FINITE DIFFERENCES AND INTERPOLATION

In Engineering , sometimes it is required to evaluate a function *f(x)* at some argument (independent variable) from a given set of tabulated values of *f(x)* within some interval. This can be done by studying a new concept called as Interpolation. Before studying interpolation, one should have an idea on the finite differences which is being used in interpolation.

The Finite Differences are:

1. **Forward Differences**
2. **Backward Differences**
3. **Central Differences**
4. **Average Differences**

If $y=f(x)$ is tabulated at equally spaced points x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh$ as $y_0=f(x_0)$,$y_1=f(x_1)$,$y_2=f(x_2)$, …..$y_n=f(x_n)$ respectively, then the different type of differences are defined by :

A) FORWARD DIFFERENCES:

1st forward differences:

$\Delta y_0 = y_1 - y_0$, $\Delta y_1 = y_2 - y_1$,…… etc

$\Delta y_i = y_{i+1} - y_i$, $i = 0,1,2,...n-1$ are called as 1st forward differences.

2nd forward differences:

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$, $\Delta^2 y_1 = \Delta y_2 - \Delta y_1$,....*etc.*

$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i$, $i = 0,1,2,....n-1$

are called as 2nd forward differences .

3rd forward differences:

$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$, $\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$, …. etc

are called as 3rd forward differences.

Similarly, the other higher order differences can be found.

TABLE OF FORWARD DIFFERENCES (For n=5)

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
x_0	y_0					
x_1	y_1	Δy_0				
x_2	y_2	Δy_1	$\Delta^2 y_0$			
x_3	y_3	Δy_2	$\Delta^2 y_1$	$\Delta^3 y_0$		
x_4	y_4	Δy_3	$\Delta^2 y_2$	$\Delta^3 y_1$	$\Delta^4 y_0$	
x_5	y_5	Δy_4	$\Delta^2 y_3$	$\Delta^3 y_2$	$\Delta^4 y_1$	$\Delta^5 y_0$

Example-1

Construct the forward difference table for $y = 3x^4 - x + 5$,given *x*= -1,1,2,3,4 and hence find the value of $\Delta f(2), \Delta^2 f(1)$ .Then Find the leading term and leading differences in the table.

x	y	Δ	Δ^2	Δ^3	Δ^4
−1	9				
1	7	−2			
2	51	44	46		
3	245	194	150	104	
4	769	524	330	180	76

From the table, $\Delta f(2) = \Delta y_2 = 194$, $\Delta^2 f(1) = \Delta^2 y_1 = 150$

B) BACKWARD DIFFERENCES

1st backward differences:

$\nabla y_1 = y_1 - y_0$, $\nabla y_2 = y_2 - y_1$, ……etc.

$\nabla y_{i+1} = y_{i+1} - y_i$, $i = 0,1,2,\ldots n-1$ are called as 1st backward differences.

2nd backward differences:

$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$, $\nabla^2 y_3 = \nabla y_3 - \nabla y_2$,....*etc.*

$\nabla^2 y_{i+1} = \nabla y_{i+1} - \nabla y_i$, $i = 1,2,\ldots.$

are called as 2nd backward differences .

3rd backward differences:

$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$, $\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$, …. etc

are called as 3rd backward differences.

Similarly, the other higher order differences can be found.

TABLE OF BACKWARD DIFFERENCES (FOR n=5)

x	y	∇	∇^2	∇^3	∇^4	∇^5
x_0	y_0					
x_1	y_1	∇y_1				
x_2	y_2	∇y_2	$\nabla^2 y_2$			
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$		
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$	
x_5	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$

