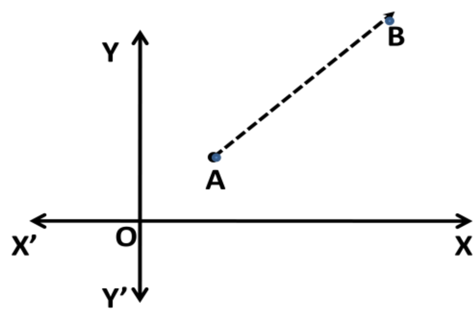
Introduction:
In physics or mechanics, while studying the motion you are using a variety of quantities such as distance, displacement, speed, velocity, acceleration, mass, force, momentum, work, power, energy etc. to describe the motion. These quantities can be classified into two categories 1) Scalar quantity 2) Vector quantity.

The quantities possessing a numerical value (or magnitude) are called as scalar quantities.
Examples: Distance, speed, mass, work etc.

The quantities possessing magnitude as well as direction are called as vector quantities.
Examples: displacement, velocity, acceleration, force, momentum etc.

Representation of vectors:



In a plane the vector directed from a point A to B is denoted as $\overrightarrow{AB}$.Here the point A is called as initial point and B is the terminal point or (Final point) .

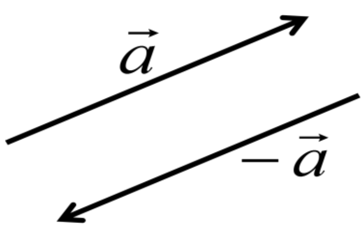
Notes: The length of a vector $\overrightarrow{AB}$ is the distance or magnitude of the vector $\overrightarrow{AB}$. It is denoted as$\left|\overrightarrow{AB}\right|$.
It is a scalar quantity , which is always positive.
Vectors are also denoted by using the lowercase bold alphabets **a, b, c** etc. or $\vec{a}, \vec{b}, \vec{c}$, etc.

Types of vectors:

Null vector:
Vector with magnitude zero (0) is called as a null vector or zero vector. It is written as $\vec{O}$.
Example: Every point is a null vector.

Unit vector: A vector with magnitude unity (1) is called as unit vector. If $\vec{a}$ be a vector, then the unit vector in the direction of $\vec{a}$ is denoted by $\hat{a}$ and given by : $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

Co-initial vectors: Vectors starting from a point are called as co-initial vectors.

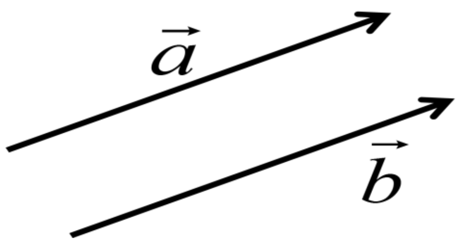


Negative of a vector: A vector having same magnitude but opposite direction is known as negative vector.

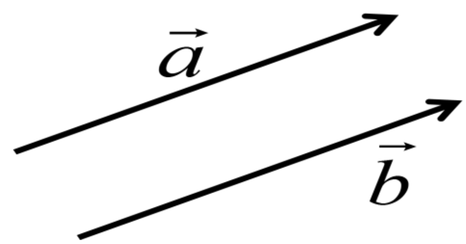
If $\vec{a}$ be a vector, then the negative of $\vec{a}$, written as -$\vec{a}$
Note: the magnitudes of $\vec{a}$ and -$\vec{a}$ are same. i.e. $|\vec{a}| = |-\vec{a}| = a$

Scalar multiplication of a vector:
Suppose k is a scalar quantity and $\vec{a}$ be a vector quantity, then the multiplication k$\vec{a}$ is a new vector called as scalar multiplication of $\vec{a}$. The vector k$\vec{a}$ is a vector whose length is $|k|$ times that of $\vec{a}$. The direction of k$\vec{a}$ will be same as of $\vec{a}$ if k is positive and opposite of $\vec{a}$ if k is negative.

Parallel vectors:
Two vectors are said to be parallel if both are of either same or opposite direction. In otherwords $\vec{a}$ and $\vec{b}$ will be parallel if $\vec{a}$ and $\vec{b}$ are scalar multiple of each other i.e. $\vec{a} = k\vec{b}$,where k is scalar.



