DETERMINANTS

Determinant will be used for solving the system of linear equations.

like $2x + y = 0$ *and* $x - y = 3$

Determinant of order 2

A determinant of order 2 can be written in the form of $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

which is defined as $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ *OR* $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - b_1a_2$

Note:

1.A determinant of order 2 contains two rows and two columns.

2.a_{11}, a_{12} *are the elements of* R_1 *and* a_{21}, a_{22} *are the elements of* R_2.

3.a_{11}, a_{21} *are the elements of* C_1 *and* a_{12}, a_{22} *are the elements of* C_2.

3.a_{11}, a_{22} *are the elements of principal diagonal and* a_{12}, a_{21} *are the elements of* sec*ondary diagonal*.

4.*It contains* $2 \times 2 = 4$ *elements.*

Determinant of order 3

A determinant of order 3 can be written in the form of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Which is defined as $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Note :

1.A determinant of order 3 can be expanded by using 6 ways (any one row or any one column).

2.A determinant of order 3 can be expanded by using the respective sign of the element in different rows or columns.i.e $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

3.*The sign of all the elements in a 2nd order* determinant *is* $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$

General element

If an element occurring in the ith row and jth column of a determinant *then it is called* (i, j)*th element.It is denoted by* a_{ij}.$(i \rightarrow ith\ row,\ j \rightarrow jth\ column)$

Ex : Construct a determinant of order 3×3 by using general element a_{ij}

$$Let\ A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

<u>*Minor of an element* a_{ij}</u>

The minor *of an element* a_{ij} *is defined as the value of a* determinant *will be obtained after deleting all the elements in the ith row and jth column.It is denoted by* $M_{ij.}$

<u>*Cofactor of an element* a_{ij}</u>

The cofactor of an element a_{ij} *is denoted by* C_{ij} *, which is defined as* $C_{ij}=(-1)^{i+j}M_{ij.}$

Ex : Find the Minor and Cofactor of a_{11}, a_{12} *and* a_{13} *in* $A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Ans : Minor of $a_{11}=M_{11}=\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

Minor of $a_{12}=M_{12}=\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

Minor of $a_{13}=M_{13}=\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

cofactor of $a_{11}=C_{11}=(-1)^{1+1}M_{11}=M_{11}$

cofactor of $a_{12}=C_{12}=(-1)^{1+2}M_{12}=-M_{12}$

cofactor of $a_{13}=C_{13}=(-1)^{1+3}M_{13}=M_{13}$

Similarly find the minor *and cofactor of other elements.*

<u>Properties of determinants</u>

P – 1: The value of the determinant remains unchanged if the rows and columns are interchanged.

$$\text{Ex:}A=\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37$$

$$Let\ B=\begin{vmatrix} 2 & 1 & -2 \\ 3 & 2 & 1 \\ -2 & 3 & -3 \end{vmatrix}=-37$$

$P-2$: If any two adjacent rows or columns of a determinant are interchanged then the numerical value is same but the sign is changed.

$$Ex: A=\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37$$

$$Let\ B=\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & -2 \\ -2 & 1 & -3 \end{vmatrix}=37$$

$P-3$: If any two rows or columns a determinant identical or same then the value of the determinant vanishes or zero.

$$Ex: A=\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

Note: If all the elements of a row or column of a determinant are zero then the value of the determinant is zero.

$$\text{Ex:}A=\begin{vmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

Note: If two rows or columns of a determinant are proportional then the value of the determinant is zero.

$$Ex: A=\begin{vmatrix} -4 & 2 & -6 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=0$$

$P-4$: If each element of a row or column of a determinant be multiplied by a constant k then the determinant is also multiplied by the same constant k.

$$Ex: A=\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37$$

$$Let\ B=\begin{vmatrix} 2k & 3k & -2k \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}=-37k=k\left(A\right)$$

$P-5$: If each element of a row or column of a determinant be the sum or difference of two or more terms then the determinant can be expressed as the sum or difference of two or more determinant.

$$Ex: A=\begin{vmatrix} a_1+\alpha_1 & a_2+\alpha_2 & a_3+\alpha_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}+\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$P-6$: If each element of a row or column of a determinant be increased or decreased by a constant multiple of the corresponding elements of another row or column then the value of the determinant remains unchanged.

$$Ex: A=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$B=\begin{vmatrix} a_1+kc_1 & a_2+kc_2 & a_3+kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}+\begin{vmatrix} kc_1 & kc_2 & kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=A+k\begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=A+0=A$$

$$Note: If\ A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} then\ |A|=a_{11}c_{11}+a_{12}c_{12}+a_{13}c_{13}\ (\text{using row})\ \ or\ \ |A|=a_{11}c_{11}+a_{21}c_{21}+a_{31}c_{31}\ (\text{using column})$$

$$Note: If\ A=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\ then\ a_{11}c_{21}+a_{12}c_{22}+a_{13}c_{23}=0$$

Method for evaluating a determinant

We should always try to make maximum number of possible zeros in any one row or column.