C) SHIFT OPERATOR

This operator is denoted by E and the inverse shift operator is denoted by E^{-1}.
If *f(x)* be any function and h = interval of differencing.
Then,

$$Ef(x) = f(x+h),$$
$$E^2 f(x) = E(E(f(x))) = E(f(x+h)) = f(x+2h),$$
$$E^n f(x) = f(x+nh), \qquad n = 1,2,3.....$$
$$E^{-1} f(x) = f(x-h),$$
$$E^{-2} f(x) = f(x-2h)$$
$$E^{-n} f(x) = f(x-nh), \qquad n = 1,2,3,......$$

Example-2

Evaluate: $E\tan x - E^{-2}e^x$, taking h=1

Answer:

$$E\tan x - E^{-2}e^x$$
$$= \tan(x+h) - e^{x-2h}$$
$$= \tan(x+1) - e^{x-2}$$

FORMULA:

1. Taylor's series expansion:

$$f(x+h) = f(x) + h\frac{f'(x)}{1!} + h^2\frac{f''(x)}{2!} + h^3\frac{f'''(x)}{3!} + \dots\dots\dots$$

2. Exponential Series:

$$e^x = 1 + \frac{x}{1!} + \frac{x}{2!} + \frac{x^3}{3!} + \dots.$$

RELATION BETWEEN THE OPERATORS

1. $\Delta = E - 1$
2. $\nabla = 1 - E^{-1}$
3. $E = e^{hD}, D = \dfrac{d}{dx}$
4. $E^{-1} = e^{-hD}, D = \dfrac{d}{dx}$

Proof:

1. For any function *f(x)*

$$\Delta f(x) = f(x+h) - f(x)$$
$$= Ef(x) - 1.f(x)$$
$$= (E-1)f(x)$$
$$\Rightarrow \Delta = E - 1$$

2. Similar to 1.

3.

$$Ef(x) = f(x+h) = f(x) + h\frac{f'(x)}{1!} + h^2\frac{f''(x)}{2!} + h^3\frac{f'''(x)}{3!} + \dots\dots\dots$$
$$= f(x) + \frac{hDf(x)}{1!} + \frac{h^2D^2f(x)}{2!} + \frac{h^3D^3f(x)}{3!} + \dots\dots\dots$$
$$= \left(1 + \frac{hD}{1!} + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots.\right)f(x)$$
$$= e^{hD}f(x)$$
$$\Rightarrow E = e^{hD}$$

4. Proceed as in 3.

Assignments:

1. Evaluate the following:

a) $\dfrac{\Delta}{E}\sin x$, b) $\dfrac{\Delta^2}{E}e^x, h = 1$ c) $(\Delta + \nabla)^2 x^2$,

2. Prove that:

a) $(1+\Delta)(1-\nabla) = 1$, b) $\Delta\nabla = \nabla\Delta = \Delta + \nabla$

Note:

1. **If a polynomial is of degree n, then nth order difference is constant and other higher order differences will be 0.**
2. **If we are given (n+1) values of y in a data, then we can find a polynomial of degree n.**

Example-6

Find the missing values in the data:

1.

x	1	1.2	1.4	1.6	1.8	2	2.2	2.4
Y=f(x)	0	0.182	-	0.47	0.587	-	0.788	0.875

Solution:

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
1	**0**						
1.2	**0.182**	**0.182**					
1.4	**A**	**A-0.182**	**A-0.364**				
1.6	**0.47**	**0.47-A**	**0.652-2A**	**1.016-3A**			
1.8	**0.587**	**0.117**	**A-0.353**	**3A-1.005**	**6A-2.021**		
2.0	**B**	**B-0.587**	**B-0.704**	**B-A-0.351**	**B-3A+0.654**	**B-9A+2.675**	
2.2	**0.788**	**0.788-B**	**1.375-2B**	**2.079-3B**	**2.43-4B+A**	**-5B+4A+1.776**	**-6B+13A-0.899**
2.4	**0.875**	**0.087**	**B-0.701**	**3B-2.076**	**6B-4.155**	**10B-A-6.585**	**15B-5A-8.361**

Since, in the data we are given only six values of y , so therefore we can find a polynomial of degree 5.Hence the 6^{th} differences will be 0.