Equal vectors:
Two vectors are said to be equal if they have same magnitude as well as same direction.



Like and Unlike vectors
Two vectors are said to be like if they are parallel but in same direction.
Two vectors are said to be unlike if they are parallel but in opposite direction.

Note: Like and Unlike vectors may be of same or different magnitudes.

Collinear of vectors:

Two vectors are said to be collinear if one is scalar multiple of other and they have a common point.
i.e $\vec{a} = k\vec{b}, where\ k\ is\ a\ scalar$

Collinear of three points:
Three points A,B and C are said to be collinear if they lie in same line and the condition is

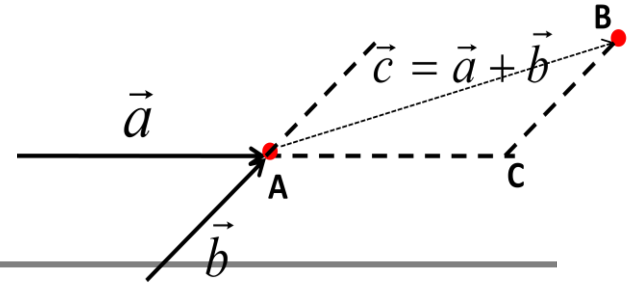
$$\overrightarrow{AB} = k\overrightarrow{AC}$$

Or

$$\overrightarrow{AB} = k\overrightarrow{BC}$$

Addition and Subtraction of vectors:

If $\overrightarrow{AC} = \vec{a}$ and $\overrightarrow{CB} = \vec{b}$ be any two vectors and join $\overrightarrow{AB}$.
Here the terminal point of $\overrightarrow{AC}$ is same as intial point of $\overrightarrow{CB}$. So $\overrightarrow{AB}$ is the sum of the vectors of $\overrightarrow{AC}$ and $\overrightarrow{CB}$.



i.e $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} = \vec{a} + \vec{b}$, which is called triangle law of vector addition.

Subtraction of vectors:
The difference of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a} - \vec{b}$ and which is defined as $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

Parallelogram law of vector addition:
Let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$ join $\overrightarrow{BC}$ and $\overrightarrow{AC}$ and make a parallelogram OACB. Join the diagonals $\overrightarrow{OC}$ and $\overrightarrow{BA}$.
Here $\overrightarrow{OA} = \vec{a} = \overrightarrow{BC}$ and $\overrightarrow{OB} = \vec{b} = \overrightarrow{AC}$
By using triangle law of vector addition we get $\overrightarrow{OC} = \vec{a} + \vec{b}$ and $\overrightarrow{BA} = \vec{a} - \vec{b}$

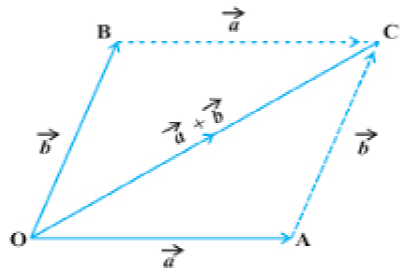


Fig. 5

Algebra of vectors:
1. Vector addition is commutative i.e. $\vec{a}+\vec{b}= \vec{b}+\vec{a}$
2. Vector addition is associative i.e. . $(\vec{a}+\vec{b}) + \vec{c}= \vec{a} + (\vec{b}+\vec{c})$
3. If m, n be any scalars, then $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}, m(n\vec{a}) = n(m\vec{a}) = mn(\vec{a})$.
4. $1\vec{a} = \vec{a}, 0\vec{a} = \vec{O}$

Position vector of a point in space:

If P be any point on the space and O be a fixed point (called origin) then the vector $\overrightarrow{OP} = \vec{r}$(say)
Is called the position vector of the point P with respect to origin O.
Which is written as P($\vec{r}$),(read as P be a point having position vector $\vec{r}$)

Note:
If A($\vec{a}$) and B($\vec{b}$) are two position vectors then $\overrightarrow{AB} = \vec{b} - \vec{a}$

Note:
If P(x,y) be any point on the plane and O be origin then

$$\boldsymbol{\overrightarrow{OP} = x\hat{\imath} + y\hat{\jmath}}$$

Where $\hat{\imath}$ *and* $\hat{\jmath}$ are the unit vectors along the direction of X-axis and Y-axis.

