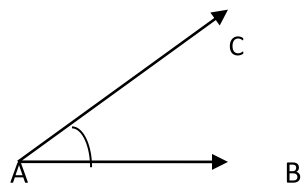
Trigonometry is a branch of mathematics that studies triangles and the relationships between the lengths of their sides and the angles.

It is derived from three small Greek words **tri + gono+ metry**

Tri---- three, Gono---- angle, Metry----- measurement

Angle: An angle can be formed by the union of two rays or two lines with a common point(called vertex).



1. If an angle measured in anticlockwise direction then it is positive .

2. If an angle measured in clockwise direction then it is negative.

Angles are measured using the following units:

1. Sexagesimal degree (°)

If the circumference of a circle is divided into 360 equal parts, the central angle corresponding to each of its 360 parts is an angle of one degree sexagesimal, (1°).

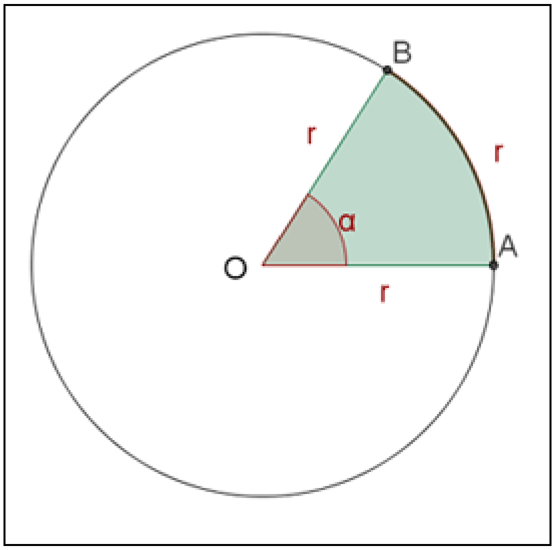
A degree has 60 minutes (') $[1^0 = 60']$ and a minute has 60 seconds ('') $[1' = 60'']$.

2. Radian (rad): It is the measure of an angle whose arc is a ratio.

$$2\pi\, radian = 360^0 \qquad and\ \pi\, radian = 180^0$$

$$1\, radian = \frac{180^0}{\pi}\ and\ 1^0 = \frac{\pi}{180} radian$$

Figure:



Radian measurement

Conversion from Degree to Radian:

$$30^0 = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$$

$$60^0 = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

$$90^0 = 90 \times \frac{\pi}{180} = \frac{\pi}{2}$$

$$150^0 = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}$$

$$270^0 = 270 \times \frac{\pi}{180} = \frac{3\pi}{2}$$

Conversion from Radian to Degree:

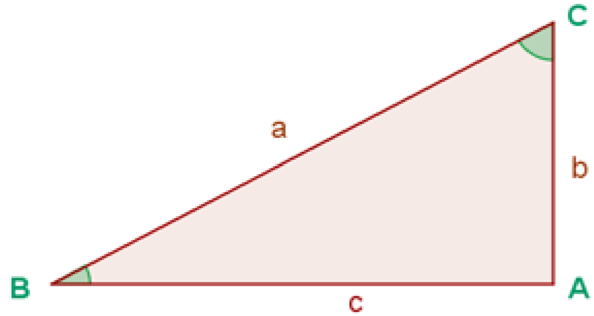
$$\frac{\pi}{2} = \frac{\pi}{2} \times \frac{180}{\pi} = 90^0$$

Trigonometric Ratios:

It is the ratio of the sides of a right angled triangle with respect to base angle.

We have six trigonometric ratios like sine, cosine, tangent, cotangent, secant and cosecant.(i.e sin, cos, tan ,cot ,sec and cosec).

How to find all trigonometric ratios from a right angled triangle.



In a right triangle, the following trigonometric ratios can be defined:

Let $\langle B = \theta, \langle A = 90, and\ \langle C = 90 - \theta,$

$$Let\ AB = c = Base, AC = b = Perpendicular\ and\ BC = a = Hypotenuse.$$

Sine

The sine of the angle θ is the ratio between the length of the opposite side(Perpendicular) and the hypotenuse of the triangle.

It is denoted by sinθ

$$sin\theta = \frac{perpendicular}{hypotenuse} = \frac{b}{a}\ or\ \frac{p}{h}$$

Cosine

The **cosine** of the angle θ is the ratio between the length of the adjacent side(base) and the hypotenuse of the triangle.

It is denoted by cosθ.

$$cos\theta = \frac{base}{hypotenuse} = \frac{c}{a}\ or\ \frac{b}{h}$$

Tangent

The tangent of the angle θ is the ratio between the length of the opposite side(perpendicular) and the adjacent side(base) of the triangle.

It is denoted by tanθ.

$$tan\theta = \frac{b}{c} \; or \; \frac{p}{b}$$

Cotangent

The cotangent of the angle θ is the inverse of the tangent of θ.

It is denoted by cotθ**.**

$$cot\theta = \frac{c}{b} \; or \; \frac{b}{p}$$

Secant

The secant of angle θ is the inverse of the cosine of θ.

It is denoted by secθ**.**

$$sec\theta = \frac{a}{c} \; or \; \frac{h}{b}$$

Cosecant

The cosecant of angle θ is the inverse of the sine of θ.