So,

$$-6B+13A-0.899=0 \text{ -------- } (1)$$
$$15B-5A-8.361=0 \text{ ---------- } (2)$$

Solving above equations we get A=21.217/55 and B=113.188/165

The missing values are A=0.386 and B=0.686 approximately.

Assignment:

Find the missing data in the tables:

1.

x	1	2	3	4	5
y	10	17	-	31	45

2.

x	1	1.5	2	2.5	3	3.5	4	4.5
y	9	-	13	21	37	53	-	82

3.

x	1	3	5	7	9
y	9	-	17	22	25

INTERPOLATION

If *y=f(x)* is tabulated at equally spaced points x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …..$y_n=f(x_n)$ respectively, then the process of getting the values of y at some intermediate value between x_0, and x_n is called as Interpolation and that of getting the values of y at some x which is outside of x_0 and x_n is called as Extrapolation.

Various Interpolation formula are:

1. Newton's Forward Interpolation
2. Newton's Backward Interpolation
3. Lagrange's Interpolation

1. Newton's Forward Interpolation Formula:

If $y=f(x)$ is tabulated at (n+1) equally spaced points x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …..$y_n=f(x_n)$ respectively, then the Newton's Forward Interpolation Formula is given by:

$$y_p = y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} + ... + p(p-1)(p-2)....(p-n+1)\frac{\Delta^n y_0}{n!}$$

which is a polynomial of degree n. Here, $p = \dfrac{x-x_0}{h}$.

2. Newton's Backward Interpolation Formula:

If $y=f(x)$ is tabulated at (n+1) equally spaced points x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …..$y_n=f(x_n)$ respectively, then the Newton's Backward Interpolation Formula is given by:

$$y_p = y_n + p\nabla y_n + p(p+1)\frac{\nabla^2 y_n}{2!} + p(p+1)(p+2)\frac{\nabla^3 y_n}{3!} + ... + p(p+1)(p+2)....(p+n-1)\frac{\nabla^n y_n}{n!}$$

which is a polynomial of degree n. Here, $p = \dfrac{x-x_n}{h}$.

3. Lagrange's Interpolation Formula:

If $y=f(x)$ is tabulated at (n+1) points x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh$ (not necessarily equally spaced) as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …..$y_n=f(x_n)$ respectively, then the Lagrange's Interpolation Formula is given by:

$$y = l_0(x)\times y_0 + l_1(x)\times y_1 + ... + l_n(x)\times y_n$$

Where $l_0(x) = \dfrac{(x-x_1)(x-x_2)....(x-x_n)}{(x_0-x_1)(x_0-x_2).....(x_0-x_n)}$, $l_1(x) = \dfrac{(x-x_0)(x-x_2)....(x-x_n)}{(x_1-x_0)(x_1-x_2).....(x_1-x_n)}$, ……………..

$$l_n(x) = \frac{(x-x_0)(x-x_1)....(x-x_{n-1})}{(x_n-x_0)(x_n-x_1).....(x_n-x_{n-1})}$$

Notes:

1. Newton's forward Interpolation method is used to find the value of y at a point x which is given near the beginning of the data.(the arguments should be equally spaced)
2. Newton's backward Interpolation method is used to find the value of y at a point x which is given near the end of the data.(the arguments should be equally spaced)
3. Lagrange's Interpolation method is used to find the value of y at any point x.