Note:
If P(x,y,z) be any point on the space and O be origin then

$\boldsymbol{\overrightarrow{OP} = x\hat{\imath} + y\hat{\jmath} + z\,\hat{k}}$

where, $\hat{\imath}\,, \hat{\jmath}\ and\ \hat{k}$ are the unit vectors along X-axis, Y-axis and Z-axis respectively.

Note:
If $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$
And
$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

Vector joining two points:
The vector joining two points A(x_1, y_1, z_1) and B(x_2, y_2, z_3) can be obtained by using the triangle law of addition of vectors
i.e. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

The coefficients x_2-x_1, y_2-y_1 and z_2-z_1 are called as the scalar components of the vector $\overrightarrow{AB}$ in the direction of X-axis, Y-axis and Z-axis respectively. And the vectors $(x_2 - x_1)\hat{\imath}, (y_2 - y_1)\hat{\jmath} and\ (z_2 - z_1)\hat{k}$ are called as vector components of the vector $\overrightarrow{AB}$ in the direction of X-axis, Y-axis and Z-axis respectively.

Problem: Find $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, $5\vec{a}$ and $2\vec{a} - 7\vec{b}$, where $\vec{a} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$.

Example: Find the position vector of the point (2, 3, -5).
Ans: $2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$.

Example: Find the vector from the point (2, -3, 4) to (6, 4, 1) and hence find the modulus of this vector.

Example: Find the value of m if the modulus of the vector joining A(0, 1, -2) and B(-2, 3, m) is $\sqrt{8}$.

Example: Find the unit vector in the direction of the vector $2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$.

Problem: Find the unit vector in the direction of the sum of the vectors $\vec{a}\ and\ \vec{b}$ where, $\vec{a} = -3\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$ and $\vec{b} = \hat{\imath} + 5\hat{\jmath} + 3\hat{k}$.
Example: Find the value of 'm' if the vectors $-3\hat{\imath} + m\hat{\jmath} - 8\hat{k}$ and $15\hat{\imath} + 2\hat{\jmath} + 40\hat{k}$ are parallel.
Example: Prove that the points (2, 1, -1), (3, -2, 1) and (8, -17, 11) are collinear.
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Questions carrying 2 marks

1. Find the position vector of the point A (2, -3). **Ans**: $2\hat{i} - 3\hat{j}$
2. Find the vector joining points A (1, -3) and B (-5, 4). **Ans**: $-6\hat{i} + 7\hat{j}$
3. Find the length of the vector joining P (2, -1) and Q (5,-4). **Ans** : $3\sqrt{2}$

4. Find the unit vector in the direction of the vector $\hat{i}+\hat{j}+\hat{k}$. **Ans** : $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$

5. For what value of 'a' the vectors $2\hat{i}-3\hat{j}$ and $a\hat{i}-6\hat{j}$ are parallel. **Ans** : 4

6. If the vectors $\vec{a}=\alpha\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ are parallel, find the value *of* α. **Ans**: α=-3

7. Find a unit vector parallel to the sum of vectors $3\hat{i}-2\hat{j}+\hat{k}$ *and* $-2\hat{i}+\hat{j}-3\hat{k}$ **Ans**: $\frac{\hat{i}}{\sqrt{6}}-\frac{\hat{j}}{\sqrt{6}}-\frac{\hat{k}}{\sqrt{6}}$

Question carrying 5 marks

1. Prove by vector method that the points A(2,6,3), B(1,2,7) and C(3,10,-1) are collinear.

2. Prove that the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\vec{i}-3\hat{j}-5\hat{k}$ and $\vec{c}=3\hat{i}-4\hat{j}-4\hat{k}$ form the sides of a right angled triangle.

Product of vectors:

The product of vectors can be defined using two special symbols i.e. (•) and (X).