It is denoted by coscecθ.

$$\boldsymbol{cosec\theta} = \boldsymbol{\frac{a}{b}} \; \boldsymbol{or} \; \frac{h}{p}$$

Formula 1:

$$\boldsymbol{sin\theta cosec\theta = 1, sin\theta = \frac{1}{cosec\theta}, cosec\theta = \frac{1}{sin\theta}}$$

$$\boldsymbol{tan\theta cot\theta = 1, tan\theta = \frac{1}{cot\theta}, cot\theta = \frac{1}{tan\theta}}$$

$$\boldsymbol{cos\theta sec\theta = 1, cos\theta = \frac{1}{sec\theta}, sec\theta = \frac{1}{cos\theta}}$$

Formula 2:

$$sin^2\theta + cos^2\theta = 1, sin^2\theta = 1 - cos^2\theta, cos^2\theta = 1 - sin^2\theta$$

$$1 + tan^2\theta = sec^2\theta, sec^2\theta - tan^2\theta = 1, sec^2\theta - 1 = tan^2\theta$$

$$1 + cot^2\theta = cosec^2\theta, cosec^2\theta - cot^2\theta = 1, cosec^2\theta - 1 = cot^2\theta$$

Formula 3: $tan\theta = \frac{sin\theta}{cos\theta}, cot\theta = \frac{cos\theta}{sin\theta}$

Value of the trigonometric ratios in different angles:

α:	0°	30°	45°	60°	90°	180°	270°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\to \infty$	0	$\to -\infty$

Define all trigonometric ratios at different angles:

$\sin(90^0 - \theta) = cos\theta$

$\cos(90^0 - \theta) = sin\theta$

$\tan(90^0 - \theta) = cot\theta$

$\cot(90^0 - \theta) = tan\theta$

$\sec(90^0 - \theta) = cosec\theta$

$\text{cosec}(90^0 - \theta) = sec\theta$

$\sin(180^0 - \theta) = sin\theta$

$\cos(180^0 - \theta) = -cos\theta$

$\tan(180^0 - \theta) = -tan\theta$

$\cot(180^0 - \theta) = -cot\theta$

$\sec(180^0 - \theta) = -sec\theta$

$\text{cosec}(180^0 - \theta) = cosec\theta$

$\sin(180^0 + \theta) = -sin\theta$

$\cos(180^0 + \theta) = -cos\theta$

$\tan(180^0 + \theta) = tan\theta$

$\cot(180^0 + \theta) = cot\theta$

$\sec(180^0 + \theta) = -sec\theta$

$\text{cosec}(180^0 + \theta) = -cosec\theta$

$\sin(-\theta) = -sin\theta$

$\cos(-\theta) = cos\theta$

$\tan(-\theta) = -tan\theta$

$\cot(-\theta) = -cot\theta$

$\sec(-\theta) = sec\theta$

$\text{cosec}(-\theta) = -cosec\theta$

$\sin(90^0 + \theta) = cos\theta$

$\cos(90^0 + \theta) = -sin\theta$

$\tan(90^0 + \theta) = -cot\theta$

$\cot(90^0 + \theta) = -tan\theta$

$\sec(90^0 + \theta) = -cosec\theta$

$\text{cosec}(90^0 + \theta) = sec\theta$

$\sin(270^0 + \theta) = -cos\theta$

$\cos(270^0 + \theta) = sin\theta$

$\tan(270^0 + \theta) = -cot\theta$

$\cot(270^0 + \theta) = -tan\theta$

$\sec(270^0 + \theta) = cosec\theta$

$\text{cosec}(270^0 + \theta) = -sec\theta$

$\sin(270^0 - \theta) = -cos\theta$

$\cos(270^0 - \theta) = -sin\theta$

$\tan(270^0 - \theta) = cot\theta$

$\cot(270^0 - \theta) = tan\theta$

$\sec(270^0 - \theta) = -cosec\theta$

$\text{cosec}(270^0 - \theta) = -sec\theta$

$\sin(360^0 - \theta) = -sin\theta$

$\cos(360^0 - \theta) = cos\theta$

$\tan(360^0 - \theta) = -tan\theta$

$\cot(360^0 - \theta) = -cot\theta$

$\sec(360^0 - \theta) = sec\theta$

$\text{cosec}(360^0 - \theta) = -cosec\theta$

$\sin(360^0 + \theta) = sin\theta$

$\cos(360^0 + \theta) = cos\theta$

$\tan(360^0 + \theta) = tan\theta$

$\cot(360^0 + \theta) = cot\theta$

$\sec(360^0 + \theta) = sec\theta$

$\text{cosec}(360^0 + \theta) = cosec\theta$

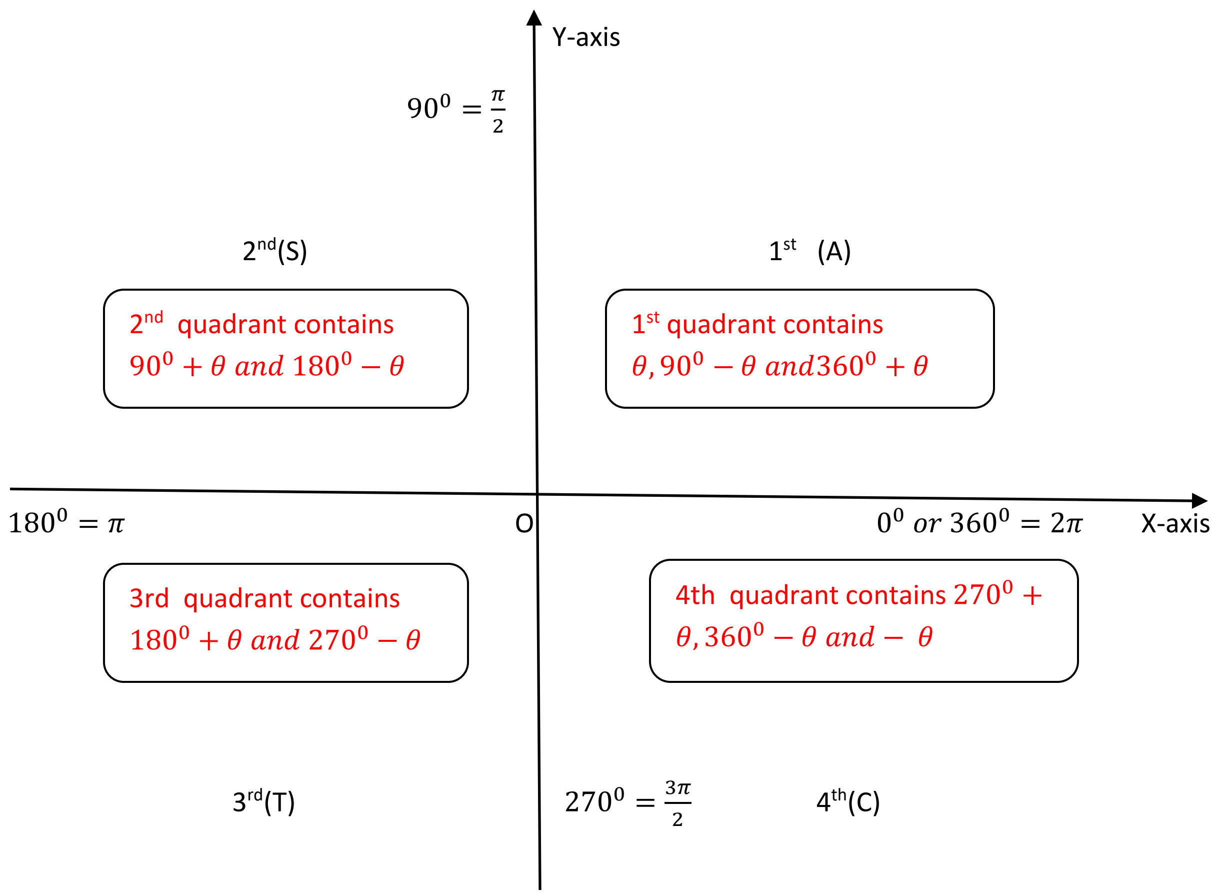
$\sin(n\pi + \theta) = (-1)^n sin\theta \quad , sin(n\pi + \theta) = \begin{cases} sin\theta, if\ n\ is\ even \\ -sin\theta, if\ n\ is\ odd \end{cases}$

$\cos(n\pi + \theta) = (-1)^n cos\theta, cos(n\pi + \theta) = \begin{cases} cos\theta, if\ n\ is\ even \\ -cos\theta, if\ n\ is\ odd \end{cases}$

$\tan(n\pi + \theta) = tan\theta$,if n is either even or odd. Where n is an integer.

Similarly we can find cot, sec and cosec

Let $0 < \theta < 90^0$



$A \Rightarrow$ *In the 1st quadrant all trigonometric ratios are positive*

$S \Rightarrow$ *In the 2nd quadrant only sine and cosec are positive but rest are negative*

$T \Rightarrow$ *In the 3rd quadrant only tan and cot are positive but rest are negative*

$C \Rightarrow$ *In the 4th quadrant only cos and sec are positive but rest are negative*

Ex: Show which of the following are positive.

$$sin271^0, cos375^0, tan99^0, cot333^0, sec159^0\ and\ cosec79^0$$

Ex: Express the following trigonometric ratios in the form of acute angle.

$$sin271^0, cos375^0, tan3333^0, cot1233^0, sec(-2349)^0\ and\ cosec1230^0$$

Ex: find the value of $sin1230^0, cos210^0, tan900^0\ and\ cot750^0$

Ex: Find the value of A if $cot3A = tan6A$

Ex: Find the value of cos1.cos2.cos3…………………cos100.

Ex: Find the value of sin1.sin2.sin3……………………sin200.

Ex: Find the value tan1.tan2.tan3…………….tan89.

Ex: find the value of tan9.tan27.tan45.tan63.tan81.

Ex: Find the value of logsin1.logsin2.logsin3…………….logsin100.

Ex: Find the value of logtan1+logtan2+…………………+logtan89.

Ex: If $sinA = \frac{3}{5}\ and\ 90^0 < A < 180^0\ then\ find\ other\ trigonometric\ ratios.$

Ans: Given $sinA = \frac{3}{5}$

$$cosA = \sqrt{1 - sin^2A} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm\frac{4}{5}$$

Due to $90^0 < A < 180^0$ $cosA$ is negative. So $osA = -\frac{4}{5}$, similarly

$tanA = \frac{sinA}{cosA} = -\frac{3}{4}$, $cotA = \frac{1}{tanA} = -\frac{4}{3}$, $secA = \frac{1}{cosA} = -\frac{5}{4}$, $cosecA = \frac{1}{sinA} = \frac{5}{3}$

Ex: If $cosA = \frac{4}{5}\ and\ 180^0 < A < 270^0\ then\ find\ the\ value\ of\ other\ trigonometric\ ratios$

<u>**Trigonometric ratios of compound angles, multiple angles and sub-multiple angles**</u>

Compound Angles: The sum or difference of two or more angles is known as multiple angles.

Ex: A+B, A-B, A+B+C……………

Multiple Angles: If an angle A is multiplied by a non zero integer then it is called multiple angles.