Examples:-

1. Construct a 3rd degree Newton's Forward Interpolating polynomial using the data:

x	1	2	3	4
y	10	17	23	32

Answer :-

X	Y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
1	**10**			
2	17	**7**		
3	23	6	**-1**	
4	32	9	3	**4**

From the above table $h=1,\ x_0=1,\ \ y_0=10,\ \Delta y_0=7,\ \Delta^2 y_0=-1,\ \Delta^3 y_0=4,\ \ p=\frac{(x-x_0)}{h}=\frac{x-1}{1}=x-1$

Newton's Forward Interpolation Formula is given by:

$$y_p = y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} \qquad ------(1)$$

which is a polynomial of degree 3.
Putting all the values in equation (1) we have,

$$y_p(x) = 10 + (x-1)\times 7 + (x-1)(x-1-1)\frac{-1}{2!} + (x-1)(x-1-1)(x-1-2)\frac{4}{3!}$$
$$= 10 + 7(x-1) - \frac{(x-1)(x-2)}{2} + 4(x-1)(x-2)(x-3)$$

2. Compute y at x = 3 using the data given below:

x	2	4	6	8	10
y	15	23	34	47	59

Answer :-

X	Y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
2	**15**				
4	23	**8**			
6	34	11	**3**		
8	47	13	2	**-1**	
10	59	12	-1	-3	**-2**

We have to compute y at x = 2.5, therefore

From the above table we take,

$h=2\ \ x_0=2,\ \ y_0=15,\ \Delta y_0=8,\ \Delta^2 y_0=3,\ \Delta^3 y_0=-1,\ \ \Delta^4 y_0=-2,\ \ p=\frac{(x-x_0)}{h}=\frac{2.5-2}{2}=0.25$

Newton's Forward Interpolation Formula is given by:

$$y_p = y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} + p(p-1)(p-2)(p-3)\frac{\Delta^4 y_0}{4!} \qquad ------(1)$$

Putting all the values in equation (1) we have,

$$y(2.5) = 15 + 0.25 \times 8 + 0.25(0.25-1)\frac{3}{2!} + 0.25 \times (0.25-1)(0.25-2)\frac{-1}{3!} + 0.25 \times (0.25-1) \times (0.25-2) \times (0.25-3)\frac{-2}{4!}$$

$$= 15 + 2 - \frac{9}{32} - 0.0546875 + 0.0752 = 17.0752 - 0.3359375 \cong 16.74$$

3. Construct Newton's Backward Interpolating polynomial using the data :

x	-1	0	1	2	3
y	9	21	42	63	87

and hence find y for x= 2.5.

Solution:

X	Y	∇	∇^2	∇^3	∇^4
-1	**9**				
0	21	**12**			
1	42	21	**9**		
2	63	21	0	**-9**	
3	87	24	3	**3**	**12**

$$p = \frac{x - x_n}{h} = \frac{x-3}{1} = x - 3$$

$$y_p = y_4 + p\nabla y_4 + p(p+1)\frac{\nabla^2 y_4}{2!} + p(p+1)(p+2)\frac{\nabla^3 y_4}{3!} + p(p+1)(p+2)(p+3)\frac{\nabla^4 y_4}{4!}$$

$$= 87 + (x-3)24 + (x-3)(x-2)\frac{3}{2} + (x-3)(x-2)(x-1)\frac{3}{6} + (x-3)(x-2)(x-1)x\frac{12}{24}$$

To find y at x=2.5 take $x_n = 2, p = \frac{x - x_n}{h} = \frac{2.5-2}{1} = 0.5$

$$y_p = 87 + 0.5 \times 24 + 0.5 \times 1.5 \times \frac{3}{2} + 0.5 \times 1.5 \times 2.5 \times \frac{3}{6} + 0.5 \times 1.5 \times 2.5 \times 3.5 \times \frac{12}{24}$$

= ?

4. Construct Lagrange's Interpolating polynomial using the data:

X	1	2	3
Y	42	63	87

and hence find y for x= 2.5.