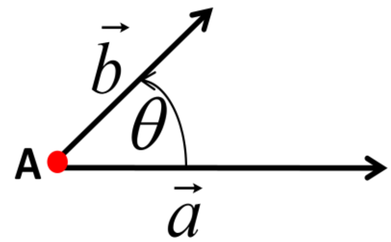
a. Scalar Product or Dot product

b. Vector Product or Cross Product

Scalar product (Dot product)

Definition:

Let $\vec{a}$ and $\vec{b}$ be any two coinitial vectors and θ be the angle between them measured from $\vec{a}$ to $\vec{b}$. Then the scalar product (or dot product) of $\vec{a}$ and $\vec{b}$ written as $\vec{a}\cdot\vec{b}$ and defined by



$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|cos\theta = abcos\theta$$

Similarly

$$\vec{b}.\vec{a} = |\vec{b}||\vec{a}|\cos(-\theta) = abcos\theta$$

Angle between two vectors

Formula:

Angle between the vectors $\vec{a}$ and $\vec{b}$ is given by:

$$cos\theta = \pm\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \theta = cos^{-1}\left(\pm\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

Condition-1:If two vectors are parallel then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| = ab$

Condition-2: If two vectors are perpendicular then $\vec{a}\cdot\vec{b} = 0 = \vec{b}.\vec{a}$

Properties of Dot product

a. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

b. $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

c. If $\vec{a}$ and $\vec{b}$ are perpendicular then $\vec{a}\cdot\vec{b} = 0$.

d. $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ (Distributive over addition)

Notes:

1. $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$ (Since $\hat{\imath},\hat{\jmath},\hat{k}$ are perpendicular)
2. $\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$
3. If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Geometrical meaning of dot product:(Scalar and vector projection)

Scalar projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Scalar projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

vector projection of $\vec{a}$ on $\vec{b} = \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2}$

vector projection of $\vec{b}$ on $\vec{a} = \frac{(\vec{a}\cdot\vec{b})\vec{a}}{|\vec{a}|^2}$

Work done:

$W = \vec{F}\cdot\vec{S}, where\ \vec{S} = \overrightarrow{AB}\ (\vec{F} = force)$

Example: Find the scalar product of the vectors $\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 3\hat{k}$
Example: Find the value of δ if the vectors $\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} + \delta\hat{\jmath} - 3\hat{k}$ are perpendicular to each other.
Example: Find the scalar projection and vector projection of $\vec{b}\ on\ \vec{a}$ where $\vec{a} = \hat{\imath} + 3\hat{\jmath} - 4\hat{k}, \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$.
Example: Find the acute angle between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.

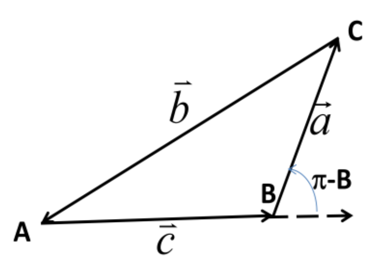
Questions carrying 2 marks

1. Find $\vec{a}.\vec{b}$ if (i) $\vec{a} = 2\hat{i} - 3\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$,(ii) $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}\ and\ \vec{b} = \hat{i} - \hat{j} + \hat{k}$
2. Find the value of 'λ' if $\vec{a} = 6\hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$ are perpendicular to each other.
3. Show that the vectors $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 3\hat{j} + \hat{k}$ are at right angles .

Question carrying 5 marks

1. Find the value of λ if $\vec{a} = (2,-2,1)$ and $\vec{b} = (0,2\lambda,1)$ are perpendicular.
2. Find the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}\ and\ \vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$
3. Find scalar and vector projections of $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.
4. Find the work done by force $\vec{F} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ which displaces a particle from A (1,-2,2) to B (3,1,5).
5. Prove by vector method that in any triangle ABC.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

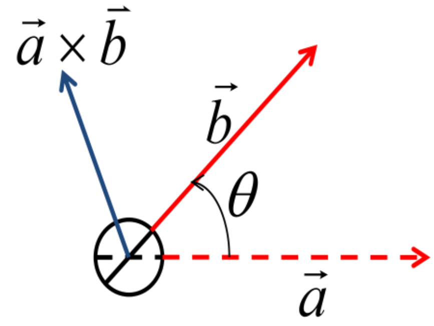


Proof: Assume the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ to represent the sides of a triangle taken in order (Refer figure).