Ex:2A, 3A,…………………

Sub-multiple Angles: If an angle A is divided by a non zero integer then it is called sub-multiple angles.

Ex: $\frac{A}{2}, \frac{A}{3} \dots \dots \dots \dots \dots \dots \dots \dots$

Formulas:

Formula-1

$$Sin(A+B) = sinAcosB + cosAsinB$$

$$Sin(A-B) = sinAcosB - cosAsinB$$

$$cos(A+B) = cosAcosB - sinAsinB$$

$$cos(A-B) = cosAcosB + sinAsinB$$

Formula-2

$$tan(A+B) = \frac{tanA + tanB}{1 - tanAtanB}$$

$$tan(A-B) = \frac{tanA - tanB}{1 + tanAtanB}$$

$$cot(A \pm B) = \frac{cotAcotB \mp 1}{cotB \pm cotA}$$

Formula-3

$$Sin(A+B) + sin(A-B) = 2sinAcosB$$

$$Sin(A+B) - Sin(A-B) = 2cosAsinB$$

$$cos(A+B) + cos(A-B) = 2cosAcosB$$

$$\begin{cases} cos(A+B) - cos(A-B) = -2sinAsinB \\ or\ cos(A-B) - cos(A+B) = 2sinAsinB \end{cases}$$

Formula-4

$$sinC + SinD = 2sin\frac{C+D}{2}cos\frac{C-D}{2}$$

$$sinC - SinD = 2cos\frac{C+D}{2}sin\frac{C-D}{2}$$

$$cosC + cosD = 2cos\frac{C+D}{2}cos\frac{C-D}{2}$$

$$cosC - cosD = \begin{cases} -2sin\frac{C+D}{2}sin\frac{C-D}{2} \\ 2sin\frac{C+D}{2}sin\frac{D-C}{2} \end{cases}$$

Formula-5

$$Sin(A+B)sin(A-B)=\begin{cases}sin^2A-sin^2B\\cos^2B-cos^2A\end{cases}$$

$$cos(A+B)cos(A-B)=\begin{cases}cos^2A-sin^2B\\cos^2B-sin^2A\end{cases}$$

Ex: Find the value of $sin15^0, cos15^0, tan15^0$ and $cot15^0$.

Ans: $sin15^0 = sin(45^0-30^0) = sin45^0cos30^0 - cos45^0sin30^0$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$cos15^0 = cos(45^0-30^0) = cos45^0cos30^0 + sin45^0sin30^0$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$$tan15^0 = tan(45^0-30^0) = \frac{tan45^0 - tan30^0}{1+tan45^0tan30^0} = \frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Or we can derive $tan15^0 = \frac{sin15^0}{cos15^0}$

$cot15^0 = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Ex: Find the value of $sin75^0, cos75^0, tan75^0$ and $cot75^0$

Hints: Expand $75^0 = 45^0 + 30^0$, use the formula you will get the answer.

Ex: if $tanA = \frac{1}{2}$ and $tanB = \frac{1}{3}$ $then\ find\ the\ value\ of\ A+B.$

Ans: Given $tanA = \frac{1}{2}$ and $tanB = \frac{1}{3}$ $, use\ tan(A+B\ )formula$

$tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1 = tan45^0 \Rightarrow A+B = 45^0$

Ex: If $tanA = \frac{13}{27}$ and $tanB = \frac{7}{20}$ $then\ find\ A+B.$

Ex: Show that $sin10^0 + sin50^0 - sin70^0 = 0$

Ans: $L.H.S\ sin10^0 + sin50^0 - sin70^0 = \sin 10^0 + sin(60^0-10^0) - sin(60^0+10^0)$

$= sin10^0 - 2cos60^0 sin10^0 = sin10^0 - sin10^0 = 0$

Formula-6

$$Sin2A = 2sinAcosA$$

$$cos2A = \begin{cases} cos^2A - sin^2A \\ 2cos^2A - 1 \\ 1 - 2sin^2A \end{cases}$$

Formula-7

$$Sin2A = \frac{2tanA}{1 + tan^2A}$$

$$cos2A = \frac{1 - tan^2A}{1 + tan^2A}$$

$$tan2A = \frac{2tanA}{1 - tan^2A}$$

Formula-8

$$sin3A = 3sinA - 4sin^3A$$

$$cos3A = 4cos^3A - 3cosA$$

$$tan3A = \frac{3tanA - tan^3A}{1 - 3tan^2A}$$

Formula-9

$$sin^2A = \frac{1 - cos2A}{2}$$

$$cos^2A = \frac{1 + cos2A}{2}$$

$$tan^2A = \frac{1 - cos2A}{1 + cos2A}$$

Formula-10

$$sinA = 2sin\frac{A}{2}cos\frac{A}{2} \ or \ \frac{2tan\frac{A}{2}}{1 + tan^2\frac{A}{2}}$$

$$cosA = cos^2\frac{A}{2} - sin^2\frac{A}{2} \ or \ 2cos^2\frac{A}{2} - 1 \ or \ 1 - 2sin^2\frac{A}{2} \ or \ \frac{1 - tan^2\frac{A}{2}}{1 + tan^2\frac{A}{2}}$$

Formula-11

$$1 + sin2A = (cosA + sinA)^2$$
$$1 - sin2A = (cosA - sinA)^2$$
$$1 + sinA = \left(cos\frac{A}{2} + sin\frac{A}{2}\right)^2$$
$$1 - sinA = \left(cos\frac{A}{2} - sin\frac{A}{2}\right)^2$$