It is possible by using

(1) properties of determinant.

(2) Row to Row addition or subtraction.

(3) column to column addition or subtraction.

(4) Any other method

Applications of determinants

Area of a triangle : The area of a triangle of whose vertices are $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3) *is*

given by $\Delta = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$

condition of collinearity : Three points $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3) *are said to be collinear if*

$$\Delta = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = 0$$

Cramer's Rule :

Solution for two unknowns:The solution of two variables x and y of two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ then $x = \frac{D_1}{D}$ and $y = \frac{D_2}{D}$.

Solution for three unknowns: The solution of two variables x and y of two linear equations $a_1x + b_1y + c_1z = d_1$,$a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$ then $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$.

Consistent: A system of linear equation is said to be consistent if it gives a solution.

Ex:$2x + 3y = -1$ and $x - 3y = 2$

Inconsistent: A system of linear equation is said to be inconsistent if it gives no solution.

Ex:$2x + 3y = -1$ and $4x + 6y = 2$

Rules for consistency and inconsistency:(For Three unknowns)

We have $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$ and $z = \frac{D_3}{D}$.

1.If $D \neq 0$ then the system of equations are consistent and gives unique solution.

2. If $D = 0, D_1 = 0, D_2 = 0$ and $D_3 = 0$then the system of equations are consistent and gives Infinite number of solutions.

3. If $D = 0$, and at least one of D_1, D_2 and D_3 is nonzero then the system of equations are inconsistent and gives no solutions.

Note: Same rule we can apply for two unknowns

Ex: Evaluate $\begin{vmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{vmatrix}$

Ex: Evaluate (a) $\begin{vmatrix} 2 & 33 \\ 4 & -25 \end{vmatrix}$,(b) $\begin{vmatrix} 1 & w \\ w^2 & 1 \end{vmatrix}$,(c) $\begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$,(d) $\begin{vmatrix} \omega^6 & \omega^4 \\ -\omega^6 & \omega^5 \end{vmatrix}, \omega^3 = 1$

Ex: Find the maximum value of $\begin{vmatrix} 1+sin\theta & cos\theta \\ -cos\theta & sin\theta \end{vmatrix}$

Ex: Solve $\begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} = 0$

Ans: $\begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} = 0$

$\Rightarrow x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Ex: Solve $\begin{vmatrix} x+1 & -2 \\ 1 & 3 \end{vmatrix} = 3$

Ex: Evaluate (a) $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 0 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{vmatrix}$,(c) $\begin{vmatrix} -6 & 0 & 0 \\ 3 & -5 & 7 \\ 2 & 8 & 11 \end{vmatrix}$.

Ex: Prove that $\begin{vmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} = 0$

Ans:

$$\begin{vmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} (After\ adding\ the\ elements\ of\ R_1, R_2, R_3\ then\ replace\ R_1)$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{vmatrix} \left(\because 1+\omega+\omega^2 = 0, you\ know\ in\ complex\ number\right)$$

$= 0$

Ex: Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Ans:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} = 1\begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix} - 0 + 0$$

$$= (b-a)(c^2-a^2) - (c-a)(b^2-a^2) = (b-a)(c-a)(c+a) - (c-a)(b-a)(b+a)$$

$$= (b-a)(c-a)\{(c+a)-(b+a)\} = (b-a)(c-a)(c-b) = (a-b)(b-c)(c-a)$$

(Taking common -sign from (b-a) and (c-b) then you will get the answer)

Ex: Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Ans:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

Multiply a in C_1, *b in* C_2, *c in* C_3 *and divide abc*

$$= \frac{1}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & cba & abc \end{vmatrix} \text{taking common } abc \text{ from } C_3$$

$$= \frac{abc}{abc}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix} C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

$$= \begin{vmatrix} a^2 & b^2-a^2 & c^2-a^2 \\ a^3 & b^3-a^3 & c^3-a^3 \\ 1 & 0 & 0 \end{vmatrix} = 1\begin{vmatrix} b^2-a^2 & c^2-a^2 \\ b^3-a^3 & c^3-a^3 \end{vmatrix} - 0 + 0 \ \text{(expand it by using row3)}$$

$$= (b^2-a^2)(c^3-a^3) - (c^2-a^2)(b^3-a^3)$$

$$= (b-a)(b+a)(c-a)(c^2+ca+a^2) - (c-a)(c+a)(b-a)(b^2+ab+a^2)$$

$$= (b-a)(c-a)\{(b+a)(c^2+ca+a^2) - (c+a)(b^2+ab+a^2)\}$$

$$= (b-a)(c-a)(bc^2+abc+a^2b+ac^2+a^2c+a^3-cb^2-abc-ca^2-ab^2-a^2b-a^3)$$

$$= (b-a)(c-a)(bc^2+ac^2-cb^2-ab^2) = (b-a)(c-a)\{bc(c-b)+a(c^2-b^2)\}$$

$$= (b-a)(c-a)(c-b)(bc+ac+ab) = (a-b)(c-a)(b-c)(bc+ac+ab)$$

Ex: solve $\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$

Ans:

$$\begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} x+3 & x+3 & x+3 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