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$
$\Rightarrow \vec{a} = -(\vec{b} + \vec{c})$
$\Rightarrow \vec{a}\cdot\vec{a} = -(\vec{b} + \vec{c})\cdot\vec{a} = (\vec{b} + \vec{c})\cdot(\vec{b} + \vec{c})$
$\Rightarrow \vec{a}\cdot\vec{a} = \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{c} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{b}$

6. Prove by vector method that an angle inscribed in a semicircle is right angle.
7. If the sum of two unit vectors is a unit vector, then prove that the magnitude of their difference will be $\sqrt{3}$.
8. In a triangle ABC prove by vector method that a=bcosC + c cosB

Vector Product (Cross Product)



Definition of vector product:

Let $\vec{a}$ and $\vec{b}$ be any two coinitial vectors and θ be the angle between them measured from $\vec{a}$ to $\vec{b}$. Then the vector product (or cross product) of $\vec{a}$ and $\vec{b}$ written as $\vec{a} \times \vec{b}$ and defined by

$\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|sin\theta)\hat{n} = (absin\theta)\hat{n}$

Here $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$.
The direction of $\vec{a} \times \vec{b}$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$.
Hence $\hat{n}$ is in the direction of $\vec{a} \times \vec{b}$.
$|\vec{a} \times \vec{b}| = \left||\vec{a}||\vec{b}|sin\theta\hat{n}\right| = absin\theta\ |\hat{n}| = absin\theta$

Now $\vec{b} \times \vec{a} = (|\vec{a}||\vec{b}|\sin(-\theta))\hat{n} = -absin\theta\ \hat{n}$
$|\vec{b} \times \vec{a}| = \left|-|\vec{b}||\vec{a}|sin\theta\hat{n}\right| = absin\theta\ |\hat{n}| = absin\theta$
So $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ but $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|$

Note: $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

Sine of the angle between the vectors $\vec{a}$ and $\vec{b}$:

$sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

Condition-1: If two vectors are parallel then $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| = 0\ or\ \vec{a} \times \vec{b} = \vec{0}$

Condition -2: If two vectors are perpendicular then $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$=ab

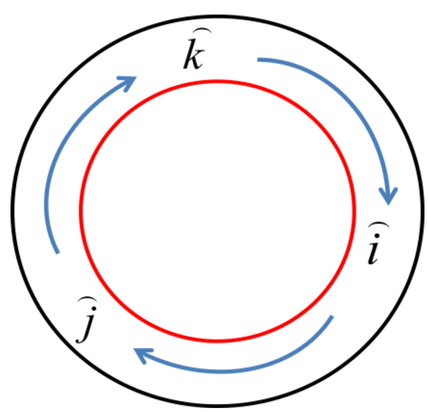
Properties of Cross product

a. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
b. $\vec{a} \times \vec{a} = \vec{0}$
c. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ (Distributive over addition)

Notes:

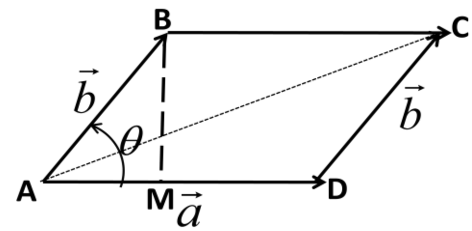
1. $\hat{\imath} \times \hat{\jmath} = \hat{k}$ (Since $\hat{\imath}, \hat{\jmath}, \hat{k}$ are perpendicular)
2. $\hat{\jmath} \times \hat{k} = \hat{\imath}$
3. $\hat{k} \times \hat{\imath} = \hat{\jmath}$
4. $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$
5. If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$,
   then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$= (a_2b_3 - b_2a_3)\hat{\imath} - (a_1b_3 - b_1a_3)\hat{\jmath} + (a_1b_2 - b_1a_2)\hat{k}$



