}{T}$, is the frequency

$\boxed{\text{velocity = wavelength x frequency}}$

<u>Ultrasonic</u>

Sound of frequency greater than the upper limit of audible range (20 Hz to 20 KHz) is known as ultrasonic .

Properties:

- Ultrasonic waves are longitudinal in nature.
- Ultrasonic waves are of high frequency Range of (2×10^4 to 10^9 Hz)
- They travel with the speed of sound.
- They constitute narrow beams.

Applications:

- Echo sounding,
- Thickness gauging
- Drilling holes.
- Ultrasonic welding

Heat and Thermodynamics

Heat :

It is a form of energy which is transferred one body to another when there is a temperature difference between them.

Unit : calorie (in C.G.S)

1 calorie is the amount of heat required to raise the temperature of 1 gram of water through 1^0C.

1 calories = 4.2 joule.

KCal (in S.I.)

1 Kcal. is amount of heat required to raise the temperature of 1 Kg.of water through 1^0C.

1 Kcal = 10^3 cal = 4200 joule.

Temperature :

Temperature is the measure of degree of hotness and coldness of a body .

Unit ; kelvin ,celsius

Specific Heat :

The specific heat of a body is defined as the amount of heat required to raise the temperature of unit mass of the body through 1^0C.

The amount of heat required.

$$\boxed{\begin{gathered} Q = mS\,\Delta\theta \\ S = \frac{Q}{m\Delta\theta} \end{gathered}}$$

If m = 1 unit, $\Delta\theta$ = 1^0C.

Then S = Q

- **Unit** : cal / gm^0C (in CGS)

  K cal/ kg ^{0}C (in S.I.)
- **Dimensions** : S = $\frac{Q}{m\Delta\theta} = \frac{M^1L^2T^{-2}}{MK} = \left[M^0L^2T^{-2}K^{-1}\right]$

→ **For water,** S = 1 cal / gm^0C.= 1 K cal /Kg^0C.

→ **For ice** S = 0.5 cal / gm^0C.= 0.5 Kcal / kg^0C.

Latent Heat :

It is defined as the amount of heat required to convert the unit mass of a substance from one state to another without change in its temperature.

$Q = mL \quad \Rightarrow \quad L = \dfrac{Q}{m}$

- **Unit** : cal/ gm, K cal/ kg
- **Dimension** : $\dfrac{M^1L^2T^{-2}}{M} = \left[M^0L^2T^{-2}\right]$

For ice $Q = m L_f$

(From ice 0^0C to water at 0^0C) L_f = 80 cal/ gm = 80 K cal/ kg

For water $Q = m L_V$.

(from water at 100^0C to steam at 100^0C) , Lv = 540 cal/ gm = 540Kcal/Kg.

Q. What is the amount of heat required to melt 5 gm of ice at 0^0C to water at 0^0C

A : 5 x 80 = 400 cal

Q Calculate the amount of heat required to raise the temp. of 50 gm of water from 10^0C to 25^0C.

A $Q = ms\Delta\theta$

Here m = 50gm. S=1, $\Delta\theta$ = 25 – 10 = 15

Q = 50 x 1 x 15 = 750 cal.

Q. What is the amount of heat required to convert 10 gm of ice at -5^0Cto water at 20^0C.

A From ice at -5^0C to ice at 0^0C

$Q_1 = ms\Delta\theta$

= 10 x 0.5 x 5 = 25 cal.

From , ice at 0^0C to water at 0^0C.

$Q_2 = mL$

= 10 x 80 = 800 cal.θ

From , water at 0^0C to water at 20^0C.

$Q_3 = ms\Delta\theta$

= 10 x 1 x 20 = 200 cal.

So, total heat required

$Q = Q_1 + Q_2 + Q_3$

= 25 + 800 + 200

= 1025 cal.

Coefficient of thermal expansion of solids :

- **Coefficient of linear expansion (α)**

  It is defined as the change in length per original length per degree rise in temperature.

  $$\boxed{\alpha = \frac{L_t - L_0}{L_0 t}}$$ Unit - $^0C^{-1}$

- **Coefficient of superficial /Areal expansion (β)**

  It is defined as the change in area per original area per degree rise in temperature.

  $$\boxed{\beta = \frac{A_t - A_0}{A_0 t}}$$ Unit - $^0C^{-1}$

- **Coefficient of cubical /volume expansion (γ)**

  It is defined as the change in volume per original volume per degree rise in temperature.

  $$\boxed{\gamma = \frac{V_t - V_0}{V_0 t}}$$ Unit - $^0C^{-1}$

Relation between α and β :

Consider a square sheet having each side of length L_0 at ^0C and L_t at t^0C.