Ans. $l_0(x)=\frac{(x-2)(x-3)}{(1-2)(1-3)}=\frac{(x-2)(x-3)}{2}$

$$l_1(x)=\frac{(x-1)(x-3)}{(2-1)(2-3)}=\frac{(x-1)(x-3)}{-1}$$

$$l_2(x)=\frac{(x-1)(x-2)}{(3-1)(3-2)}=\frac{(x-1)(x-2)}{2}$$

$$y(x)=l_0(x)\times y_0+l_1(x)\times y_1+l_2(x)\times y_2$$

$$=\frac{(x-2)(x-3)}{2}\times 42+\frac{(x-1)(x-3)}{-1}\times 63+\frac{(x-1)(x-2)}{2}\times 87$$

$$=simplify$$

$$y(2.5)=l_0(2.5)\times y_0+l_1(2.5)\times y_1+l_2(2.5)\times y_2$$

$$=\frac{(2.5-2)(2.5-3)}{2}\times 42+\frac{(2.5-1)(2.5-3)}{-1}\times 63+\frac{(2.5-1)(2.5-2)}{2}\times 87$$

$$=simplify$$

Numerical Integration

We can evaluate the definite integrals of the type: $\int_a^b f(x)dx$ provided we know the integration $\int f(x)dx$. But it is always not possible to evaluate all the definite integrals. In that case, we can implement a new method as discussed below.

Definition: Suppose a function y=f(x) is continuous in the interval [a, b] and tabulated at equally spaced points a= x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh=b$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …..$y_n=f(x_n)$, then the process of evaluating the approximate value of the integral $\int_a^b f(x)dx$ is called as Numerical Integration.

Techniques of Numerical Integration:

1. Newton-Cote's Rule
2. Trapezoidal Rule
3. Simpson's 1/3rd Rule

1. Newton-Cote's Rule

If y=f(x) is continuous in the interval [a, b] and tabulated at equally spaced points a= x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh=b$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$,…..$y_n=f(x_n)$, then we can construct the Newton's Forward Interpolating Polynomial as:

$$y_p=y_0+p\Delta y_0+p(p-1)\frac{\Delta^2 y_0}{2!}+p(p-1)(p-2)\frac{\Delta^3 y_0}{3!}+......+p(p-1)(p-2)....(p-n+1)\frac{\Delta^n y_0}{n!} \text{ ------ (1)}$$

which is a polynomial of degree n. Here $p=\frac{x-x_0}{h}$.

Now, $\int_a^b f(x)dx$ can be evaluated by integrating the function approximated by the interpolating polynomial in Eq. (1) over [a, b].

$$\int_a^b f(x)dx = \int_a^b (y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} + ..)dx$$

$$= \int_0^n (y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} + ..)ndp \text{ as } p = \frac{x - x_0}{h} \Rightarrow hp = x - x_0$$

$$\Rightarrow dx = hdp$$

$$= \left[n(y_0 p + \frac{p^2}{2}\Delta y_0 + (\frac{p^3}{3} - \frac{p^2}{2})\frac{\Delta^2 y_0}{2!} + ...) \right]_0^n = n^2(y_0 + \frac{n}{2}\Delta y_0 + (\frac{n^2}{3} - \frac{n}{2})\frac{\Delta^2 y_0}{2!} + ...)$$

2. Trapezoidal Rule

Formula:

If y=f(x) is continuous in the interval [a, b] and tabulated at equally spaced points a= x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh=b$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$,…..$y_n=f(x_n)$, then the approximate value of $\int_a^b f(x)dx$ will be

$$\int_a^b f(x)dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + ...) + y_n\right]$$

Here, h = (b-a)/n. This is called as trapezoidal formula.

Example:

Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule taking h=1/4 . Hence find an approximate value of π.

Answer: Here a=0, b=1, h=1/4=0.25
¼ = (1-0)/n
⇒n=1/(1/4)=4.