Geometrical meaning of cross product:

Area of a parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$



If $\vec{a}$ and $\vec{b}$ represent the two sides of a parallelogram taken in order, then the area of the triangle will be $\frac{1}{2}|\vec{a} \times \vec{b}|$

Unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$:

The unit vector $perpendicular$ to the vectors $\vec{a}$ and $\vec{b}$ will be $\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

Note:The vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ will be $\vec{a} \times \vec{b}$

Note:Area of a triangle with vertices A, B and C is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Example: Find the vector product of the vectors $\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ and $-\hat{\imath} + \hat{\jmath} - 3\hat{k}$.

Ex: Find $\vec{a} \times \vec{b}$ where $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 7\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

Example: Find the vector perpendicular to both $5\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$.

Example: Find the unit vector perpendicular to both $\hat{\imath} + 2\hat{\jmath}$ and $\hat{\imath} + 3\hat{\jmath} + \hat{k}$.

Example: Find the area of the parallelogram whose adjacent sides are $\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + \hat{k}$.

Example: Find the area of the triangle whose vertices are at A(1, -1, 3), B(2, 1, 0) and C(3, 1, -1)

Example: Find the sine of the angle between the vectors $3\hat{\imath} - 4\hat{\jmath}$ and $6\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$.

<u>Question carrying 2 marks</u>

1. Find the vector perpendicular to both of the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$. **Ans**: $\hat{i} - \hat{j} + \hat{k}$
2. Find the unit vector perpendicular to both the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{k}$.

   Ans: $\frac{-1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$
3. Find the unit vector perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$

   Ans $\frac{2}{\sqrt{110}}\hat{i} - \frac{9}{\sqrt{110}}\hat{j} - \frac{5}{\sqrt{110}}\hat{k}$
4. Find area of parallelogram whose adjacent sides are given by vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ **Ans**: $8\sqrt{3}$
5. Find area of the triangle whose sides are given by vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

   Ans: $4\sqrt{3}$
6. Find area of the triangle whose vertices are A(1,-2,3), B(3,1,2), C(2,3,-1).

   Ans: $\frac{7\sqrt{3}}{2}$
7. Find sine angle between the vectors $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$

   Ans: $\frac{\sqrt{171}}{14}$

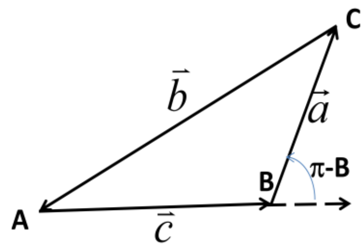
8. Find the area of the parallelogram whose diagonals are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -3\hat{i} + 4\hat{j} - \hat{k}$

Ans: $\dfrac{3\sqrt{30}}{2}$

9. Prove that $\left(\vec{a} \times \vec{b}\right)^2 = a^2b^2 - \left(\vec{a}.\vec{b}\right)^2$

10. Prove that by vector method in any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Hints: Assume the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ to represent the sides of a triangle taken in order (Refer figure).

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \vec{a} = -(\vec{b} + \vec{c})$ ------------ (a)

$\Rightarrow \vec{a} \times \vec{a} = -(\vec{b} + \vec{c}) \times \vec{a} = -(\vec{b} \times \vec{a}) - (\vec{c} \times \vec{a})$

$\Rightarrow \vec{0} = -(\vec{b} \times \vec{a}) - (\vec{c} \times \vec{a})$

$\Rightarrow (\vec{a} \times \vec{b}) = (\vec{c} \times \vec{a})$ ------------ (b)

Similarly, take the cross product with $\vec{b}$ on both sides of equation (a) and get

$\Rightarrow (\vec{a} \times \vec{b}) = (\vec{b} \times \vec{c})$ ------------ (c)

Comparing Equation (b) and (c) in getting

$(\vec{a} \times \vec{b}) = (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{a})$

$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

$\Rightarrow absin(\pi - C) = bcsin(\pi - A) = casin(\pi - B)$

$\Rightarrow absinC = bcsinA = casinB$

Now divide 'abc' on both sides to get the answer.