Formula-12

$$tan\frac{A}{2} = \frac{1 - cosA}{sinA} \; or \; \frac{sinA}{1 + cosA}$$
$$cot\frac{A}{2} = \frac{1 + cosA}{sinA} \; or \; \frac{sinA}{1 - cosA}$$

Ex: find the value of $sin105^0 . cos105^0$

Ans: $sin105^0 . cos105^0 = \frac{2sin105^0 . cos105^0}{2} = \frac{sin210^0}{2} = \frac{sin(180^0+30)}{2} = \frac{-sin30^0}{2} = -\frac{1}{4}$

Ex: Show that $tan75^0 + cot75^0 = 4$

Ans: L.H.S $tan75^0 + cot75^0 = \frac{sin75^0}{cos75^0} + \frac{cos75^0}{sin75^0} = \frac{sin^2 75^0 + cos^2 75^0}{sin75^0 cos75^0} = \frac{1}{sin75^0 cos75^0} = \frac{2}{2sin75^0 cos75^0} = \frac{2}{sin150^0} = \frac{2}{\left(\frac{1}{2}\right)} = 4$

Note :The value of $sin18^0 = \frac{\sqrt{5}-1}{4}$ $and\ cos36^0 = \frac{\sqrt{5}+1}{4}$

Ex: Find the value of $sin^2 18^0 + cos^2 36^0$.

Ans: $sin^2 18^0 + cos^2 36^0 = \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2}{16} = \frac{2(5+1)}{16} = \frac{3}{4}$

Ex: Prove that $\frac{cos9^0 + sin9^0}{cos9^0 - sin9^0} = tan54^0$

Ans: L.H.S $\frac{cos9^0 + sin9^0}{cos9^0 - sin9^0} = \frac{\frac{cos9^0}{cos9^0} + \frac{sin9^0}{cos9^0}}{\frac{cos9^0}{cos9^0} - \frac{sin9^0}{cos9^0}} = \frac{1 + tan9^0}{1 - tan9^0} = \frac{tan45^0 + tan9^0}{1 - tan45^0 tan9^0} = tan(45^0 + 9^0) = tan54^0$

Ex: Prove that $sec2A + tan2A = tan\left(\frac{\pi}{4} + A\right)$

Ans: L.H.S $sec2A + tan2A = \frac{1}{cos2A} + \frac{sin2A}{cos2A} = \frac{1 + sin2A}{cos2A} = \frac{(cosA + sinA)^2}{cos^2 A - sin^2 A} = \frac{cosA + sinA}{cosA - sinA}$

$$divide\ cosA\ in\ both\ N^r\ and\ D^r$$

$$= \frac{1 + tanA}{1 - tanA} = \frac{tan\frac{\pi}{4} + tanA}{1 - tan\frac{\pi}{4} tanA} = tan\left(\frac{\pi}{4} + A\right)$$

Assignment-1

Prove that

1.$\frac{tan\theta}{1+tan^2\theta} = sin\theta\ cos\theta$

2.$\frac{1-tan^2\theta}{1+tan^2\theta} = cos^2\theta - sin^2\theta$

3.$tan\theta + cot\theta = cosec\theta\ sec\theta$

4.$(1 - cos^2\theta)cosec^2\theta = 1$

5.$sec\theta\sqrt{(1 - sin^2\theta)} = 1$

6.$tan\theta sin\theta + cos\theta = sec\theta$

7.$(1 - cos\theta)(1 + cos\theta)(1 + cot^2\theta) = 1$

8.$\frac{cos\theta}{1-tan\theta} + \frac{sin\theta}{1-cot\theta} = sin\theta + cos\theta$

9.$(1 - sin\theta)(1 + cosec\theta) = cos\theta cot\theta$

10.$sin\theta cos\theta tan\theta = sin^2\theta$

11.$sec^4\theta - tan^4\theta = 2sec^2\theta - 1$

12.$sin\theta + sin\theta cot^2\theta = cosec\theta$

13.$\frac{1}{cosecA-cotA} + \frac{1}{cosecA+cotA} = \frac{2}{sinA}$

14.$\frac{tan\theta+sec\theta-1}{tan\theta-sec\theta+1} = \frac{1+sin\theta}{cos\theta}$

Assignment-2

15.$\frac{cot\theta+cosec\theta-1}{cot\theta-cosec\theta+1} = \frac{1+cos\theta}{sin\theta}$

16.$\sqrt{\frac{1+sin\theta}{1-sin\theta}} = sec\theta + tan\theta$

17.$\sqrt{\frac{1-cos\theta}{1+cos\theta}} = cosec\theta - cot\theta$

18.$\frac{sin\theta-cos\theta}{sin\theta-cos\theta} = \frac{cos2\theta}{1+sin2\theta}$

19.$\frac{sin\theta}{1-cos\theta} + \frac{1-cos\theta}{sin\theta} = 2cosec\theta$

20.$sec^4A - sec^2A = tan^2A + tan^4A$

21.$(1 + cot\theta - cosec\theta)(1 + tan\theta + sec\theta) = 2$

22.$(cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta) = 1$

23.$\frac{sin\theta sec\theta}{cos^2\theta} = tan^3\theta + tan\theta$

24.$\frac{1-sinA}{1+sinA} = (secA - tanA)^2$

25.$\sqrt{\frac{secA-1}{secA+1}} = coseA - cotA$

Assignment-3

Prove that

1.$\frac{1+cos\theta+sin\theta}{1+cos\theta-sin\theta} = \frac{cos\theta}{1-sin\theta}$