Taking common x+3 from row1(R_1)

$$\Rightarrow (x+3)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

either $x+3=0$ or $\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$

since $x+3=0 \Rightarrow x=-3$

$$Again\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & x \end{vmatrix} = 0 \Rightarrow 1\begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} - 0 + 0 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

$Hence\ x=0\ and\ x=-3$

Ex: Solve by Cramer's rule $2x+3y=-1\ and\ x-2y=3$

Ans: Given equations are $2x+3y=-1\ and\ x-2y=3$

Here $a_1 = 2, b_1 = 3, c_1 = -1, a_2 = 1, b_2 = -2, c_2 = 3$

Let $D = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$

$D_1 = \begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} = -7$

$D_2 = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7$

$x = \frac{D_1}{D} = \frac{-7}{-7} = 1 \; and \; y = \frac{D_2}{D} = \frac{7}{-7} = -1$

Ex: Solve by cramer's rule $2x + y + 2z = 2, 3x + 2y + z = 2 \; and -x + y + 3z = 6$

Ans:

Let $D = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} = 2(5) - 1(10) + 2(5) = 10$

$D_1 = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 6 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 6 & 1 \end{vmatrix} = 2(5) - 1(0) + 2(-10) = -10$

$D_2 = \begin{vmatrix} 2 & 2 & 2 \\ 3 & 2 & 1 \\ -1 & 6 & 3 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} = 2(0) - 2(10) + 2(20) = 20$

$D_3 = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ -1 & 1 & 6 \end{vmatrix} = 2\begin{vmatrix} 2 & 2 \\ 1 & 6 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix} = 2(10) - 1(20) + 2(5) = 10$

$x = \frac{D_1}{D} = \frac{-10}{10} = -1, y = x = \frac{D_2}{D} = \frac{20}{10} = 2, z = x = \frac{D_3}{D} = \frac{10}{10} = 1$

Questions carrying 2 marks

1. Find the value of $\begin{vmatrix} 5 & -2 & 1 \\ 3 & 0 & 2 \\ 8 & 1 & 3 \end{vmatrix}$

Ans :7

2. Find the value of $\begin{vmatrix} sec\theta & tan\theta \\ tan\theta & sec\theta \end{vmatrix}$

Ans : 1

3. If $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ x & 0 & 0 \end{vmatrix}$, then find the value of x

Ans : x = -1

4. Find the minimum value of $\begin{vmatrix} sinx & cosx \\ -cosx & 1+sinx \end{vmatrix}$ where $x \in R$

Ans: 0

5. Find the maximum value of $\begin{vmatrix} sin^2x & sinx.cosx \\ -cosx & sinx \end{vmatrix}$ where $x \in R$

Ans : 1

6. If ω is the cube root of unity, find the value of the determinan t $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$

Ans : 0

7. Find the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

Ans : 0

8. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$, then find x

Ans : x=a

9. Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$

Ans : x=8

10. Solve $\begin{vmatrix} 2 & -3 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} x & 1 \\ -2 & x \end{vmatrix}$

Ans : ± 4

Questions carrying 5 marks

1. Solve $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

Ans: x=0 or x= -3

2. Solve $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+c \end{vmatrix} = 0$

Ans: x= $-(a+b+c)$ or $\sqrt{a^2+b^2+c^2-ab-bc-ca}$

3. Solve $\begin{vmatrix} x & a & a \\ m & m & m \\ b & x & b \end{vmatrix} = 0$

Ans: $x = a$ or $x = b$

4. Solve $\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix} = 0$ where ω is cube root of unity.

Ans: x=0

5. Prove without expanding that $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

6. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

7. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

8. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

9. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

10. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

11. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (b-c)(c-a)(a-b)$

12. Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

13. Solve by Cramer's rule:

(i) $4x-y=9, 5x+2y=8$

Ans: $x=2, y=-1$

(ii) $2x-y=2, 3x+y=13$

Ans: $x=3, y=4$

(iii) $x-y+z=1, 2x+3y-5z=7, 3x-4y-2z=-1$

Ans : $x=\frac{35}{16}, y=\frac{53}{32}, z=\frac{15}{32}$

(iv) $x+y+z=3, 2x+3y+4z=9, x+2y-4z=-1$

Ans: $x=1, y=1, z=1$