So, it, area at $0\ ^0C$, $A_0 = L_0^2$.

area at $t\ ^0C$, $A_t = L_t^2$

We have , $\alpha = \frac{L_t - L_0}{L_0 t}$

$\Rightarrow \quad L_t - L_0 = \alpha L_0 t \qquad \Rightarrow \quad L_t = L_0 + \alpha L_0 t$

$\Rightarrow \quad L_t = L_0(1 + \alpha t)$

Again, $\beta = \frac{A_t - A_0}{A_0 t} \qquad \Rightarrow \quad \beta = \frac{L_t^2 - L_0^2}{L_0^2 t}$

$\Rightarrow \quad \beta = \frac{L_0^2(1+\alpha t)^2 - L_0^2}{L_0^2 t} \qquad \Rightarrow \quad \beta = \frac{1 + 2\alpha t + \alpha^2 t^2 - 1}{t}$

$\Rightarrow \quad \beta = \frac{2\alpha t + \alpha^2 t^2}{t} \qquad \Rightarrow \quad \beta = \frac{(2\alpha + \alpha^2 t)t}{t}$

$\Rightarrow \quad \beta = 2\alpha + \alpha^2 t$

Since, α is very small the higher powers of α can be neglected.

So, $\boxed{\beta = 2\alpha}$

Relation Between α and γ

Consider a cubical body having each side of lenth L_0 at ^{0}C and L_t at t^0C

So, it's volume at 0^0C, $V_0 = L_0^3$

at t^0C, $V_t = L_t^3$

we have,

$$\alpha = \frac{L_t - L_0}{L_0 t}$$

$\Rightarrow$ $L_t – L_0 = \alpha L_0 t$ $\Rightarrow$ $L_t = L_0 (1+\alpha t)$

Now $\gamma = \frac{V_t - V_0}{V_0 t}$ $\Rightarrow$ $\gamma = \frac{L_t^{\ 3} - L_0^{\ 3}}{L_0^{\ 3} t}$

$\Rightarrow$ $\gamma = \frac{L_t^{\ 3}(1+\alpha t)^3 - L_0^{\ 3}}{L_0^{\ 3} t}$ $\Rightarrow$ $\gamma = \frac{L_t^{\ 3}(1+\alpha t)^3 - L_0^{\ 3}}{L_0^{\ 3} t}$

$\Rightarrow$ $\gamma = \frac{(1+\alpha t)^3 - 1}{t}$ $\Rightarrow$ $\gamma = \frac{1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3 - 1}{t}$

$\Rightarrow$ $\gamma = \frac{3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3}{t}$ $\Rightarrow$ $\gamma = \frac{t\left(3\alpha + 3\alpha^2 t + \alpha^3 t^2\right)}{t}$

$\Rightarrow$ $\gamma = 3\alpha + 3\alpha^2 t + \alpha^3 t^2$

Since, α is very small the higher powers of α can be neglected.

So $\boxed{\gamma \ \textbf{-= 3}\boldsymbol{\alpha}}$

- Since , β = 2α

γ **-= 3α**

$\Rightarrow$ $\boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$

Q The length of a red at 0^0c is 50cm and 52cm at 100^0C. Find the co-efficient of linear expansion.

A $\alpha = \frac{L_t - L_0}{L_0 t}$

Here, L_0 = 50cm, L_t = 52 cm, t = 100^0C

$$\alpha = \frac{52 - 50}{50 \times 100} = \frac{2}{5 \times 10^3} = 0.4 \times 10^{-3}$$

$\Rightarrow$ $\alpha = 4 \times 10^{-4}\ {}^0C^{-1}$

So $\alpha = 0.0004\ ^0C^{-1}$

First law of Thermodynamics :

Statement : If some amount of heat given to a system is capable of doing some work, then amount of heat is equal to the sum of the change in internal energy and the amount of work done.

$$Q = (U_2 - U_1) + W$$

Amount of heat = (change in internal energy) +(work done)

Mechanical Equivalent of Heat (J) :

$W \propto H \quad \Rightarrow \quad W = JH$

$\Rightarrow \quad J = \dfrac{W}{H}$, If H = 1 unit, then J = W

i.e Mechanical equivalent of heat is defined as the amount of work done to produce unit amount of heat.

- $J = 4.2$ joule / cal $= 4.2 \times 10^7$ erg/ cal.