2.$sin^6A + cos^6A = 1 - 3sin^2Acos^2A$

3.$\frac{tanA}{1-cotA} + \frac{cotA}{1-tanA} = secAcosecA + 1$

4.$(sec\theta - 1)cosec\theta - (1 - cos\theta)cot\theta = tan\theta - sin\theta$

5. If $cos\theta = -\frac{1}{2}$ $and\ 180^0 < \theta < 270^0$ then find all the trigonometric ratios.

6.$\frac{cos(180^0-A)cot(90^0+A)cos(-A)}{tan(180^0+A)tan(270^0+A)sin(360^0-A)} = cosA$

7. $\frac{cos(180^0-A)sin(90^0+A)tan(90^0+A)}{tan(270^0+A)sin(270^0-A)sin(90^0-A)} = 1$

8. $\frac{cot(180^0+A)cot(90^0+A)cos(360-A)}{tan(90^0+A)cose(180^0-A)sin(-A)} = sinA$

Assignment:4

1.Find the value of $cos210^0$, $sin(-1125^0)$, $\tan 480^0$

2.Express as the trigonometric ratio of an acute angle
$\tan 1129^0, \cot 1239^0, cosec\ 3333^0, \cos(-439^0), \sin(-1339^0)$

3.Prove that $\tan 225^0 \cot 405^0 + \tan 765^0 \cot 675^0 = 0$

4.Prove that $\tan 720^0 - \cot 270^0 - \sin 150^0 \cos 120^0 = \frac{1}{4}$

5.Prove that $cos^2\frac{\pi}{6} + cosec\frac{5\pi}{6} + 3tan^2\frac{\pi}{6} = 6$

6.$Prove\ that\ \frac{2sin\theta cos\theta - sin\theta}{1 - sin\theta + sin^2\theta - cos^2\theta} = cot\theta$

7.$cos\theta(tan\theta + 2)(2tan\theta + 1) = 2sec\theta + 5sin\theta$

Assignment-5

Ex: Find the value of $\tan 105^0\ and\ tan\frac{13\pi}{12}$

Ex: Prove that $sin(n + 1)Asin(n + 2)A + cos(n + 1)Acos(n + 2)A = \cos A$

Ex: Prove That $\frac{tanx + tany}{tanx - tany} = \frac{\sin(x+y)}{\sin(x-y)}$

Ex: If $A + B = \frac{\pi}{4}\ then\ prove\ that\ (1 + tanA)(1 + tanB) = 2$

Ex: Prove that $\frac{cos9^0 + sin9^0}{cos9^0 - sin9^0} = tan54^0$

Ex. Prove that $sin^4\frac{\pi}{8} + sin^4\frac{3\pi}{8} + sin^4\frac{5\pi}{8} + sin^4\frac{7\pi}{8} = \frac{3}{2}$

Ex. Prove that $cotA - cosec2A = cot2A$

Ex. Prove that $tanA(1 + sec2A) = tan2A$

Ex. Find the value of $2sin67\frac{1}{2}^0\ cos22\frac{1}{2}^0$

Ex. Prove that $tan75^0 + cot75^0 = 4$

Ex. Prove that $tan70^0 = tan20^0 + 2\ tan50^0$

Ex: $.Prove\ that\ \frac{sin5x + sin3x}{cos5x + cos3x} = tan4x$

Ex: $If\ tanA = xtanB\ then\ prove\ that\ \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$

Ex: $If\ A + B = \frac{\pi}{4}\ prove\ that\ (1 + tanA)(1 + tanB) = 2$

Ex: $If\ tanA - tanB = x\ and\ cotB - cotA = y\ then\ prove\ that\ \cot(A - B) = \frac{1}{x} + \frac{1}{y}$

Ex: $Prove\ that\ \frac{cos4x + cos3x + cos2x}{sin4x + sin3x + sin2x} = cot3x$

Ex: $Prove\ that\ \frac{1 + sinx - cosx}{1 + sinx + cos} = tan\frac{x}{2}$

Ex: $Prove\ that\ \frac{cosx}{1 + sinx} = tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

Ex: $prove\ that\ \frac{1 + sin2x + cos2x}{1 + sin2x - cos2x} = cotx$

Ex: $Prove\ that\ cos^4\frac{\pi}{8} + cos^4\frac{3\pi}{8} + cos^4\frac{5\pi}{8} + cos^4\frac{7\pi}{8} = \frac{3}{2}$

INVERSE TRIGONOMETRIC FUNCTION

Inverse Function : A function $f: X \to Y$ is said to be invertible (i.e f^{-1} $exist$) if it is bijective i.e one-one and on-to.

Here X is the domain and Y is the range of the function ,so $f^{-1}: Y \to X$.

If $y = f(x) \Rightarrow x = f^{-1}(y)$,Here the domain is Y and range is X.

Now consider a trigonometric function $y = sinx$ which is not bijective in R.

So we have to restrict its domain in $\left[-\frac{\pi}{2}.\frac{\pi}{2}\right]$ $corresponding\ to\ the\ range$ $[-1,1]$ then the function sinx is bijective .