X	$X_0=0$	$X_1=0.25$	$X_2=0.5$	$X_3=0.75$	$X_4=1$
y	$1=y_0$	$0.9411=y_1$	$0.8=y_2$	$0.64=y_3$	$0.5=y_4$

$$\int_a^b f(x)dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + ...) + y_n\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2}dx = \frac{0.25}{2}\left[1 + 2(0.9411 + 0.8 + 0.64) + 0.5\right]$$

$$= 0.782775$$

To find the value of **π**

$$\int_0^1 \frac{1}{1+x^2}dx = 0.782775$$

$$\Rightarrow \left[\tan^{-1}x\right]_0^1 = 0.782775$$

$$\Rightarrow \tan^{-1}1 - \tan^{-1}0 = 0.782775$$

$$\Rightarrow \frac{\pi}{4} = 0.782775$$

$$\Rightarrow \pi = 4\times 0.782775 = 3.1311$$

Assignment:

i) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule taking h=1/5 . Hence find an approximate value of π.

ii) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule taking h=1/10. Hence find an approximate value of π.

3. Simpson's 1/3rd rule:

Formula:

If y=f(x) is continuous in the interval [a, b] and tabulated at equally spaced points a= x_0, $x_1=x_0+h$, $x_2=x_0+2h$, ….$x_n=x_0+nh=b$ as $y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$,…..$y_n=f(x_n)$, then the approximate value of $\int_a^b f(x)dx$ will be

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5+.........) + 2(y_2+y_4+y_6+.........) + y_n\right]$$

Here, h = (b-a)/n and n is always **even**. This is called as **Simpson's 1/3rd rule**.

Example:

Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Simpson's 1/3rd rule taking 11 ordinates. Hence find an approximate value of π.

Answer:

Here a=0, b=1, n=10. h=(b-a)/n=(1-0)/10=0.1

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	1	0.99	0.961	0.917	0.862	0.8	0.735	0.671	0.609	0.552	0.5

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5+.........) + 2(y_2+y_4+y_6+.........) + y_n\right]$$

$$\int_0^1 \frac{1}{1+x^2}dx = \frac{0.1}{3}\left[1 + 4(0.99+0.917+0.8+0.671+0.552) + 2(0.961+0.862+0.735+0.609) + 0.5\right]$$

$$\approx 0.78513$$

To find the value of **π**

$$\int_0^1 \frac{1}{1+x^2}dx = 0.78513$$

$$\Rightarrow \left[\tan^{-1} x\right]_0^1 = 0.78513$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} 0 = 0.78513$$

$$\Rightarrow \frac{\pi}{4} = 0.78513$$

$$\Rightarrow \pi = 4 \times 0.78513 = 3.14052$$

Example:

Evaluate $\int_0^{\pi/2} \sqrt{\cos x}dx$ using Simpson's 1/3rd rule taking h=π/12(or 7 ordinates). (Keep your calculator in "rad" mode)

Answer:

Here a=0, b= $\pi/2$, n=6.

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{6} = \frac{\pi}{12}$$

x	0	$\frac{\pi}{12}$	$\frac{2\pi}{12} = \frac{\pi}{6}$	$\frac{3\pi}{12} = \frac{\pi}{4}$	$\frac{4\pi}{12} = \frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12} = \frac{\pi}{2}$
y	1	0.98281	0.93061	0.84089	0.70711	0.50874	0

$$\int_a^b f(x)dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + ...) + 2(y_2 + y_4 + y_6 + ...) + y_n\right]$$

$$\int_0^1 \sqrt{\cos x}dx = \frac{\frac{\pi}{12}}{3}\left[1 + 4(0.98281 + 0.84089 + 0.50874) + 2(0.93061 + 0.70711) + 0\right] = 1.18723$$ (ans)

Assignments

i. Evaluate $\int_0^1 x^3 dx$ using a) Trapezoidal rule b) Simpson's 1/3rd rule taking a suitable h.

ii. Evaluate $\int_0^5 \frac{dx}{4x+5}$ using a) Trapezoidal rule b) Simpson's 1/3rd rule considering 10 sub intervals.

iii. Evaluate $\int_0^1 \cos x dx$ using a) Trapezoidal rule taking 5 equal parts.