Now sin: $\left[-\frac{\pi}{2}.\frac{\pi}{2}\right] \to [-1,1]$ is bijective and its inverse will exist,which can be written as $sin^{-1}: [-1,1] \to \left[-\frac{\pi}{2}.\frac{\pi}{2}\right]$

Hence $y = sinx \Rightarrow x = sin^{-1}y$

Note :$sin^{-1}x \neq (sinx)^{-1} = \frac{1}{sinx}$

Note: $f^{-1}Of(x) = f^{-1}f(x) = f^{-1}(y) = x$

$fOf^{-1}(y) = f(f^{-1}(y)) = f(x) = y$

Let us consider a function $y = sinx$ then its inverse exist and which is written as $x = sin^{-1}y$

Function	**Domain**	**Range (Principal value)**
sin^{-1}	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
cos^{-1}	$[-1,1]$	$[0,\pi]$
tan^{-1}	R	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
cot^{-1}	R	$(0,\pi)$
$cosec^{-1}$	R-$(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right] - \{0\}$
sec^{-1}	R-$(-1,1)$	$[0,\pi] - \left\{\frac{\pi}{2}\right\}$

Formula-1

$\sin(sin^{-1}x) = x \qquad x \in [-1,1]$

$\cos(cos^{-1}x) = x \qquad x \in [-1,1]$

$\tan(tan^{-1}x) = x \qquad x \in R$

$\cot(cot^{-1}x) = x \qquad x \in R$

$\sec(sec^{-1}x) = x \qquad x \in \mathrm{R} - (-1,1)$

$\mathrm{cosec}(cosec^{-1}x) = x \quad , x \in \mathrm{R} - (-1,1)$

Formula-2

$sin^{-1}(sinx) = x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$cos^{-1}(cosx) = x \qquad x \in [0, \pi]$

$tan^{-1}(tanx) = x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$cot^{-1}(cotx) = x \qquad x \in (0, \pi)$

$sec^{-1}(secx) = x \qquad x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$cosec^{-1}(cosecx) = x \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Formula-3

$$sin^{-1}x = cosec^{-1}\frac{1}{x} \quad , cosec^{-1}x = sin^{-1}\frac{1}{x} \quad , cos^{-1}x = sec^{-1}\frac{1}{x} \quad , sec^{-1}x = cos^{-1}\frac{1}{x}$$

$$tan^{-1}x = cot^{-1}\frac{1}{x} \quad , cot^{-1}x = tan^{-1}\frac{1}{x}$$

Formula-4

$sin^{-1}x + cos^{-1}x = \frac{\pi}{2}, \ x \in [-1,1]$

$tan^{-1}x + cot^{-1}x = \frac{\pi}{2}, x \in R$

$sec^{-1}x + cosec^{-1}x = \frac{\pi}{2}, x \in R — (1,1)$

Formula-5

$$sin^{-1}(-x) = -sin^{-1}x \quad , cos^{-1}(-x) = \pi - cos^{-1}x \, , tan^{-1}(-x) = -tan^{-1}x \, ,$$

$$cot^{-1}(-x) = -cot^{-1}x \ , sec^{-1}(-x) = \pi - sec^{-1}x, cosec^{-1}(-x) = -cosec^{-1}x$$

Formula-6

$$sin^{-1}x = cos^{-1}\sqrt{1-x^2} = tan^{-1}\frac{x}{\sqrt{1-x^2}} = cot^{-1}\frac{\sqrt{1-x^2}}{x} = sec^{-1}\frac{1}{\sqrt{1-x^2}} = cosec^{-1}\frac{1}{x}$$

$$cos^{-1}x = sin^{-1}\sqrt{1-x^2} = cot^{-1}\frac{x}{\sqrt{1-x^2}} = tan^{-1}\frac{\sqrt{1-x^2}}{x} = cosec^{-1}\frac{1}{\sqrt{1-x^2}} = sec^{-1}\frac{1}{x}$$

$$tan^{-1}x = sec^{-1}\sqrt{1+x^2} = sin^{-1}\frac{x}{\sqrt{1+x^2}} = cosec^{-1}\frac{\sqrt{1+x^2}}{x} = cos^{-1}\frac{1}{\sqrt{1+x^2}} = cot^{-1}\frac{1}{x}$$

Formula-7

$tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}$

$tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1+xy}$

Formula-8

$$sin^{-1}x + sin^{-1}y = sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}$$

$$sin^{-1}x - sin^{-1}y = sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}$$

$$cos^{-1}x + cos^{-1}y = sin^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}$$

$$cos^{-1}x - cos^{-1}y = sin^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}$$

Formula-9

$$2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2} = sin^{-1}\frac{2x}{1+x^2} = cos^{-1}\frac{1-x^2}{1+x^2}$$

Formula-10

$$2\ sin^{-1}x = sin^{-1}2x\sqrt{1-x^2}$$

$$2cos^{-1}x = cos^{-1}(2x^2-1) = sin^{-1}(1-2x^2)$$

Formula-11

$$3sin^{-1}x = sin^{-1}\{3x - 4x^3\}$$

$3cos^{-1}x = cos^{-1}\{4x^3 - 3x\}$ $3tan^{-1}x = tan^{-1}\left\{\frac{3x-x^3}{1-3x^2}\right\}$

Ex: Find the principal value of $sin^{-1}\frac{1}{2}$ $and\ sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Ans: let $sin^{-1}\frac{1}{2} = \theta \Rightarrow sin\theta = \frac{1}{2} = sin\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$ $which\ is\ the\ principal\ value$

$sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \theta \Rightarrow sin\theta = -\frac{1}{\sqrt{2}} = -sin\frac{\pi}{4} = sin\left(-\frac{\pi}{4}\right) \Rightarrow \theta = -\frac{\pi}{4}$ $which\ is\ the\ principal\ value$

Ex: Evaluate $sin^{-1}\left(sin\frac{2\pi}{3}\right)$

Ans: $sin^{-1}\left(sin\frac{2\pi}{3}\right) = sin^{-1}\left(sin\left(\pi - \frac{\pi}{3}\right)\right) = sin^{-1}\left(sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

Ex: Prove that $tan^{-1}\frac{2}{11} + tan^{-1}\frac{7}{24} = tan^{-1}\frac{1}{2}$

Ans: $tan^{-1}\frac{2}{11} + tan^{-1}\frac{7}{24}$

$= tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right)$

$= tan^{-1}\left(\frac{48+77}{264-14}\right) = tan^{-1}\left(\frac{125}{250}\right) = tan^{-1}\left(\frac{1}{2}\right)$

Ex: Prove that $sin^{-1}\frac{3}{5} + sin^{-1}\frac{8}{17} = sin^{-1}\frac{77}{85}$

Ans: $sin^{-1}\frac{3}{5} + sin^{-1}\frac{8}{17}$

$= sin^{-1}\left(\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$

$= sin^{-1}\left(\frac{3}{5}\times\frac{15}{17} + \frac{8}{17}\times\frac{4}{5}\right) = sin^{-1}\frac{77}{85}$

Assignment-1

1.Find the principal value of $sin^{-1}\left(-\frac{1}{2}\right), cos^{-1}\left(-\frac{1}{2}\right), tan^{-1}(-\sqrt{3})$ $and\ sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

2. Evaluate $cos^{-1}\left(\cos\left(-\frac{\pi}{8}\right)\right)$, $sin^{-1}\left(sin\frac{5\pi}{6}\right), cos^{-1}\left(cos\frac{4\pi}{3}\right)$ $and\ tan^{-1}\left(\frac{3\pi}{4}\right)$

3. True or False

(a) $cos\left(cos^{-1}\frac{5}{13}\right)$ $(b) tan(tan^{-1}3)$ $(c) sin\left(sin^{-1}\frac{3}{2}\right)$

4. True or False

(a)$sin^{-1}\frac{1}{2} + cos^{-1}\frac{1}{2} = \frac{\pi}{2}$ $(b) tan^{-1}5 + cot^{-1}5 = \frac{\pi}{2}$ $(c) sec^{-1}\frac{1}{4} + cosec^{-1}\frac{1}{4} = \frac{\pi}{2}$

5.Find the value of

(a)$sin cos^{-1} tan sec^{-1}\sqrt{2}$ $(b) cos tan^{-1} cot cos^{-1}\frac{\sqrt{3}}{2}$ $(c) cos tan^{-1} cot sin^{-1} x$ $(d) \sin\left(tan^{-1}x + tan^{-1}\frac{1}{x}\right)$

6. Prove that $tan^{-1}\frac{1}{7} + tan^{-1}\frac{1}{13} = tan^{-1}\frac{2}{9}$

7. $Prove\ that\ tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

8. $Prove\ that\ tan^{-1}\frac{1}{7} + tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{8} + tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

9. $Prove\ that\ tan^{-1}\frac{3}{4} + tan^{-1}\frac{3}{5} - tan^{-1}\frac{8}{9} = \frac{\pi}{4}$

10.Find the value of $tan\left(\frac{\pi}{4} + tan^{-1}\frac{3}{4}\right)$

11. Prove that $sin^{-1}\frac{5}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$

12. Prove that $sin^{-1}\frac{4}{5} + sin^{-1}\frac{5}{13} + sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

13. Prove that $cos^{-1}\frac{4}{5} - cos^{-1}\frac{15}{17} = cos^{-1}\frac{84}{85}$

14. Prove that $cos^{-1}\frac{3}{5} + cos^{-1}\frac{12}{13} + cos^{-1}\frac{63}{65} = \frac{\pi}{2}$

15. Find the value of $cos\left(sin^{-1}\frac{5}{13} + sin^{-1}\frac{3}{5}\right)$

Assignment-2:

1. $Prove\ that\ 4tan^{-1}\frac{1}{5} - tan^{-1}\frac{1}{70} + tan^{-1}\frac{1}{99} = \frac{\pi}{4}$

2. $Prove\ that\ sec^{-1}\frac{5\sqrt{2}}{7} + 2tan^{-1}\frac{1}{5} + 2tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

3. Evaluate $sin(2sin^{-1}0.6)$, $sin(2sin^{-1}0.8), and\ sin(3sin^{-1}0.4)$

4.Prove that $2tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{7} = tan^{-1}\frac{31}{17}$

5.Prove that $tan\left\{\frac{\pi}{4} + \frac{1}{2}cos^{-1}\frac{a}{b}\right\} + tan\left\{\frac{\pi}{4} - \frac{1}{2}cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

6. Prove that $sec^2(tan^{-1}2) + cosec^2(cot^{-1}3) = 15$

7.$Evaluate\ tan\left(2tan^{-1}\frac{1}{5}\right)$

8. $Evaluate\ tan\left(2tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$

9. $Evaluate\ tan\left(\frac{1}{2}cos^{-1}\frac{\sqrt{5}}{3}\right)$

10.$Solve\ tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$

11. $Solve\ sin^{-1}(1 - x) - 2sin^{-1}x = \frac{\pi}{2}$

12. $Solve\ sin^{-1}x + sin^{-1}2x = \frac{\pi}{3}$

13. $Solve\ cos^{-1}x + sin^{-1}\frac{x}{2} = \frac{\pi}{6}$

14. Prove that $sin^2(sin^{-1}x + sin^{-1}y + sin^{-1}z) = cos^2(cos^{-1}x + cos^{-1}y + cos^{-1}z)$

15.$If\ sin^{-1}x + sin^{-1}y + sin^{-1}z = \pi\ \ then\ prove\ that\ x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} = 2xyz$